Suppression of Superconductivity by Disorder

T. R. Kirkpatrick
Institute for Physical Science and Technology, University of Maryland, College Park, Maryland 20742

D. Belitz
Department of Physics and Materials Science Institute, University of Oregon, Eugene, Oregon 97403
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Renormalization group arguments are used to give a complete parametrization of the disorder dependence of the mean-field superconducting T_c within a BCS model. The theory describes destruction of superconductivity in bulk materials well before the metal-insulator transition is reached, and rapid degradation of T_c in thin films with increasing sheet resistance. Comparison of explicit calculations with experiments yields good agreement.

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It is an established experimental fact that strong nonmagnetic disorder is detrimental to superconductivity (SC) [1]. In order to describe this phenomenon, it is useful to distinguish between two idealized classes of disordered SC's, viz., "granular" and "homogeneous" systems [2]. In the former, phase fluctuations are important and ultimately destroy phase coherence, while in the latter the amplitude of the order parameter is suppressed, and a mean-field (MF) approach is reasonable. Here we will restrict ourselves to the homogeneous case. Theoretically, the degradation and eventual destruction of SC in these systems pose a very hard problem. One has to deal with strong disorder in the presence of strong electron-electron interactions, a task which is very difficult even in the absence of SC. Nevertheless, there has been substantial effort devoted to this problem, and a number of effects that contribute to a degradation of the SC transition temperature T_c have been identified [3–9]. This makes one wonder whether there are still more that have been overlooked.

In this Letter we consider a model, which we will call a BCS model, where the phonon-mediated part of the effective electron-electron interaction potential is a phenomenological constant. The Coulomb part of the interaction is treated realistically. For this model we use general renormalization group (RG) arguments concerning the structure of the pair propagator (PP) to show that the destruction of SC is driven by disorder-induced changes in three quantities: the single-particle density of states (DOS), a quasiparticle DOS, and the Coulomb pseudopotential. We obtain this result from a microscopic derivation of the Landau-Ginzburg theory for the SC transition in a disordered system. Other derivations are possible and will be discussed elsewhere [10].

Our starting point is the action for a general Fermi system [11],

$$S=\int_0^\beta d\tau \int d\mathbf{x} \sum_i \bar{\psi}^i(\mathbf{x},\tau)\partial_\tau \psi^i(\mathbf{x},\tau) - \int_0^\beta d\tau\, H'(\tau)\,. \tag{1}$$

Here $\beta=1/T$ is the inverse temperature, τ denotes imaginary time, $\bar{\psi}^i$ and ψ^i are Grassmann fields with spin index i, and $H'(\tau)$ is the Hamiltonian in imaginary time representation. We use units such that $k_B=\hbar=1$. H' contains a part describing free electrons with mass m in a static random potential, and an electron-electron interaction part, $H_{\rm int}$. The random potential is assumed to be delta correlated in space and to have a Gaussian distribution. $H_{\rm int}$ can be written in terms of three interaction amplitudes K_s, K_t, and K_c for the particle-hole spin singlet, particle-hole spin triplet, and particle-particle channel, respectively [12],

$$H_{\rm int}(\tau)=\frac{1}{2N_F V}\sum_{\mathbf{k},\mathbf{p},\mathbf{q}}{}'\sum_{\substack{i,j\\k,l}}\{-[K_s\delta_{il}\delta_{jk}+K_t\boldsymbol{\sigma}_{il}\cdot\boldsymbol{\sigma}_{jk}]\bar{\psi}^i(\mathbf{k},\tau)\bar{\psi}^j(\mathbf{p}+\mathbf{q},\tau)\psi^k(\mathbf{p},\tau)\psi^l(\mathbf{k}+\mathbf{q},\tau)$$
$$+K_c\delta_{il}\delta_{jk}\bar{\psi}^i(\mathbf{k},\tau)\bar{\psi}^j(-\mathbf{k}+\mathbf{q},\tau)\psi^k(\mathbf{p},\tau)\psi^l(-\mathbf{p}+\mathbf{q},\tau)\}\,. \tag{2}$$

Here V is the system volume, N_F is the bare DOS per spin at the Fermi level, $\boldsymbol{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$ denotes the Pauli matrices, and the prime on the wave-vector sum indicates a restriction to wave numbers small compared to the Fermi wave number. In what follows we will construct an effective, long-wavelength, small-frequency theory for the excitations in the particle-particle channel. We therefore interpret K_s and K_t as already containing Fermi liquid corrections. Likewise, K_c will be interpreted as containing the bare Coulomb pseudopotential, μ^*, in addition to the attractive BCS interaction.

With an attractive interaction in the Cooper channel, $K_c<0$, the usual sigma-model approach to disordered electronic systems is complicated by the fact that in addition to disorder fluctuations of the spin and charge density the fluctuations in the Cooper channel have a critical singularity at finite temperature due to the SC phase transition. In order to separate the disorder fluctuations [which lead to the metal-insulator transition (MIT)]

from the SC fluctuations, we derive an effective theory for the SC order parameter $\Delta(\mathbf{x},\tau)$. Δ is a classical field which serves as an auxiliary field in a Hubbard-Stratonovich decoupling of the four-fermion term in the Cooper channel in Eq. (2). Expansion of the action in powers of Δ yields an action of Landau-Ginzburg-Wilson (LGW) type that describes a phase transition from a Fermi liquid (FL) to a SC. For a MF theory of SC, we can restrict ourselves to the Gaussian term in the LGW action, which reads

$$S_{\mathrm{LGW}}=\sum_{\mathbf{q}}\sum_{m}\bar{\Delta}(\mathbf{q},m)\left(\frac{1}{|K_c|}-T\sum_{n_1,n_2}C_{n_1n_2}(\mathbf{q},m)\right)\Delta(\mathbf{q},m)+O(\Delta^4)\,. \tag{3a}$$

In our case, $\bar{\Delta}=\Delta$. n and m are Matsubara frequency indices. C denotes the PP,

$$C_{n_1n_2}(\mathbf{q},m)=\frac{1}{V}\sum_{\mathbf{k},\mathbf{p}}\langle\psi^{\uparrow}_{-n_1+m}(-\mathbf{k}+\mathbf{q})\psi^{\downarrow}_{n_1}(\mathbf{k})\bar{\psi}^{\downarrow}_{n_2}(\mathbf{p})\bar{\psi}^{\uparrow}_{-n_2+m}(-\mathbf{p}+\mathbf{q})\rangle_0\,. \tag{3b}$$

$\langle\cdots\rangle_0$ denotes an average performed with an action given by Eqs. (1) and (2) with $K_c=0$, and includes the disorder average. We note that our approach is very general in that (1) the coefficients of the LGW theory are correlation functions for a fully interacting electron system, and (2) thermal fluctuations can be incorporated by going to higher order in Δ. The latter would require evaluation of higher-order correlation functions, and will be investigated elsewhere [10]. The MF critical temperature T_c is given by

$$\frac{1}{|K_c|}=T\sum_{n_1,n_2}C_{n_1n_2}(\mathbf{q}=0,m=0)\,. \tag{3c}$$

The PP, Eq. (3b), is difficult to calculate. In a field theoretic description of the system, the bare parameters entering the PP are the disorder G, the interaction constants K_s and K_t, and a frequency or temperature renormalization constant whose bare value is $H_F=\pi N/2$ [13,14]. In previous approaches [3,8] the authors resorted to perturbation theory. However, recent advances in a RG description of the theory in the absence of superconductivity have provided us with the general structure of the PP [14]. An explicit one-loop RG calculation has been given in Ref. [14]. In terms of renormalized parameters g, k_s, k_t, and h (due to a compressibility sum rule one has $h=-k_s$) and a wave function renormalization parameter Z, one finds for the T_c equation

$$\frac{1}{|K_c|}=\frac{Z}{h}\frac{\ln(\omega_D/T)}{1+(\delta k_c/h)\ln(\omega_D/T)}\,. \tag{4}$$

Here ω_D is an upper frequency cutoff on the order of the Debye frequency, and $\delta k_c=k_c-K_c$ is the (repulsive) interaction in the Cooper channel which is generated by the RG even if the bare K_c vanishes [14]. The $\ln(\omega_D/T)$ terms come from a frequency integration over a diffusion propagator at zero wave number. For T_c we obtain

$$T_c=\omega_D\exp\left[-\frac{h}{Z|K_c|-\delta k_c}\right]. \tag{5}$$

While Eq. (5) is convenient for a physical discussion to be given below, and correctly describes how T_c vanishes, for quantitative evaluations at finite temperature it is important to acknowledge the fact that as renormalized quantities h, δk_c, and Z are scale and therefore temperature or frequency dependent and should be kept under the integrals leading to $\ln(\omega_D/T)$. Doing so yields the T_c equation in implicit form,

$$1=\int_{T_c}^{\omega_D}d\omega\,\gamma_c(\omega)/\omega\,, \tag{6a}$$

with

$$\gamma_c(\omega)=[Z(\omega)|K_c|-\delta k_c(\omega)]/h(\omega)\,. \tag{6b}$$

Before we proceed to calculate $\gamma_c(\omega)$, let us discuss the physical interpretation of this result. The coupling constant K_c is given by the DOS squared times an attractive effective potential. $Z^{1/2}$ renormalizes the single-particle DOS [13]. Its presence in Eq. (5) thus reflects an effect which one would expect on general physical grounds [15]. h has been interpreted as the quasiparticle DOS in a disordered FL [16], and it is reasonable that it also appears. It is interesting to note that if one writes the BCS coupling constant as $\lambda-\mu^*=N_F^2(v-w)/N_F$ with attractive and repulsive potentials v and w, respectively, then the DOS factors N_F^2 and N_F represent different physical quantities which happen to coincide for noninteracting electrons. δk_c is a disorder-induced renormalization of the Coulomb pseudopotential μ^*. It has been stressed in Ref. [4] that the retarded nature of the effective electron-electron interaction is crucial for an enhancement of $\mu^*=\mu/[1+\mu\ln(\varepsilon_F/\omega_D)]$, since an increase of the instantaneous potential $\mu=N_Fw$ would have little effect. This can also be seen from Eq. (6b): The bare value $\delta k_c^0=\delta k_c(\omega\cong\omega_D)$ is zero, and δk_c increases with decreasing ω. From the structure of the RG analysis of the Cooper propagator we know that the three quantities Z, h, and δk_c represent a complete parametrization of disorder effects on T_c. This statement holds for our BCS model where K_c and ω_D are phenomenological constants *independent of disorder*. It is well known that in a strong coupling theory the Eliashberg function *is* disorder dependent, which tends to increase T_c [17]. This effect is neglected in our model.

In order to determine $\gamma_c(\omega)$, we have to solve the appropriate RG flow equation. We have done so for three different physical situations: the case of strong spin-orbit

coupling [18] and the cases of approaching a frozen spin phase [19] and an insulator [20], respectively, from a FL in the absence of spin-orbit coupling, magnetic impurities, or magnetic fields. We have found qualitatively similar behavior of T_c versus disorder in all three cases. Here we choose the spin-orbit case to compare with some experiments. We will report on results for the other universality classes elsewhere [10]. For the spin-orbit case, the flow equations to one-loop order are

$$d\gamma_c/d\ln b = -\gamma_c g f(b) - g(1-\gamma_c)/4\,, \tag{7a}$$

$$dg/d\ln b = -\varepsilon g + g^2/4\,. \tag{7b}$$

Here $\varepsilon = d-2$ for a d-dimensional system, and the function f reads

$$f(b) = \begin{cases} 1/\varepsilon \quad (d>2)\,, & \text{(7c)} \\ \dfrac{\pi/2}{1+(x_D b)^2} + \dfrac{\ln(bx_D)}{1+(x_D b)^{-2}} \quad (d=2)\,, & \text{(7d)} \end{cases}$$

where $x_D = \kappa/q_0$ with κ the screening wave number and q_0 on the order of the Debye wave number. The RG length scale factor b is related to the temperature or frequency by

$$T/T_0 = H/b^d h(b)\,, \tag{7e}$$

with

$$dh/d\ln b = -hg/4\,. \tag{7f}$$

$T_0 = T(b=1)$ is the initial temperature scale which is on the order of ω_D.

Figure 1 shows results for $d=3$ together with experimental data from Ref. [21]. The relation between the RG parameter $g_0 = g(b=1)$ and the resistivity ρ we have taken to be $\rho = \rho^* g_0/g_c$, where $g_c = 4$ is the critical disorder for the MIT, and ρ^* is a resistivity scale on the order of the Mott number. Our parameters are then $\gamma_c^0 = \gamma_c(b=1)$, $T_c^0 = T_c(\rho=0)$, and ρ^*. Good agreement is obtained with reasonable parameters (because of the schematic nature of our scaling theory, we do not expect γ_c^0 to coincide precisely with the value obtained from a McMillan inversion procedure). Notice that the curvature of the curves changes sign as a function of γ_c^0.

Let us also mention two experimental results [22] on homogeneous 3D systems which the present theory *cannot* explain. First, in materials which have a low T_c^0 (≤ 7 K), T_c generally *increases* with disorder [1]. Presumably, this is due to the disorder dependence of the electron-phonon coupling mentioned above [8,17]. Reference [8] argued that effects which increase T_c dominate for small T_c^0, while T_c degradation dominates for high T_c^0. Second, three experiments on amorphous solid solutions of Si and Ge with metals (Nb, Au, and Al) showed that T_c vanishes only very close to the MIT, while we find suppression of SC well in the metallic phase. In these systems physical mechanisms favoring SC must be present which render our simple model inapplicable. This is obvious in the case of SiAu, where neither of the constituents is superconducting and our model is clearly inadequate.

Figure 2 compares results for $d=2$ with experiments on MoGe [23] and Pb [16] films. The relation between g_0 and the observed resistance just above T_c is

$$R_\square(T_c) = f\frac{\pi^2}{2}\frac{g_0}{1-(g_0/4)\ln b(T_c)}\,. \tag{8}$$

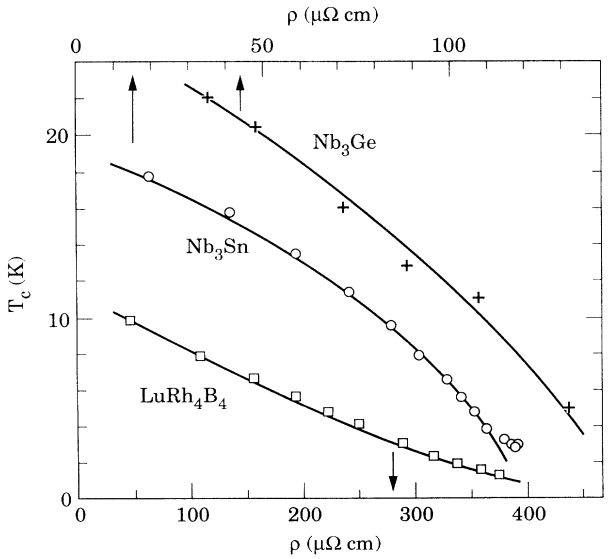

FIG. 1. Theory (solid lines) with experimental data redrawn from Ref. [21]. Parameters chosen for Nb_3Ge, Nb_3Sn, and $LuRh_4B_4$ are $\gamma_c^0 = 0.60$, 0.70, and 0.294; $T_c^0 = 26.3$, 19.4, and 11.5 K; $\rho^* = 433$, 305, and 3070 $\mu\Omega$ cm.

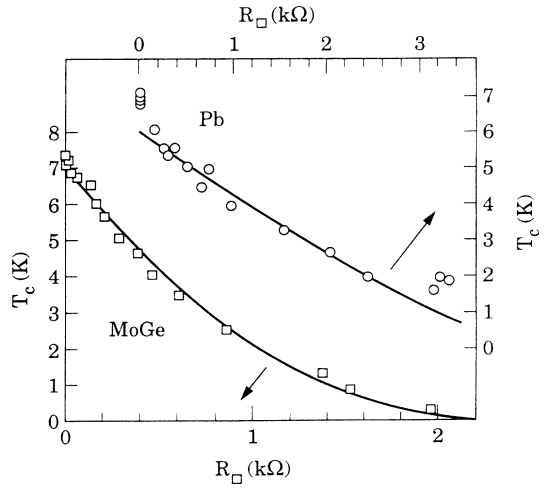

FIG. 2. Theory (solid lines) with experimental data redrawn from Ref. [23] (MoGe) and Ref. [16] (Pb). Parameters chosen for MoGe,Pb are $\gamma_c^0 = 0.15, 0.28$; $x_D = 6.0, 6.0$; $T_c^0 = 7.0, 6.0$ K; $f = 3.1, 1.58$. In both cases a fit of slightly lower quality is possible with $f=1$.

For free electrons, $f=1$, but in real systems one expects $f\neq 1$. Again we get good agreement with reasonable parameter values (i.e., within a factor of 2 of what one would reasonably expect). Two other experiments on homogeneous films are Refs. [24] and [25]. The results of Ref. [24] on Mo-C films are very similar to those of Ref. [23] on MoGe, and can be fitted with similar parameters. Data on Pb in Ref. [25] show a substantially slower decrease of T_c than Ref. [16]. We do not know the origin of this discrepancy.

Finally, we mention the relation of our theory to previous approaches. Our theory is a generalization of earlier work by Maekawa *et al.* [3], Anderson, Mattalib, and Ramakrishnan [4], and one of us [8]. Reference [3] calculated the PP in perturbation theory. Reference [4] considered localization corrections to the Coulomb pseudopotential, and Ref. [8] calculated the leading disorder correction to T_c for small disorder, including the disorder dependence of the electron-phonon coupling which we have neglected here. Notice that due to an approximation made, Ref. [8] did not distinguish between the two DOS renormalizations denoted by h/H and $Z^{1/2}$ above. Finkelshtein [9] considered a Cooper channel interaction amplitude Γ_c which includes the ladder summation. Therefore his condition for T_c is a divergence of the renormalized Γ_c, while his flow equations are valid only for small Γ_c. As a consequence, Ref. [9] misses the DOS effects discussed above.

In summary, we have given a complete parametrization of the disorder-induced suppression of the mean-field superconducting transition temperature within a model where the phonon-mediated electron-electron interaction is constant. An evaluation of our results at one-loop order yields good agreement with experiments.

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