Constraints on Almost-Dirac Neutrinos from Neutrino-Antineutrino Oscillations

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If a neutrino has both Dirac and Majorana masses (m,μ), as some recent models predict, $\nu\leftrightarrow\bar\nu$ oscillations within a single generation may cause a violation of the nucleosynthesis bound on the number of light neutrino species. Taking into account the effects of finite temperature and density, we find that μm must be less than 3×10^{-5} eV^2, which is too small to explain atmospheric neutrino observations. The bound can be softened to 0.05 eV^2 if the right-handed components couple to massless bosons (Majorons). We introduce a simple but quantitatively accurate formalism for tracking the approach of the sterile state to thermal equilibrium.

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The successful predictions of big bang nucleosynthesis put stringent limits on lepton number violating oscillations between light, weakly interacting neutrinos ν_L and noninteracting antineutrinos $\bar\nu_R$, which would arise if the neutrino mass term has both Dirac (m) and Majorana $(\mu_{L,R})$ entries,

$$(\nu_L\ \bar\nu_R)\begin{pmatrix}\mu_L & m\\ m & \mu_R\end{pmatrix}\begin{pmatrix}\nu_L\\ \bar\nu_R\end{pmatrix}. \tag{1}$$

Some models [1,2] which predict neutrino masses of this form have been recently proposed to explain reports of a 17-keV neutrino mixing with ν_e in beta decay experiments [3]. In these models, the tau neutrino is mostly Dirac, but gets a small left-handed Majorana mass μ_L of order m_{ν_μ}, assumed to be $\sim10^{-3}$ eV as required for the Mikheyev-Smirnov-Wolfenstein (MSW) solution of the solar neutrino puzzle [4]. We show that such a large Majorana mass is ruled out since it would imply the existence of a fourth, sterile species of neutrinos in thermal equilibrium at temperatures of 1 MeV, in contradiction to the nucleosynthesis bound that the effective number of ν species should be no more than [5]

$$N_\nu=3.3\,. \tag{2}$$

Even if the 17-keV neutrino should be disproved, it has focused attention on the theoretical attractiveness of pseudo Dirac neutrinos, resulting from an almost-conserved lepton number that also naturally explains how one can have maximally large mixing angles. Such large angles are suggested by the apparent deficit in the flux of ν_μ from the atmosphere [6]. The significance of our bound thus transcends Simpson's neutrino.

The oscillation of active and sterile species was examined in Refs. [7] and [8], with somewhat conflicting results for the nonresonant case. We note that they underestimated the rate of sterile neutrino production by using too small a value for the total thermal rate of active neutrino interactions, defined below. Consequently, they find that resonant oscillations always give stronger limits for ν_μ and ν_τ mixing with a sterile neutrino, whereas we will show that nonresonant oscillations are in fact the dominant process, and significantly improve their bound on $(\mu_L+\mu_R)m$. We similarly tighten bounds obtained by Babu and Rothstein [9] for the case when the neutrinos have large interactions with Goldstone bosons.

Our results are derived using a new formalism for damped oscillations, in which the incoherent interactions are incorporated into the imaginary part of the Hamiltonian. This gives a more rigorous description of the approach of the sterile neutrino species toward thermal equilibrium than the heuristic procedure of multiplying the oscillation probability times the ordinary interaction rate to obtain the interaction rate of the sterile species. It is also simpler to use than the density-matrix approach.

Starting from the Dirac equation for a neutrino of momentum p propagating in a thermal bath [10], we have derived the Hamiltonian for ν_L-$\bar\nu_R$ oscillations,

$$\mathcal{H}(p)=\begin{pmatrix}A_L+i\Gamma/2 & B/2\\ B^*/2 & A_R\end{pmatrix}, \tag{3}$$

where $A_{L,R}=-V_{L,R}+|\mu_{L,R}|^2/2p$, $B=(m\mu_L+m^*\mu_R)/p$, the thermal potential for left-handed mu and tau neutrinos is given by

$$V_L=(7\pi/90\alpha)\sin^2(2\theta_W)G_F^2T^4p\,, \tag{4}$$

and we have allowed for the possibility that the right-handed component may also get a potential through exotic interactions. Because a "test beam" of neutrinos will be attenuated by incoherent interactions with the plasma, the Hamiltonian has an imaginary part $\Gamma/2$, where

$$\Gamma=(7\pi/24)[1+\tfrac{1}{15}(1-4\sin^2\theta_W)^2]G_F^2T^4p \tag{5}$$

is the total interaction rate of the test particle, including elastic scattering. It is not correct to use the rate of annihilation processes only, as has been commonly done in the literature, since a sterile neutrino can also be produced through elastic scattering in which the final-state neutrino oscillates. The difference is significant: Γ is approximately 38 times faster than the annihilation rate. We have computed Γ from the individual cross sections to verify (5), which was originally calculated from the two-loop thermal self-energy diagrams [10,11].

From (3) one finds the mixing angle in matter,

$$\sin^2(2\theta_m) = \frac{|B|^2}{|B|^2 + (A_L - A_R)^2}, \tag{6}$$

valid when the oscillation frequency

$$\omega_{\rm osc} = \frac{1}{2\pi}[|B|^2 + (A_L - A_R)^2]^{1/2} \tag{7}$$

is much greater than Γ. We have assumed that the lepton asymmetry is of order the baryon asymmetry, so that we could omit finite-density effects in (4) at the temperatures of interest. With this assumption, the entire analysis is identical for the charge conjugate ($\bar{\nu}_L$-ν_R) system. For simplicity we are ignoring the possibility of mixing with other flavors which might give rise to oscillations between three states.

In the nonresonant case, it is common to estimate the rate Γ_s at which oscillations bring the sterile state into equilibrium as the time-averaged oscillation probability times the interaction rate of the left-handed species, and to then require that Γ_s never exceed the Hubble expansion rate H:

$$\Gamma_s = \tfrac{1}{2}\sin^2(2\theta_m)\Gamma < H. \tag{8}$$

We propose a more accurate approach which is nonetheless simple. Let N_a and N_s, respectively, be the number of active and sterile neutrinos in a comoving volume of phase space ($d^3x\,d^3p$). If $P(t_0;t)$ is the probability that a ν_L created by thermal processes at time t_0 oscillates into $\bar{\nu}_R$ by time t, and $dN_{\rm prod}/dt$ is the rate at which new ν_L states are being produced thermally, then at any time t, N_s is given by

$$N_s(t) = \int_{t_i}^{t} dt_0\, P(t_0;t)\, dN_{\rm prod}/dt_0 + P(t_i;t)N_a(t_i), \tag{9}$$

where the initial time t_i is arbitrary and drops out of all results if $N_s(t_i) \cong 0$. By solving the Schrödinger equation for the Hamiltonian (3), with the notation $s_t = \sin\theta_m(t)$, $c_t = \cos\theta_m(t)$, we find the nonunitary time evolution operator $U_{ij}(t_0;t)$ from which one can compute the oscillation probability

$$P(t_0;t) = |U_{RL}(t_0;t)|^2 = s_t^2 c_{t_0}^2 \exp\left(-\int_{t_0}^{t}\Gamma c_\tau^2\, d\tau\right) + s_{t_0}^2 c_t^2 \exp\left(-\int_{t_0}^{t}\Gamma s_\tau^2\, d\tau\right) \tag{10}$$

plus oscillatory terms that approximately average to zero in the integral (9) as long as the oscillations are undamped ($\omega_{\rm osc} \gg \Gamma$). Because the active neutrinos are in equilibrium, their thermal production rate must be the same as the rate at which they are knocked out of the test beam,

$$dN_{\rm prod}/dt = \Gamma N_a(t). \tag{11}$$

Using this together with the fact that $N_a(t)$ is conserved in a comoving volume (we are assuming that the oscillations do not greatly perturb the active states from thermal equilibrium), the integral (9) can be done exactly with the result

$$\frac{N_s(T)}{N_a(T)} \cong 1 - \exp\left[-\int_{T}^{T_i}\left(\frac{s_T^2\Gamma}{HT}\right)dT\right], \tag{12}$$

where we have reparametrized the time integrals by temperature, and also made the approximation of small matter mixing angle. The nucleosynthesis constraint becomes $N_s(T)/N_a(T) < 0.3$ at the decoupling temperature $T_{\rm dec} = 3.5$ MeV [12,13]. Notice the qualitative similarity of this criterion to the naive one (8): Both of them depend on the ratio $\theta_m^2\Gamma/H$ when the matter mixing angle is small. Equation (12) should not be used below $T_{\rm dec}$ since oscillations thereafter will conserve the total neutrino density $N_s + N_a$, and hence the total energy density [14]. One advantage of (12) over the usual criterion using (8) is that it shows explicitly how the limits on the oscillation parameters depend on the fraction of a full species of additional neutrinos that one is willing to tolerate—the dependence is only logarithmic. We have computed the limits both ways and found that they agree to within a factor of 3.

In the resonant case, where $\sin(2\theta_m)$ goes through unity because of the vanishing of $A_1 - A_2$ in (3), the exponential in (12) will become very small below the resonance temperature $T_{\rm res}$, bringing the sterile state into full equilibrium, as long as the following conditions are satisfied: (1) Obviously, the resonance must occur before decoupling of the active species if it is to have any effect on nucleosynthesis. (2) Adiabaticity: The duration of the resonance, which we define as the time to go through its half-width, must exceed the oscillation period in order for the resonance to be effective [15]. That is,

$$\frac{\pi}{2}\left|\frac{d}{dt}(A_L - A_R)\right| < |B|^2. \tag{13}$$

(3) Damping: The oscillation frequency must be real; otherwise the system is critically overdamped. Solving for the eigenvalues of (3), this translates into the requirement that

$$B = 2\pi\omega_{\rm osc} > \tfrac{1}{2}\Gamma. \tag{14}$$

It is now straightforward to apply these criteria and obtain limits on $m\mu_L, m\mu_R$ in the standard model, where the right-handed neutrino potential V_R vanishes. From (6) one sees that there is a resonance in the case that $|\mu_L|^2 > |\mu_R|^2$; for simplicity assume $\mu_R = 0$ and $|\mu_L| = \mu_L$. From conditions (1)-(3) above, we obtain the following bounds:

$$\begin{aligned} &\mu_L < \mu_{\rm crit} \equiv 9\times10^{-3}\ \text{eV}, \\ &m < 4\times10^{-4}(\mu_L/\mu_{\rm crit})^{1/2}\ \text{eV}, \\ &m < 9\times10^{-5}(\mu_L/\mu_{\rm crit})\ \text{eV}. \end{aligned} \tag{15}$$

If any of these are violated, the resonance is unimportant for populating the sterile state. Note that the second condition would not apply for Simpson's neutrino, and the third would not apply for an almost Dirac neutrino, leaving the first as the interesting bound. But regardless of whether a resonance occurs, there may be nonresonant oscillations in addition, which can give a more stringent bound. Indeed, from Eq. (12), we find

$$m\mu \equiv |m^* \mu_R + m\mu_L| < 3\times 10^{-5}\,\mathrm{eV}^2\,, \qquad (16)$$

which for $m=17$ keV is always more stringent than the resonant bound (15); it constrains $\mu < 10^{-9}$ eV.

It was recently observed by Babu and Rothstein [9] that (16) can be weakened if the right-handed potential V_R is nonzero. This happens naturally in models where some of the neutrinos get their masses from the seesaw mechanism, when one or more lepton number symmetries are spontaneously broken. Then the neutrinos would couple to a massless Goldstone boson (the Majoron, ϕ) via

$$ih\phi\bar{\psi}\gamma_5\psi\,, \qquad (17)$$

where $\psi=(\nu_R,\nu_L)^T$ is a Dirac spinor. A ν_R of momentum p acquires a potential due to its forward scatterings off background ν_L's, given by

$$V_R = h^2 T^2/48p\,, \qquad (18)$$

in the limiting case that the Majoron temperature is zero. [Equation (18) can be easily calculated from the one-loop self-energy diagram for ν_R using the finite-temperature ν_L propagator.] We have reanalyzed the effect of V_R on the oscillations, using the correct interaction rate (5), to obtain limits about 60 times stronger than were found in Ref. [9]. For simplicity we assume that the right-handed neutrino and the Majoron are at zero temperature. Otherwise, the left-handed neutrino would also receive a Majoron-induced contribution δV_L to its potential, but this can only weaken the effects of the Majorons since the sign of δV_L is the same as in (18), but it is always the combination $V_L - V_R$ which matters.

To understand the effects of Majorons one must know how large a value of the coupling h is allowed. We have computed the production rate for Majorons and ν_R's in the early Universe to be $\sim 10^{-3}h^4T$, which would bring them into equilibrium before 3.5 MeV unless $h<10^{-5}$. A more stringent bound of $h<10^{-6}$ was inferred from the fact that Majoron emission did not shorten the neutrino signal from supernova 1987A [16]. We will therefore assume that

$$\hat{h} \equiv h/10^{-6} < 1\,. \qquad (19)$$

From (6) it follows that a resonance will occur, regardless of the sign of δm^2, at the temperature

$$T_{\mathrm{res}} = (28.5\hat{h}^{1/2}\,\mathrm{MeV})\{\sqrt{4/3}\cos[\tfrac{1}{3}\arccos(7.7\times 10^{-2}\mu^2\hat{h}^{-3})]\}^{1/2}\,, \qquad (20)$$

where μ is in eV and the factor in curly brackets approaches 1 for $\mu \ll \hat{h}^{3/2}$ eV. (One should replace cos with cosh in the opposite limit.) Paralleling the $V_R=0$ case, we obtain the constraints

$$h < 6\times 10^{-8}\,,\quad m\mu < 0.06\hat{h}^{9/4}\,\mathrm{eV}^2\,,\quad m\mu < 0.3\hat{h}^3\,\mathrm{eV}^2\,. \qquad (21)$$

In addition, the bound from nonresonant oscillations is

$$m\mu < 0.05\hat{h}^2\,\mathrm{eV}^2\,. \qquad (22)$$

Thus in the most optimistic case, Majorons could weaken the bound on μ by 4 orders of magnitude. Note that for the 17-keV neutrino, this would still imply that the light neutrino masses, in models where they are naturally of order μ [1,2], are too small for the MSW effect to work in the Sun.

In conclusion, we have shown that the nucleosynthesis constraint requires an approximately Dirac mu or tau neutrino to have $\delta m^2 < 6\times 10^{-5}$ eV^2 in the standard model, or $\delta m^2 < 0.1(h^2/10^{-12})$ eV^2 in models where the two components couple with strength h to a Goldstone boson. It is interesting to note that the deficit of atmospheric ν_μ's observed by the Kamioka detector (and to a lesser extent that of Irvine-Michigan-Brookhaven) [6] suggests that ν_μ oscillates into ν_τ or a sterile neutrino, with a large vacuum mixing angle and δm^2 near 10^{-2} eV^2. Thus $\nu \leftrightarrow \bar{\nu}$ oscillations might account for the atmospheric ν_μ flux, but only if Majorons are playing an important role in suppressing such oscillations in the early Universe, or the primordial ^{4}He abundance is larger than is presently believed.

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