Two-Stage Equilibration in High Energy Heavy Ion Collisions

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Using the (lowest-order) perturbative QCD, we argue that high energy heavy ion collisions proceed via two stages: *equilibration of gluons* takes time $\tau_g \sim \frac{1}{2}$ fm/c, while *production and equilibration of quarks* needs time at least $\tau_q \sim 2$ fm/c. If so, the initial gluon plasma is much hotter than usually estimated, $T_g \sim 400$ MeV, which leads to enhanced charm production and significant modifications of other proposed signals.

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The main objective of the future experimental heavy ion program at the BNL Relativistic Heavy Ion Collider (RHIC) and the CERN Large Hadron Collider (LHC) is production of a new form of matter, the so-called *quark-gluon plasma* (QGP) [1-3]. Dynamics of these collisions during the first few fm/c remains very uncertain. One approach is based on "soft" processes (e.g., the dual string model), extrapolating properties of the *pp* and *pA* collisions to the *AA* case [4]. Another approach focuses at the "semihard" processes, with momenta transfer ~ 1-3 GeV, which can in principle be described by *perturbative QCD* (PQCD). The relatively large *gg* cross section leads to the idea [5] that it is the gluonic component of the hadrons which intersect the most, and that was supplemented by the proposal [6] that it should also lead to very "hot glue." In the present paper we specify some details of this scenario.

During the last decade, scattering of few-GeV partons (the "minijets") was related to spectra observed in *pp* collisions [7,8], producing evidence that this component of the collision processes is indeed reproduced by perturbative QCD. For nuclear collisions the picture obtained depends on the boundary of the perturbative description. If it is set at $p > 2$ GeV [8,9], then even a central Au-Au collision produces a dilute system of partons. Recuperation of their color field (or "gluonic branching") multiplies their number by about a factor of 3 until time ~ 0.4 fm/c [9], when the system becomes dense and interacting. Another (and more optimistic) scenario [10] appears if the parton cutoff is set by the Gribov-Levin-Ryskin "saturation condition." This leads to a dense cloud of partons with $p > 1$ GeV: so even without scattering one produces enough gluons for the total entropy needed. "Partonic cascades" [11] can provide more details, but it seems very plausible that (i) *entropy is produced very early* and (ii) it appears mainly as few-GeV *gluons* [12]. With these assumptions, we discuss below the issue of partonic equilibration, in terms of both their *momentum distribution* and *composition.*

First, let us recall the *standard scenario à la* Bjorken [13], used as a benchmark. For central *AA* collisions in the central region we take [14]

$$\frac{dN_{AA}}{dy} = A^{\alpha} 0.8 \ln E_{\text{c.m.}}$$

with [15] $\alpha = 1.1$ and $E_{\text{c.m.}}$ being the center-of-mass energy per nucleon. Using *entropy conservation* one evaluates the entropy density at time τ_0 as

$$s_i = \frac{3.6\, dN/dy}{\pi R_A^2 \tau_0}, \tag{1}$$

where R_A is the nuclear radius and 3.6 comes from the entropy/number density ratio for the *pion gas.* If one simply takes $\tau_0 = 1$ fm/c, for *central* collisions at RHIC (Au-Au, $\sqrt{s} = 200A$ GeV) and LHC (Pb-Pb, $\sqrt{s} = 6300A$ GeV), one gets the initial entropy $s_i \approx 35$ and 60 fm^{-3}, corresponding for *equilibrated QGP* to the following initial temperatures:

$$T_i \approx 240\ \text{MeV (RHIC)}, \quad T_i \approx 290\ \text{MeV (LHC)}. \tag{2}$$

Second, let us recall the relevant cross sections. In the lowest order [16] the matrix elements squared M^2 [defined by $d\sigma/dt = (\pi\alpha_s^2/s^2)M^2$] are

$$\begin{aligned} M^2_{gg\to gg} &= \frac{9}{2}\left(3 - \frac{ut}{s^2} - \frac{us}{t^2} - \frac{st}{u^2}\right), \\ M^2_{gg\to \bar{q}q} &= \frac{1}{6}\frac{(u^2+t^2)}{ut} - \frac{3}{8}\frac{u^2+t^2}{s^2}, \\ M^2_{qg\to qg} &= -\frac{4}{9}\frac{u^2+s^2}{us} + \frac{u^2+s^2}{t^2}, \\ M^2_{q_1q_2\to q_1q_2} &= \frac{4}{9}\frac{s^2+u^2}{t^2}. \end{aligned} \tag{3}$$

Each process has (i) *large-* and (ii) *small-angle* parts, which we discuss subsequently.

(i) The large-angle cross sections are very different: at 90° the M^2 are related as 30.4/0.14/5.4/2.2, thus the *gg* scattering is by far the most important. (Note, however, that in the *gg* case integrating over t one should not take into account the same final state twice.)

For the ideal gas of gluons at temperature T the mean kinematical invariants are $-\bar{u} = -\bar{t} = \bar{s}/2 \sim (3T)^2$, leading to an *effective large-angle scattering rate* at least

$$1/\tau_g^{\text{large angle}} \geq 5\alpha_s^2 T. \tag{4}$$

(ii) The small-angle scattering leads to divergent cross sections, which are finite in QGP due to finite "Debye mass" $t_{\min} = k_D^2 = g^2T^2$ [17]. The effective small-angle scattering rate is then

$$1/\tau_g^{\text{small angle}} \approx 2.2\alpha_s T\,. \tag{5}$$

For completeness, several results from the literature are to be mentioned here. Baym *et al.* [18] have determined the relaxation time for *small deviations from the equilibrium* to be $1/\tau_g^{\text{viscous}} = 4.2\alpha_s^2 \ln(1/\alpha_s)T$. *Soft gluons* ($p \ll T$) have the damping rate [19] $1/\tau_g^{\text{soft gluon}} = 2.5\alpha_s T$. The *energy loss* [8,20,21] due to scattering is $dE/dx = (4\pi/3)C\alpha_s^2 T^2 \ln(p/k_D)$, where p is the parton momentum and $C = 4/3$, 3, 9/4 for gg, gq, qq collisions.

Now we proceed to "equilibration time," a concept which can be defined as the time at which *each parton* has been in average scattered N_{coll} times. The value of N_{coll} depends on the accuracy considered: For rough features (like anisotropy of momentum distribution) $N_{\text{coll}} \sim 1$ is enough.

Note that it is essentially the same condition as is traditionally used for defining a system of *breakup*: Its size becomes comparable to the mean free path of constituents. One may imagine the system moving backward in time, from collision to many secondaries into hot partonic plasma, and then ask at what moment the time becomes too short for any more collisions to take place.

Our main approximation is that the collision rate is evaluated for the *equilibrated* plasma: The reason is that it depends on screening and other details, which was not yet worked out for the nonequilibrium case.

This *self-consistency condition* can be written as follows:

$$\tau_0 = \frac{3.6\,dN/dy}{\pi R_A^2}\frac{1}{7.0T_i^3} \approx N_{\text{coll}}\tau_g = \frac{N_{\text{coll}}}{\text{const}\times T_i}\,. \tag{6}$$

The factor 7.0 came from the entropy of the gluonic plasma at $T = T_i$. The constant on the right-hand side should be taken from the scattering rates discussed above.

For large-angle scattering the magnitude of the relevant coupling constant should be fixed from the asymptotic freedom formula $\alpha_s(Q) = 0.7/\ln(Q/\Lambda) \approx 0.3$ for $Q \sim 2$ GeV, $\Lambda = 0.2$ GeV, and one may check it with minijets. We do not use the K factors ~ 2, as is usually done for hard scattering: The corresponding correction *in matter* remains to be worked out. For small-angle processes we deal with the Debye scale, and, based on lattice studies (e.g., [22]), one can presumably use the lowest-order formula quite close to T_c.

The initial gluonic temperatures and the equilibration time following from the *self-consistency condition* (6) are shown in the Table I. For comparison, we use either the large-angle part only or the sum $1/\tau_g = 1/\tau_g^{\text{small angle}} + 1/\tau_g^{\text{large angle}}$. The numbers in the table correspond to $N_{\text{coll}} = 1$, for other values $T_i \sim 1/N_{\text{coll}}^{1/2}$. It seems clear that even the *leading-order estimates* lead to the conclusion that the gluonic equilibration time is short. Other expressions lead to similar results. In particular, under these conditions the total gluon energy loss is $\tau_g\, dE/dx \approx 2$–3 GeV, quite consistent with the idea that typical minijet gluons are *effectively trapped*. Perturbative corrections (K factors and $2 \to N$ processes) and nonperturbative ones can probably make this statement only stronger.

Now we proceed to the *quark equilibration* issue, related with relaxation of (i) momentum distribution and (ii) quark number.

(i) We get the following estimates for the rates of large- and small-angle quark scattering on glue:

$$1/\tau_q^{\text{large angle}} \geq 1.8\alpha_s^2 T\,, \tag{7}$$

$$1/\tau_q^{\text{small angle}} \approx \alpha_s T\,. \tag{8}$$

Using the same condition (6) for *quarks*, one finds that if only the large-angle part is used, quarks *are not equilibrated at all*: the obtained $T_i < T_c$. However, including *both small and large angles*, one gets the numbers listed in the last pair of columns in Table I.

(ii) The *quark production rate* can be estimated similarly to the small-angle gluonic scattering. Neglecting the quark masses, one has a divergent total cross section, regularized in QGP by *thermal quark mass* [23] $t_{\min} = m_q^2 = g^2T^2/6$:

$$\sigma_{gg \to \bar{q}q} \approx \frac{\pi\alpha_s^2}{3s}\ln(s/m_q^2) \tag{9}$$

(for *each* quark flavor). Thus (unless large next-order contributions or some nonperturbative mechanisms are found), one has to conclude that quark degrees of freedom are out of equilibrium for the time of about 2 fm/c.

Now we proceed to *observable consequences* of the proposed "hot glue" scenario, starting with *enhanced production of charm* due to the $gg \to \bar{c}c$ reaction, suggested as a signature for QGP in [6] and studied in detail by Shor [24]. Since its rate is $\sim 2m_c \approx 2.5$–3 GeV, it is a very sensitive "thermometer" [25]; see Fig. 1. Comparing thermal production to that due to direct process (the region between the two dashed lines in Fig. 1), one can see that our hot glue scenario suggests charm production to be enhanced by about a factor of 2 compared to the standard parton model prediction [26].

Production of strangeness is another (and much more

TABLE I. Initial gluonic and quark temperatures and equilibration times, estimated with only large-angle scattering (l) or both small- and large-angle ones ($l+s$).

	T_g (MeV) l	τ_g (fm/c) l	T_g (MeV) $l+s$	τ_g (fm/c) $l+s$	T_q (MeV) $l+s$	τ_q (fm/c) $l+s$
RHIC	340	1	500	0.3	200	2
LHC	440	0.8	660	0.25	260	1.7

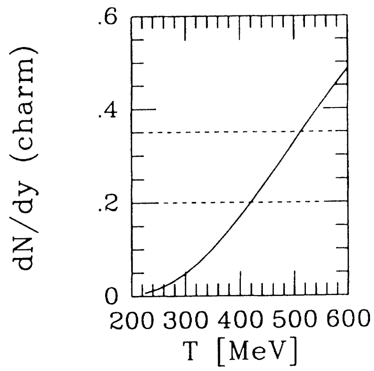

FIG. 1. The number of charmed pairs per unit rapidity, dN/dy, in the central region for Au-Au central collisions at RHIC, according to the quark-gluon plasma model (the solid line), vs the initial temperature T_0. The region between the two dashed lines corresponds to direct charm production, evaluated in the parton model.

discussed) QGP signal [1], and we have a comment on that. Although even the standard scenario suggests relatively large gluon energies $3T \sim 1$ GeV $\gg m_s$, it is not obvious that s quarks are as numerous as u,d ones: $\bar{q}q$ production is dominated by the small-angle process, so one should compare m_s to the "thermal mass," used as a cutoff. Only in our scenario the latter is large enough, so that there should be *no significant difference* between u, d, and s quarks produced (at time ~ 2 fm/c).

Spectra of the produced *photons and dileptons* should also be significantly modified in this scenario: During the "transitory period" ($\tau_g < \tau < \tau_q$) one has *smaller* number of quarks, but those are *hotter*. The reason is again that $gg \rightarrow \bar{q}q$ is dominated by *small angles*, so the produced quarks have the same momentum distribution as gluons. As most photons and dileptons to be observed actually correspond to the *tails of the distribution functions*, it is important that their relaxation happens *from above*.

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[25] In [24] a much more dramatic increase with T_i is given, but it corresponds to fixed $\tau_0 = 1$ fm/c and variable T_i while our Fig. 1 is for *fixed entropy*, $\tau_0 T_i^3 =$const, which much better corresponds to the experimental situation.

[26] After completion of this paper we saw the work [B. Muller and Xin-Nian Wang, Phys. Rev. Lett. **68**, 2437 (1992)], which also discusses charm enhancement. The difference is that in our scenario the additional charm production is *thermal*, it comes from equilibrated glue, while Muller and Wang consider the *preequilibrium* era, before gluonic rescattering. Although some double-counting may appear, roughly one should probably add both.