Reinterpretation of Jordan-Brans-Dicke Theory and Kaluza-Klein Cosmology

Y. M. Cho
Department of Physics, Seoul National University, Seoul 151-742, Korea
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We emphasize that it is the Pauli metric, not the Jordan metric, which describes the massless spin-two graviton in the Brans-Dicke theory. Similarly in the "Jordan-Brans-Dicke theory" based on Kaluza-Klein unification, only the Pauli metric can correctly describe Einstein's theory of gravitation. This necessitates a completely new reinterpretation of the "old" Kaluza-Klein cosmology as well as the Brans-Dicke theory. More significantly our analysis shows that the Kaluza-Klein dilaton must generate a fifth force which could violate the equivalence principle.

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Recently the cosmology based on the Brans-Dicke theory [1,2] has been studied by many authors [3,4] in an attempt to construct a realistic alternative to the standard model. A characteristic feature of the Brans-Dicke theory is that the Brans-Dicke scalar field plays the role of Newton's constant. As a result, the Brans-Dicke theory makes Newton's constant time dependent, and thus can naturally realize Dirac's conjecture [5]. A less well appreciated but nevertheless very important fact is that *the Brans-Dicke Lagrangian admits two space-time metrics, the Jordan metric [1] and the Pauli metric [6], which can actually describe two different physics. In fact it allows infinitely many space-time metrics which are logically acceptable but physically distinct*, as we will show in the following. This poses a serious problem, because one must decide which metric (and for what reason) should be identified as physical before one can discuss the physics.

Exactly the same problem arises in the Kaluza-Klein theory [7,8], because in this higher-dimensional unification the Kaluza-Klein dilaton plays the role of the Brans-Dicke scalar field which makes the Jordan metric (the precise meaning of which we will define shortly) couple to different matter fields with different strengths [9]. So in this unification a central issue to settle is which metric one must identify as the physical space-time metric (i.e., the metric which is responsible for Einstein's theory of gravitation). The purpose of this Letter is to settle the issue and to clarify the existing confusion in the literature.

We first discuss an inherent ambiguity which exists in the Brans-Dicke Lagrangian, and show how the Lagrangian allows infinitely many logically acceptable metrics which describe different physics. But we prove that *only the Pauli metric can represent the massless spin-two graviton, and thus can correctly describe Einstein's theory of gravitation.* This means that, as long as one wants to interpret the theory as a generalization of Einstein's theory, one must treat the Pauli metric as physical. In the Kaluza-Klein theory we arrive at essentially the same conclusion, but for a different reason. Here one cannot treat the Jordan metric as physical, because it violates the positivity of the Hamiltonian. At the same time one must accept the Pauli metric as physical [9,10], as long as one wishes to achieve the unification of Einstein's gravitation with other interactions from the Kaluza-Klein theory. This necessitates a completely new reinterpretation of the "old" Kaluza-Klein cosmology which identifies the Jordan metric as physical [11,12]. More significantly our analysis shows that *the Kaluza-Klein dilaton must create a fifth force which could violate the equivalence principle* [13]. This is because the dilaton makes the Pauli metric couple to the matter fields "abnormally" in the Kaluza-Klein theory.

Let us begin with the following Brans-Dicke Lagrangian [2]:

$$\begin{aligned} \mathcal{L}_{BD} &= \mathcal{L}_0 + \mathcal{L}_1 , \\ \mathcal{L}_0 &= -\sqrt{\gamma}[\phi R + \omega \gamma^{\mu\nu}(\partial_\mu \phi)(\partial_\nu \phi)/\phi] , \\ \mathcal{L}_1 &= -\sqrt{\gamma}[\tfrac{1}{2}\gamma^{\mu\nu}(\partial_\mu \Psi)(\partial_\nu \Psi) + V(\Psi)] , \end{aligned} \tag{1}$$

where $\gamma = |\det \gamma_{\mu\nu}|$ and $\gamma_{\mu\nu}$ is what we call the Jordan metric [1], ϕ is the Brans-Dicke scalar field, ω is the Brans-Dicke coupling constant, and $\mathcal{L}_1$ is the Lagrangian for the matter field Ψ which we choose to be a scalar field for simplicity. Notice that here $\mathcal{L}_1$ does *not* depend on ϕ, which is one of the original assumptions of Brans and Dicke. However, it is very important for our purpose to keep in mind that the theory can easily be generalized in such a way that $\mathcal{L}_1$ does depend on ϕ, to make the Jordan metric couple to different matter fields with different strengths. Indeed this generalization is precisely what one finds in the Kaluza-Klein theory [8,9], or in the generalized Brans-Dicke theory proposed recently by Damour, Gibbons, and Gundlach [4]. Now let us introduce the Brans-Dicke dilaton field σ by

$$\phi = (1/16\pi G)e^{\alpha\sigma} , \tag{2}$$

where α is a nonvanishing constant. In terms of the dilaton one has

$$\begin{aligned} \mathcal{L}_{BD} = &-\frac{\sqrt{\gamma}}{16\pi G} e^{\alpha\sigma}[R + \omega\alpha^2 \gamma^{\mu\nu}(\partial_\mu \sigma)(\partial_\nu \sigma)] \\ &- \sqrt{\gamma}[\tfrac{1}{2}\gamma^{\mu\nu}(\partial_\mu \Psi)(\partial_\nu \Psi) + V(\Psi)] . \end{aligned} \tag{3}$$

This shows that, in the limit that the Jordan metric becomes flat, the dilaton has a positive kinetic energy if and only if ω is positive. So *if one wants to treat the Jordan metric as physical, one must require ω to be positive to satisfy the positivity of the Hamiltonian.* Indeed this is precisely what Brans and Dicke did in their original paper [2].

To discuss an inherent ambiguity in the Brans-Dicke theory we consider the following conformal transformation:

$$\gamma_{\mu\nu} \to \gamma'_{\mu\nu} = e^{\beta\sigma}\gamma_{\mu\nu}, \tag{4}$$

where β is an arbitrary constant. Under the conformal transformation we have (up to a total divergence)

$$\mathcal{L}_0 = -\frac{\gamma'^{1/2}}{16\pi G}e^{-\alpha'\sigma}[R' + \omega'\alpha'^2\gamma'^{\mu\nu}(\partial_\mu\sigma)(\partial_\nu\sigma)],$$
$$\mathcal{L}_1 = -\gamma'^{1/2}e^{-\beta\sigma}[\tfrac{1}{2}\gamma'^{\mu\nu}(\partial_\mu\Psi)(\partial_\nu\Psi) + e^{-\beta\sigma}V(\Psi)], \tag{5}$$

where

$$\alpha' = \alpha - \beta,$$
$$\omega' = (\omega\alpha^2 + 3\alpha\beta - \tfrac{3}{2}\beta^2)/(\alpha-\beta)^2.$$

This clearly shows that *as far as $\mathcal{L}_0$ is concerned the Lagrangian is form invariant under the conformal transformation* (when $\alpha \neq \beta$). Only $\mathcal{L}_1$ breaks the conformal invariance. This tells us two things. First, in the absence of any matter field, one cannot tell which conformal frame one is in. There is simply no way of telling the difference between $\gamma_{\mu\nu}$ and $\gamma'_{\mu\nu}$. Second, in the presence of the matter field, one has infinitely many logically acceptable metrics which can describe different physics. Indeed any $\gamma'_{\mu\nu}$ can be chosen as "physical" as long as ω' remains positive. This confirms the fact that the theory has an inherent ambiguity. The only way to resolve the ambiguity is to specify how the physical metric should couple to the matter field. Brans and Dicke have chosen the Jordan metric as physical by insisting that the physical metric must couple to the matter field "normally" without any conformal factor (insisting that the Brans-Dicke scalar field should not couple to the matter field). Similarly in the generalized Brans-Dicke theory [4], the authors have chosen the Jordan metric as physical by insisting that the physical metric must couple normally only to the ordinary matter.

Now we show that the physical metric so chosen does *not* represent the massless spin-two graviton, and thus cannot be identified as the metric which is responsible for Einstein's theory of gravitation. To show this we expand the Jordan metric around the vacuum and let

$$\gamma_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{6}$$

where $\eta_{\mu\nu}$ is the flat Minkowski metric. With this we find

$$\mathcal{L}_0 = -\frac{1}{16\pi G}\left[\frac{1}{4}(\partial_\mu\rho_{\alpha\beta}) - \frac{1}{4}(\partial_\mu\rho_{\alpha\alpha})(\partial_\mu\rho_{\beta\beta}) + \frac{1}{2}(\partial_\mu\rho_{\mu\nu})(\partial_\nu\rho_{\alpha\alpha}) - \frac{1}{2}(\partial_\mu\rho_{\mu\alpha})(\partial_\nu\rho_{\nu\alpha}) + \frac{2\omega+3}{2}\alpha^2(\partial_\mu\sigma)(\partial_\mu\sigma)\right]$$
$$+\text{interactions}, \tag{7}$$

where all the contractions of the space-time indices are made with the flat metric and

$$\rho_{\mu\nu} = h_{\mu\nu} + \alpha\sigma\eta_{\mu\nu}. \tag{8}$$

With the normalization $\alpha = 1/\sqrt{2\omega+3}$ the Lagrangian (7) describes the generally invariant gravitational interaction between the massless spin-two field $\rho_{\mu\nu}$ and the (normalized) dilaton field σ. This means that it is $\rho_{\mu\nu}$, but not $h_{\mu\nu}$, which describes the massless graviton. Notice that $h_{\mu\nu}$ does not even describe a mass eigenstate. From this we conclude that *the metric which is responsible for Einstein's theory of gravitation is not $\gamma_{\mu\nu}$, but $g_{\mu\nu}$, given by*

$$g_{\mu\nu} = e^{(\sigma/\sqrt{2\omega+3})}\gamma_{\mu\nu}, \tag{9}$$

which we call the Pauli metric [6].

In terms of the Pauli metric the Brans-Dicke Lagrangian is neatly written as

$$\mathcal{L}_{BD} = -\frac{\sqrt{g}}{16\pi G}[R + \tfrac{1}{2}g^{\mu\nu}(\partial_\mu\sigma)(\partial_\nu\sigma)]$$
$$+\sqrt{g}e^{-\alpha\sigma}g^{\mu\nu}[\tfrac{1}{2}(\partial_\mu\Psi)(\partial_\nu\Psi) + e^{-\alpha\sigma}V(\Psi)]. \tag{10}$$

Notice that *the Pauli metric can be defined even when ω is negative, as long as ω becomes larger than $-\frac{3}{2}$.* More importantly *the gravitational coupling to the dilatonic matter becomes "normal," but to the ordinary matter "abnormal," after the conformal transformation.* This means that the Lagrangian (10) describes a "new" Brans-Dicke theory. In the "old" Brans-Dicke theory the gravitational coupling to the matter field is such that the motion of an ordinary particle should follow the geodesic determined by the Jordan metric [2]. In contrast in the new theory it is the dilatonic particle (not the ordinary particle) which should move along the geodesic, but now along the geodesic determined by the Pauli metric.

It has often been claimed that the Brans-Dicke theory can be derived from Kaluza-Klein unification. This claim has served to provide a (much needed) "philosophical" justification for the Brans-Dicke theory [11]. However, we emphasize that this claim should be taken with a great care, because this claim becomes true *only if* the Pauli metric is identified as physical. This means that the Brans-Dicke theory in the strict sense does *not* follow from Kaluza-Klein unification. To see this consider a $(4+n)$-dimensional Kaluza-Klein unification with the unified metric γ_{AB} $(A,B=1,2,\ldots,4+n)$, where the internal space is made of an n-dimensional isometry group which acts freely on the $(4+n)$-dimensional unified manifold [8,9]. Let $\gamma_{AB} = \gamma_{\mu\nu} \oplus \phi_{ab}$ ($\mu,\nu = 1,2,3,4;$

$a,b=5,6,\ldots,4+n$) in a block-diagonal basis, where $\gamma_{\mu\nu}$ is what we identify as the four-dimensional Jordan metric and ϕ_{ab} is the n-dimensional internal metric. Now let us define the Kaluza-Klein scalar field $\tilde{\phi}$ and the normalized internal metric ρ_{ab} by

$$\tilde{\phi}=|\det\phi_{ab}|,\quad \rho_{ab}=\tilde{\phi}^{-1/n}\phi_{ab}\quad (|\det\rho_{ab}|=1)\,. \tag{11}$$

Then the isometry automatically reduces the $(4+n)$-dimensional Einstein-Hilbert Lagrangian to the following Cho-Freund Lagrangian [8,9]:

$$\mathcal{L}_{\rm CF}=-\frac{\sqrt{\gamma}}{16\pi G}\tilde{\phi}^{1/2}\left[R+4\pi G\tilde{\phi}^{1/n}\rho_{ab}\gamma^{\mu\alpha}\gamma^{\nu\beta}F_{\mu\nu}{}^{a}F_{\alpha\beta}{}^{b}-\frac{n-1}{4n}\gamma^{\mu\nu}\frac{(\partial_\mu\tilde{\phi})(\partial_\nu\tilde{\phi})}{\tilde{\phi}^2}-\frac{1}{4}\gamma^{\mu\nu}(D_\mu\rho^{ab})(D_\nu\rho_{ab})\right.$$
$$\left.+R(\rho_{ab})+\Lambda+\lambda(|\det\rho_{ab}|-1)\right], \tag{12}$$

where $R(\rho_{ab})$ is the internal curvature obtained with ρ_{ab}, $F_{\mu\nu}{}^{a}$ is the Kaluza-Klein gauge field, Λ is a $(4+n)$-dimensional cosmological constant, and λ is a Lagrange multiplier. Notice that as far as the Kaluza-Klein scalar field is concerned, (12) looks very much like the Brans-Dicke Lagrangian. Indeed with

$$\phi=(1/16\pi G)\tilde{\phi}^{1/2},\quad \omega=-(n-1)/n\,,$$

one finds

$$\mathcal{L}_{\rm CF}=-\sqrt{\gamma}\left[\phi R+\omega\gamma^{\mu\nu}\frac{(\partial_\mu\phi)(\partial_\nu\phi)}{\phi}+\text{other terms}\right]. \tag{13}$$

However, there are in fact two fundamental differences between (1) and (12). First, *the Jordan metric in (12) couples abnormally to the matter fields.* Indeed none of the matter fields couples normally to the Jordan metric. Second, *the Jordan metric couples to the Kaluza-Klein scalar field with a negative* ω. This shows us that there is no way to guarantee the unitarity (i.e., the positive definiteness of the Hamiltonian) of the Lagrangian (12), if one tries to identify the Jordan matrix as physical. This clearly rules out the Jordan metric as unphysical [9,10].

Notice that although $\omega=-(n-1)/n$ is not positive definite, it remains larger than -1 for any positive integer n. This means that the unitarity of the Lagrangian (12) can be assured with a simple conformal transformation of the metric. To see this we introduce the Pauli metric $g_{\mu\nu}$ and the Kaluza-Klein dilaton σ by

$$g_{\mu\nu}=\tilde{\phi}^{1/2}\gamma_{\mu\nu},\quad \sigma=\tfrac{1}{2}\sqrt{(n+2)/n}\,\ln\tilde{\phi}\,, \tag{14}$$

and find in terms of the new metric [9]

$$\mathcal{L}_{\rm CF}=-\frac{\sqrt{g}}{16\pi G}\Big[R+4\pi G e^{[(n+2)/n]^{1/2}\sigma}\rho_{ab}F_{\mu\nu}{}^{a}F_{\mu\nu}{}^{b}+\tfrac{1}{2}(\partial_\mu\sigma)^2+R(\rho_{ab})e^{-[(n+2)/n]^{1/2}\sigma}+\Lambda e^{-[n/(n+2)]^{1/2}\sigma}$$
$$-\tfrac{1}{4}(D_\mu\rho^{ab})(D_\mu\rho_{ab})+\lambda(|\det\rho_{ab}|-1)\Big]\,. \tag{15}$$

This proves that the Pauli metric not only restores the unitarity, but more importantly describes the massless spin-two graviton.

We conclude with the following remarks.

(A) It has been a tradition to treat the Jordan metric as physical in the Brans-Dicke theory, ever since Brans and Dicke proposed to do so. The justification for this is that by construction the Jordan metric is the one which makes the ordinary matter (and *only* the ordinary matter) satisfy the so-called weak equivalence principle [2]. This means that the ordinary matter must follow the geodesic determined by the Jordan metric. But we emphasize that the weak equivalence principle based on the Jordan metric is *not* a first principle and is subject to the following criticisms. First of all it is *not* universal, because it does not apply to all matter fields equally. Obviously the dilatonic matter (as well as the dark matter in the generalized Brans-Dicke theory) does not follow the geodesic determined by the Jordan metric. Second, it is *ad hoc*, because *a priori* there is no reason why only the ordinary matter should follow the geodesic determined by the physical metric. It is perfectly possible that the dilatonic matter (or the dark matter) could play a more "fundamental" role than the ordinary matter in our Universe. If so, the physical metric could turn out to be the one which makes the dilatonic matter (or the dark matter), not the ordinary matter, to satisfy the weak equivalence principle.

(B) A fundamental assumption in the theory of gravitation is that *the gravitational interaction is generated by the massless spin-two graviton. Once one accepts this proposition, one must accept the Pauli metric as the physical space-time metric in the Brans-Dicke theory.* This is unavoidable especially when one wishes to compare Einstein's theory with the Brans-Dicke theory. This necessitates a completely new reinterpretation of the Brans-Dicke theory, in particular the "old" Brans-Dicke cosmology [3,4] which identifies the Jordan metric as physical. As importantly our analysis shows that in the Brans-Dicke theory there must exist a fifth force acting on the ordinary matter which could violate the equivalence principle. This is so because the ordinary particles in the Brans-Dicke theory must follow the geodesic determined by the Jordan metric.

(C) In the Kaluza-Klein theory the unitarity rules out the Jordan metric as unphysical. More importantly, here again *one must identify the Pauli metric as physical, as long as the purpose of the higher-dimensional unification is to unify Einstein's theory of gravitation with other interactions.* This means that all the results of the "old" Kaluza-Klein cosmology which identifies the Jordan metric as physical [11,12] should be reanalyzed in terms of the Pauli metric. Remarkably the "new" unified cosmology [14] provides us with an attractive alternative to the big-bang cosmology which could circumvent the major defects of the standard model without necessarily compromising its successes. First of all, Dirac's conjecture can naturally be realized in the unified cosmology. Second, the dilatonic matter could couple more strongly to gravitation than the ordinary matter, and thus could play the role of the dark matter of the universe. Finally the unique dilatonic potential could allow us to have a *generalized inflation* [14] in which a universe without any horizon becomes possible. In fact, if necessary, the dilaton could play the role of the "inflaton."

(D) A most remarkable consequence of our analysis is that it confirms the existence of a fifth force in the Kaluza-Klein theory which could violate the equivalence principle [13]. To find out what kind of fifth force one can expect from the Kaluza-Klein theory, we notice first that the dilaton must have a very small mass because it appears as a pseudo Goldstone particle in the unified theory [15]. Furthermore the dilaton modifies gravitation because it modifies Newton's constant. This implies that *the Kaluza-Klein dilaton is most likely to create a "medium-range" fifth force which couples "weakly" to the matter field.* We notice that this prediction is perfectly compatible with the present experimental limit on the fifth force [16].

(E) Finally we emphasize that the above conclusions on the Kaluza-Klein cosmology and the fifth force must apply to all the unified theories which are based on higher-dimensional unification, including the supergravity and the superstring. So the prediction on the Kaluza-Klein fifth force could provide a straightforward means to test the validity of the very idea of higher-dimensional unification.

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