Measurement of Collisional Anisotropic Temperature Relaxation in a Strongly Magnetized Pure Electron Plasma

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The rate at which the temperatures parallel and perpendicular to a magnetic field relax to a common value has been measured in a pure electron plasma. This rate was measured over the temperature range 28 to 10^4 K and for magnetic fields in the range 30 to 60 kG. When the cyclotron radius r_c is large compared to the classical distance of closest approach b, the measured rates are described by conventional scattering theory. When $r_c/b < 1$, the measured rate drops precipitously as r_c/b is decreased, in agreement with the adiabatic-invariant theory of O'Neil and Hjorth.

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Collisional Coulomb scattering in velocity space is an important process in plasma physics. Examples include the equipartition of energy among the various degrees of freedom (important in many fusion heating schemes), the scattering of charged particles into a specific region of velocity space (e.g., the loss cone of a magnetic mirror), and momentum exchange between electrons and ions (plasma resistivity). Many theoretical studies on collisional velocity scattering have been made during the last fifty years [1]. However, there are only a few direct experimental tests of these theories and, with the exception of the Hyatt, Driscoll, and Malmberg [1] experiment, the relative uncertainties in these tests are of order unity. Also, unlike the prior experiments, we investigate collisional scattering in the strong magnetic-field regime where the magnetic field greatly affects collision dynamics.

We have measured the rate ν at which electron-electron collisions equilibrate parallel and perpendicular temperatures in a pure electron plasma. Here parallel and perpendicular refer to directions relative to an applied uniform magnetic field. Our methods are closely related to the work of Hyatt, Driscoll, and Malmberg, but cover a different parameter range, and we employ a different procedure to measure ν. The plasma is confined axially by an electric field and radially by a magnetic field. The confining electric field is modulated, producing a modulation in the plasma length parallel to the magnetic field. Collisions make the process irreversible and the plasma is heated. Maximum heating per cycle is predicted to occur when $2\pi f = 3\nu$, where f is the modulating frequency. We determine ν by measuring the modulating frequency which produces the most heating per cycle.

At high temperatures the plasmas we studied were in the weakly magnetized regime (i.e., $r_c/b > 1$, where r_c is the cyclotron radius and $b = e^2/\kappa T$ is the classical distance of closest approach). In this regime the measured ν is proportional to $T^{-3/2}$ (Fig. 3). For temperatures where $r_c/b \sim 1$, the measured ν peaks. As the temperature is lowered below this point, the plasmas entered the strongly magnetized regime, $r_c/b < 1$. In this latter regime ν drops precipitously as the temperature is decreased. This precipitous drop is in agreement with a theoretical prediction of O'Neil and Hjorth [2] who argue that the collisional dynamics is constrained by a many-electron adiabatic invariant in the regime $r_c/b \ll 1$.

In the weakly magnetized regime we compare our results to an unmodified and a modified Ichimaru and Rosenbluth (IR) prediction [3],

$$\nu = (8\sqrt{\pi}/15) n b^2 \bar{v} \ln[\Lambda] , \qquad (1)$$

where $\bar{v} = \sqrt{\kappa T/m}$. For the unmodified prediction the Coulomb interaction is cut off at a distance of λ_D so that $\Lambda = \lambda_D/b$. However, when $r_c \ll \lambda_D$, as is the case for the data in this paper, theoretical work by Silin [4] and theoretical and numerical work by Montgomery, Joyce, and Turner [5] show that Coulomb interactions should be cut off at a distance of about r_c. This modified prediction has $\Lambda = r_c/b$. In the strongly magnetized regime we compare our results to an O'Neil and Hjorth [2] (OH) calculation which gives

$$\nu \simeq 2.48 n b^2 \bar{v} (r_c/b)^{1/5} \exp[-2.34(b/r_c)^{2/5}] . \qquad (2)$$

A section of our electron trap is schematically shown in Fig. 1. This section consists of three cylindrical electrodes (called gates and labeled G_1 to G_3) and five charge

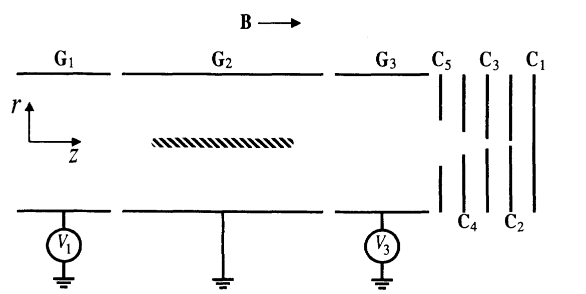

FIG. 1. A schematic of part of the apparatus. The gate radius is $R_G = 1.27$ cm, and $r_3/2 = r_2 = 0.040$ cm where r_j is the radius of the hole in collector C_j.

collectors (C_1 to C_5). Radial confinement of the plasma is provided by the magnetic field [6]. Biasing G_1 and G_3 sufficiently negative relative to G_2 confines the plasma axially within G_2. The complete trap includes an electron source and additional gates. The source, gates, collectors, and magnetic field are all coaxial. The electron trap resides in a sealed, evacuated vessel which is cooled to liquid-helium temperature (4.2 K).

To measure the plasma density the potential of G_3, V_3, is quickly ramped to ground. This allows the electrons to stream out along the magnetic field lines and onto the charge collectors. The number of electrons collected by C_i ($i=1-5$) is the number of electrons between radius r_i and r_{i+1} where r_j is the radius of the hole in C_j. Using these data and the potentials of the cylindrical electrodes, an average plasma density $\langle n\rangle$ and axial length $\langle l\rangle$ can be calculated from Poisson's equation [7]. We believe these values are good to about $\pm 15\%$.

The temperature is determined by slowing ramping V_3 to ground, and measuring as a function of V_3 the number of electrons which have sufficient axial energy to escape past G_3. For simplicity, we ignore here the fact that the actual confinement potential is less than the applied voltage V_3 since G_3 is finite in length. There is also a correction due to the fact that the plasma expands during this process. Both corrections are included in the actual analysis of the data. An electron can escape only when its parallel kinetic energy is sufficient to overcome the potential on G_3. This requirement is

$$mv_\parallel^2/2 > -e[V_3-\phi(r,V_3)] \equiv mv_e^2(r,V_3)/2\,, \qquad (3)$$

where $\phi(r,V_3)$ is the total potential due to the confining potentials and electron space charge. We evaluate $\phi(r,V_3)$ at the axial midplane of the plasma where the confining potentials are ignorable. However, $\phi(r,V_3)$ is still a function of V_3 because the part of the potential due to space charge changes as electrons escape. If V_3 is ramped slowly [8], essentially all electrons at r with $v_\parallel > v_e(r,V_3)$ escape while all other electrons are still confined. Thus, for a slow ramp the total number of electrons which escape past G_3 as a function of V_3 is given by

$$N_e[V_3] = 2\pi\int_0^{R_G} N(r)\,\mathrm{erfc}[v_e(r,V_3)/\sqrt{2}\bar{v}_\parallel]\,r\,dr\,, \qquad (4)$$

where R_G is the electrode radius and $\bar{v}_\parallel = (\kappa T_\parallel/m)^{1/2}$. Here erfc is the complementary error function and comes from integrating the Maxwellian distribution from $v_\parallel = v_e(r,V_3)$ to $v_\parallel = \infty$. We assume a radial density profile and fit the data with Eq. (4) to obtain $\bar{v}_\parallel$ and thus the parallel temperature. For $T_\parallel \gtrsim 200$ K we believe that the error in the measured temperature is about 10%. At $T_\parallel = 30$ K the random error is about 30%, and there may be a systematic error of about 30%.

To determine ν we modulate V_1 sinusoidally and measure the plasma heating which results. The modulation of V_1 modulates the axial plasma length which, in turn, modulates $T_\parallel$. Collisions make this process irreversible and thus the modulation, averaged over a cycle, puts heat into the plasma. We model the rate of change of $T_\perp$, $T_\parallel$, and $\langle l\rangle$ as

$$\frac{dT_\perp}{dt} - \nu(T_\parallel - T_\perp) = -\frac{3}{2}\frac{T_\perp}{\tau_r}\,, \qquad (5)$$

$$\frac{dT_\parallel}{dt} + 2\nu(T_\parallel - T_\perp) = -2\frac{T_\parallel}{\langle l\rangle}\frac{d\langle l\rangle}{dt}\,, \qquad (6)$$

$$\langle l\rangle = l_0[1+\varepsilon\sin(2\pi f t)]\,, \qquad (7)$$

where τ_r is the cyclotron radiation cooling time. The right-hand side of Eq. (5) is derived from Larmor's radiation formula. For a transparent, high-temperature plasma, Larmor's formula yields

$$\tau_r = 3.9\times 10^8/B^2\ \mathrm{sec} \qquad (8)$$

(e.g., for $B=40$ kG, $\tau_r = 0.24$ sec). Experimentally, when the plasma is not modulated (i.e., $d\langle l\rangle/dt=0$), it is observed to cool exponentially in agreement with Eqs. (5), (6), and (8). Despite the fact that the plasma is in a complicated waveguide, the observed cooling time agrees with Eq. (8) to within 30%. The right-hand side of Eq. (6) is derived from the ideal-gas law and from the work done on an ideal gas when its volume is changed, $dW = P_\parallel A\,d\langle l\rangle$. The ideal-gas approximation is valid since the plasma is weakly correlated. Heating due to plasma waves can be ignored since f is much less than the lowest plasma mode frequency.

We solve Eqs. (5)-(7) by expanding $T_\perp$ and $T_\parallel$ in powers of ε (e.g., $T_\perp = \sum T_{\perp,i}\varepsilon^i$) and then equate each order in ε separately to zero. To order ε^2 we find that

$$\left\langle\frac{dT}{dt}\right\rangle_{\mathrm{cycle}} = \left[\frac{4}{3}\varepsilon^2\nu\frac{\beta^2}{1+\beta^2} - \frac{1}{\tau_r}\right]T\,, \qquad (9)$$

where $T=(T_\parallel + 2T_\perp)/3$, $\beta = 2\pi f/3\nu$, and $\langle dT/dt\rangle_{\mathrm{cycle}}$ means an average of dT/dt over one modulation cycle. Thus, there is a gradual change in T resulting from the competition between the average heating due to modulation and the cyclotron cooling.

From Eq. (9), it follows that maximum average heating per cycle occurs when $f=3\nu/2\pi$. This maximum in the heating per cycle can be observed by modulating the plasma for a fixed number of cycles H. After the modulation is over, we measure the parallel temperature of the plasma, T_e. We measure T_e at a fixed time after the start of the heating, to make the cyclotron cooling duration independent of f. By repeating the experiment at a series of frequencies, we obtain T_e vs f as shown by the diamonds in Fig. 2(a). The solid line in Fig. 2(a) is a least-squares fit by Eq. (9). This solid line is obtained by numerically integrating Eq. (9) using a $\nu[T] = c\nu_1[T]$ where $\nu_1[T]$ is given by Eq. (1), which is appropriate in this temperature range. In the fit c and ε are free parameters. The fit in Fig. 2(a) yields $\nu = 36.3\times 10^3\ \mathrm{sec}^{-1}$ for $T=1400$ K.

To determine ν more accurately, we modify the heating

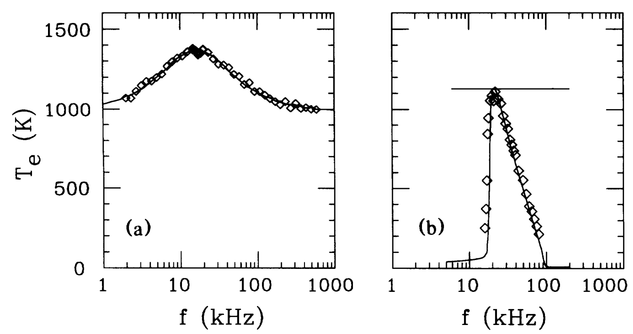

FIG. 2. Experimentally measured T_e vs modulation frequency f (diamonds). (a) Heating comprised of 80 contiguous cycles compared to prediction; (b) heating comprised of 301 sets of 24 cycles, with the total heating process lasting about ten cyclotron radiation times. The horizontal line at $T_e = 1129$ K indicates the temperature of the plasma at the beginning of the heating process.

process discussed in the previous paragraphs to make the total process last many radiation times τ_r. The modified heating process consists of S intervals, each having H cycles, with each interval lasting a fixed time. This insures that both the total time and the total number of cycles are held constant (i.e., independent of f). Furthermore, we take curves of T_e vs f for various ε until an ε is found such that at the peak in net heating the plasma maintains a nearly constant temperature. That is, at $f \simeq 3\nu/2\pi$ the heating balances cyclotron cooling; for lower or higher f the heating is insufficient and the plasma cools by a large amount. This makes the peak of the curve much sharper. An example of T_e vs f from this modified process is shown in Fig. 2(b). In Fig. 2(b) the solid horizontal line at $T=1129$ K is the initial temperature. For the data in Fig. 2(b), $S=301$, $H=24$, and each interval lasted 5 msec. The solid curve in Fig. 2(b) is obtained by numerically integrating Eqs. (5)-(7) with a $\nu[T]$ provided in part by Glinsky and O'Neil [9]. The solid line is present only to show how well Eqs. (5)-(7) describe the experiment, and is not used to determine ν. Instead, ν is measured by visually determining the peak of T_e vs f and employing the formula $\nu = 2\pi f_{max}/3$. This procedure is accurate to about 5%. This modified heating process was used to determine ν for all the data in Figs. 3 and 4.

Figure 3 shows a plot of the measured ν vs T (and r_c/b) for $B=61.3$ kG and $\langle n\rangle = 8\times10^8$ cm^{-3}. The dashed and dash-dotted curves are plots of the modified IR ($\Lambda = r_c/b$) and the unmodified IR ($\Lambda = \lambda_D/b$) predictions, respectively. For high temperatures the modified IR prediction is in much better agreement with our data than is the unmodified prediction. The solid curve is a plot of the OH prediction [Eq. (2)]. For low temperatures our data agree with the OH prediction.

Dividing Eqs. (1) and (2) by $nb^2\bar{v}$ produces normalized rates which are a function only of r_c/b. In Fig. 4 we plot $\nu/nb^2\bar{v}$ vs r_c/b for various magnetic fields. For the data in Fig. 4, $\langle n\rangle \simeq 8\times10^8$ cm^{-3}, $\langle l\rangle \simeq 3.5$ cm, and $(T_\parallel - T_\perp)/T_\parallel \lesssim 4\%$. This figure shows that the normalized data for various magnetic fields can be described by a single curve to within experimental error.

The existence of our experimental results for the intermediate regime $r_c/b \sim 1$ has motivated further theoretical work [9]. Our data are in good agreement with this theory also.

Because of the strong dependence of ν on r_c/b in the regime $r_c/b < 1$, the possible systematic error of 30% in the measured temperature in this region is much more important when comparing theory and experiment than other uncertainties. Furthermore, we can determine the

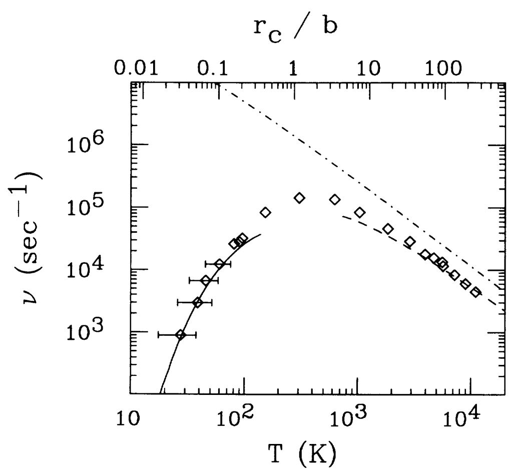

FIG. 3. Measured relaxation rate for $B=61.3$ kG. The solid curve is the OH prediction. The dashed and dash-dotted curves are plots of the modified IR ($\Lambda = r_c/b$) and the unmodified IR ($\Lambda = \lambda_D/b$) predictions, respectively.

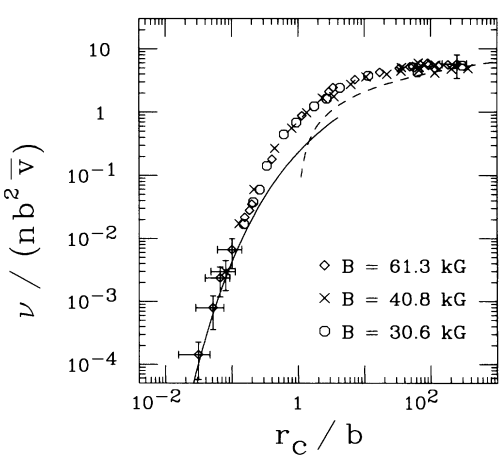

FIG. 4. Normalized rate vs r_c/b. For the ◇, ×, and ○ data the magnetic fields are 61.3, 40.8, and 30.6 kG, respectively. For all the data $\langle n\rangle \simeq 8\times10^8$ cm^{-3}. The solid curve is the OH prediction. The dashed curve is the modified IR prediction.

frequency which produces the most heating per cycle to within about 5%. If the average heating per electron at radius r were independent of r and if the density were uniform, then ν would also be determined to about 5%. However, since $\nu \propto n$ and heating depends on the ratio ν/f, radial density variations cause radial variation in the heating per electron and thus additional uncertainty to our determination of ν. We have analyzed our heating method for various density profiles. Since we do not know the radial thermal conductivity of our plasmas, we studied two cases: zero and infinite radial thermal conductivity. The error bars in Figs. 3 and 4 include the uncertainty implied by this analysis.

In deriving Eq. (9) it is assumed that ε is small. For the modified heating process, ε is easily estimated by using the fact that when $2\pi f = 3\nu$ the average heating $\simeq 4\pi H\varepsilon^2 T/9$ balances the average cooling $\approx t_i T/\tau_r$ for each interval. Here t_i is the time per interval, and we have employed the fact that $t_i/\tau_r \ll 1$ for all our data. We conclude that $\varepsilon \lesssim 6\%$ and that corrections to ν due to finite ε are at most 5%.

In summary, we have measured the rate of anisotropic temperature relaxation for a pure electron plasma in the regime $1/32 \lesssim r_c/b \lesssim 410$. For $r_c/b \gg 1$ our results are consistent with the prediction by Ichimaru and Rosenbluth as modified by Silin and Montgomery, Joyce, and Turner. For $r_c/b \ll 1$ our results are consistent with a prediction by O'Neil and Hjorth that the collisional dynamics is constrained by a many-electron adiabatic invariant.

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