Radio-Frequency Beam Conditioner for Fast-Wave Free-Electron Generators of Coherent Radiation

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A method for conditioning electron beams is proposed to enhance gain in resonant electron-beam devices by introducing a correlation between betatron amplitude and energy. This correlation reduces the axial-velocity spread within the beam, and thereby eliminates an often severe constraint on beam emittance. Free-electron-laser performance with a conditioned beam is examined and analysis is performed of a conditioner consisting of a periodic array of FODO channels and idealized microwave cavities excited in the TM_{210} mode. Numerical examples are discussed.

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In recent years there has arisen quite a variety of concepts for the generation of coherent radiation from electron beams, including the free-electron laser (FEL), the cyclotron autoresonant maser, and others [1]. In such devices, the primary limitation on gain, and the subject of an extensive literature, is the spread in axial velocity within the beam [2]. For this reason much effort has been invested in producing highly monoenergetic beams [3], with the result that performance is most typically limited by *emittance*, which produces a nonzero angular divergence of the beam and a distribution in axial velocities [4-7].

In this Letter, we outline and demonstrate the efficacy of a beam "conditioner" which can greatly enhance gain by *removing* the axial-velocity spread of a finite-emittance nearly monoenergetic beam.

Before describing and analyzing the conditioner, we consider first the kind of conditioning required and its effect on gain. For definiteness we examine a short-wavelength planar FEL with curved pole faces, following the theoretical treatment of Refs. [5] and [6]. The beam will be characterized by some initial distribution f_0, in betatron amplitude and energy, with central energy $mc^2\gamma_0$, where m is the electron mass and c is the speed of light. In traversing the wiggler, electrons oscillate transversely with wiggler period λ_w, and undergo betatron oscillations about the design orbit with betatron period λ_β. We take the electromagnetic signal to be characterized by angular frequency ω and electric field amplitude E varying with the radial coordinate R, and the axial coordinate z. The dispersion relation for linear gain is determined from the radial mode equation

$$[\nabla_\perp^2+\mu+U(\Omega,q,r)]E=0\,, \qquad (1)$$

where $\mu=\Omega-q$, with Ω and q the Fourier-transform variables conjugate to k_wz and $\xi=(k_r+k_w)z-\omega t$. The wiggler wave number $k_w=2\pi/\lambda_w$, the normalized radial coordinate $r=R(2k_rk_w)^{1/2}$, the Laplacian is $\nabla_\perp^2$, and time is t. The axial wave number at resonance, k_r, is determined by the peak wiggler magnetic field B. The susceptibility U is given by

$$U=4\gamma_0^2\rho^3\int d\hat{\gamma}\,d^2p\,\frac{\partial f_0(\hat{\gamma},J)}{\partial\hat{\gamma}}\,\frac{1}{\Omega-2(1+q)(\hat{\gamma}-J)}\,, \qquad (2)$$

where ρ is the Pierce parameter [8], $\hat{\gamma}=(\gamma-\gamma_0)/\gamma_0$ is the fractional deviation in energy, and $J=(\kappa^2r^2+p^2)/8$ is a measure of the energy of the betatron motion. Here $p=dr/d(k_wz)$ is the normalized canonical momentum, $\kappa=k_\beta/k_w$, and $k_\beta=2\pi/\lambda_\beta$. Equations (1) and (2) assume that betatron oscillations are slow on the scale of a gain length $L_G=[k_w\,\mathrm{Im}(\mu)]^{-1}$.

As one can see from the denominator of Eq. (2), growth is reduced by an uncorrelated spread in $\hat{\gamma}$ and J. This effect is small when $\hat{\gamma},J\ll\rho$, i.e., when the beam-energy spread $mc^2\sigma$ and emittance ε satisfy the well-known constraints $\sigma/\gamma\ll\rho$, and

$$\varepsilon\ll\frac{\lambda_r}{2\pi}\left(\frac{\rho k_w}{k_\beta}\right), \qquad (3)$$

where $\lambda_r=2\pi/k_r$ is the resonant signal wavelength.

In practice, while small σ represents a tolerable constraint, Eq. (3) is exceedingly stringent at short wavelengths. On the other hand, the form of the denominator in Eq. (2) suggests that one could eliminate the constraint, Eq. (3), by transforming or "conditioning" the distribution f_0 according to

$$f_c(\hat{\gamma},J)=f_0(\hat{\gamma}-J,J)\,. \qquad (4)$$

With such a conditioned beam, FEL gain may be computed directly from Eqs. (1) and (2), or, more generally, following the treatment of Ref. [6], which takes into account energy spread, emittance, and focusing of the electron beam [9], and the diffraction and guiding [10] of the radiation. In fact, the only change to the work of Ref. [6] is to delete the term $3is(\kappa/D)(k_r\varepsilon)$ in their Eq. (10). The resulting growth rate is given by

$$\mathrm{Im}(\mu)=DG\left(k_r\varepsilon,\frac{\sigma}{\gamma D},\frac{k_\beta}{k_wD},\frac{\omega-\omega_r}{\omega_rD}\right), \qquad (5)$$

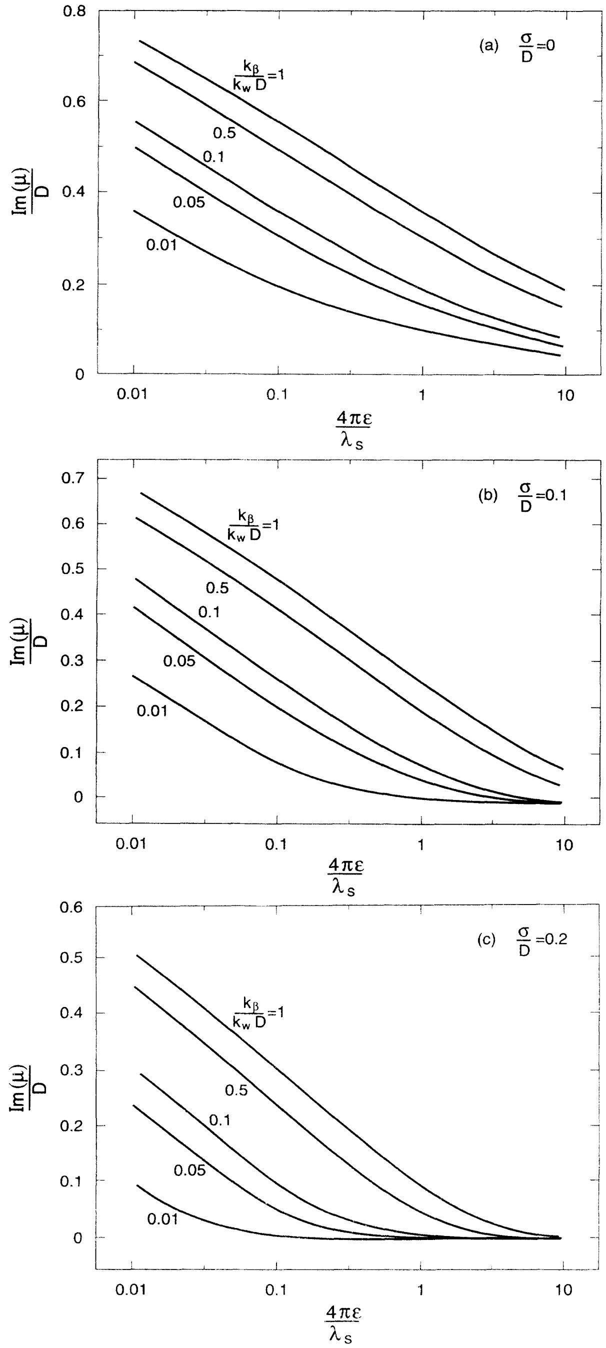

FIG. 1. Normalized gain vs emittance for several values of $k_\beta/k_w D$ corresponding to energy spreads (a) $\sigma/D=0$, (b) $\sigma/D=0.1$, and (c) $\sigma/D=0.2$.

where $\omega_r = ck_r$ is the resonant angular frequency, and $D=(2\rho)^{3/2}a$, with a the normalized beam radius $a^2 = 12k_r\varepsilon k_w/k_\beta$. The dependence of G upon its first three arguments is given in Fig. 1, which in effect replaces Fig. 1 of Ref. [6] [11].

Next we consider a device or conditioner which will produce the transformation of Eq. (4), i.e., a net "boost" in energy proportional to the squared betatron amplitude.

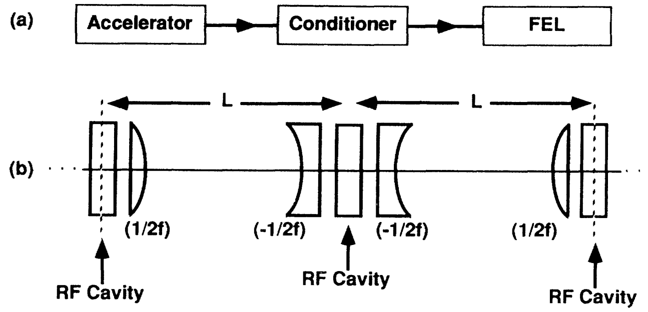

FIG. 2. (a) The beam conditioner is a periodic microwave and magnetic lattice located between the accelerator and the FEL. As depicted in (b), each period consists of two focusing lenses, two defocusing lenses, and two rf cavities operated in the TM_{210} mode.

(Note that this is not beam "cooling," but merely a volume-conserving redistribution in phase space.) This device is to be placed between the accelerator producing the beam and the FEL, as depicted in Fig. 2(a). We consider the simplest example of such a conditioner corresponding to a periodic lattice, with period as depicted in Fig. 2(b), consisting of a FODO array and suitably phased microwave cavities operating in the TM_{210} mode.

In the thin-lens approximation a matched beam will have maximum squared waist $\langle x^2\rangle_+$ at the focusing lenses and minimum $\langle x^2\rangle_-$ at the defocusing lenses, where, in terms of the focal length f and separation L,

$$\langle x^2\rangle_\pm = 2f\varepsilon_x\left(\frac{2f\pm L}{2f\mp L}\right)^{1/2} = \varepsilon_x\beta_\pm \,, \qquad (6)$$

with ε_x the rms beam emittance in x. The analogous result holds for $\langle y^2\rangle_\pm$ with the signs reversed; i.e., $\langle y^2\rangle_+$ will be largest at the defocusing lenses.

To determine the effect of the cavities we consider first a point bunch phased to pass through each cavity at a null in the deflecting fields. A particle in this bunch is unperturbed transversely, but gains energy $\Delta\gamma_+ = \alpha(x^2 - y^2)$, with α a constant that depends on cavity parameters. Similarly, with a 180° phase change, at the defocusing lens, $\Delta\gamma_- = -\alpha(x^2-y^2)$. In passing through N periods of a conditioner, a particle of initial amplitude of oscillation characterized by emittance ε and initial phases φ_{0x} and φ_{0y} will gain energy:

$$\Delta\gamma = \alpha\varepsilon\sum_{n=1}^{N}\{\beta_+\sin^2(n\delta+\varphi_{0x}) - \beta_-\sin^2(n\delta+\varphi_{0y})\}$$
$$-\alpha\varepsilon\sum_{n=1}^{N}\{\beta_-\sin^2([n+\tfrac{1}{2}]\delta+\varphi_{0x})$$
$$-\beta_+\sin^2([n+\tfrac{1}{2}]\delta+\varphi_{0y})\}\,, \qquad (7)$$

where δ, the phase advance for a period, is given by $\cos\delta = 1 - L^2/2f^2$. In the limit of large N there results an averaging over oscillations and $\Delta\gamma \to \Delta\gamma_c = \alpha\varepsilon(\beta_+$

$-\beta_-)N$. For finite N, one may show that the *maximum* deviation in $\Delta\gamma$ from the ideal converges fairly rapidly,

$$\left|\frac{\Delta\gamma}{\Delta\gamma_c}-1\right| \leq \left|\frac{\sin(N\delta)}{N\sin(\delta)}\right| \left[\cos^2\left(\frac{\delta}{2}\right)+\frac{4f^2}{L^2}\sin^2\left(\frac{\delta}{2}\right)\right]^{1/2}. \tag{8}$$

Thus the periodic lattice of Fig. 2 will in fact produce a boost in energy proportional to emittance. The cavity parameter α is then determined from Eq. (4):

$$\alpha=\left(\frac{\gamma k_r k_\beta}{2k_w}\right)\frac{1}{N(\beta_+-\beta_-)}. \tag{9}$$

Here the first factor contains only FEL parameters, and the second only conditioner parameters. The average boost is $\Delta\gamma_c \sim k_r k_\beta \varepsilon_n/2k_w$ with $\varepsilon_n=\gamma\varepsilon$ the rms normalized emittance.

These results assume a point bunch. For a finite bunch length l, there will in addition be a small sweep in energy from head to tail, $\Delta\gamma_l=-(\theta^2/2)\Delta\gamma_c$, where $\theta=\omega_c l/c$ and ω_c is the cavity angular frequency. More significantly, from the Panofsky-Wenzel theorem one expects a radio-frequency quadrupole (RFQ) effect with a phase-dependent focal length of order $f_l \sim \gamma/2\alpha l$. As a result the beam head and tail will have slightly different lattice parameters and will be mismatched upon injection. We will consider only the limit $f_l \gg f$, where this effect is small. In general one expects that this effect can be eliminated with proper matching at the conditioner entrance and exit, for example, with an RFQ [12].

In addition, a finite bunch will experience a coupling of head to tail due to "beam breakup" (BBU) modes [13] of the conditioner cavities. To bound this effect, we require that the BBU growth length L_B be larger than the conditioner length. Moreover, alignment errors should be small compared to the beam spot size in the conditioner. To avoid multibunch beam breakup the bunch separation should be several microseconds.

Finally, to minimize the device length, it is useful to consider two modifications to Fig. 2. First, the treatment given assumes a small transit angle through the cavity. For the examples, we will fix $\omega d/c \sim 30°$, where d is the cavity length. However, this corresponds to a rather short interaction length ($d\sim 5$ mm, for a 5-cm cavity radius) and favors increasing the number of cavities per half period N_c. The treatment of the beam optics goes through as before provided $N_c d \ll L$. In addition, beam coupling to the TM_{210} mode depends on the beam spot size and favors conditioning at a lower energy $mc^2\gamma_c < mc^2\gamma_0$, and larger spot size.

Next, to appreciate the improvement in FEL performance and the conditioner scalings, we consider several examples, as listed in Table I. The beam current, energy spread, and emittance correspond to much less demanding performance than that of existing photocathodes [3]. The FEL data derive from the scaling laws of Fig. 1 and have been checked with a many-particle FEL simulation, which gives results in close agreement with the scaling laws [14]. In the infrared example the gain length is reduced by a factor of 5. In the ultraviolet example the energy of the beam is reduced by a factor of 3 (reducing the cost of the accelerator by this factor) while the gain length is reduced by more than a factor of 2 (reducing the cost of the wiggler by the same factor). The saving in cost is even larger than these numbers indicate, for with the shorter wiggler, magnet errors are less important and manufacturing tolerances are reduced. The effect of conditioning is quite dramatic.

For the conditioner parameters we have somewhat arbitrarily fixed the operating frequency at $\omega_c/2\pi \sim 5$ GHz, the quality factor $Q\sim 10^4$, $L\sim 50$ cm, $\delta\sim 150°$, and the bunch length $l\sim 1$ mm. The number of periods has been chosen to insure that the total power input per cavity is less than 5 MW, and $f_l \gg f$. Head-tail beam breakup will be acceptable for a beam aperture of 1 cm diameter.

Finally, note that in this analysis k_β has been determined by the "natural" focusing of the FEL. However,

TABLE I. Parameters for several example FEL designs, with and without a conditioned beam. Current is fixed at $I\sim 300$ A, with energy spread $\sigma/\gamma \sim 4.4\times10^{-4}$. In each case k_w was varied to minimize L_G.

	10 μm	Conditioned	3000 Å	Conditioned	500 Å	Conditioned
$mc^2\gamma_0$ (MeV)	54	54	483	153	1004	304
ε_n (m)	$8\times10^{-4}\pi$	$8\times10^{-4}\pi$	$5\times10^{-5}\pi$	$5\times10^{-5}\pi$	$2\times10^{-5}\pi$	$2\times10^{-5}\pi$
λ_β (m)	8.9	8.9	20	12	34	19
λ_w (cm)	8.0	8.0	4.8	2.8	3.7	2.0
B (T)	0.25	0.25	1.0	0.52	1.26	0.66
$L_G/2$ (m)	8.0	1.6	3.1	1.4	4.6	2.1
$mc^2\Delta\gamma_c$ (MeV)		3.6		2.0		2.1
$mc^2\gamma_c$ (MeV)		54		51		51
N		10		20		50
N_c		1		10		10
L_c (m)		10		20		50

having removed the constraint on emittance there is every reason to consider other, stronger focusing mechanisms, for example, ion focusing [6,15]. We have explored this possibility at a variety of wavelengths and even in the "water window" where wet biological samples are transparent and where an intense coherent source would be of great interest for imaging [16]. We find that while the potential improvement in FEL performance is great, the simple conditioner we have considered here is inadequate. For example, for a 30-Å FEL, with $I \sim 80$ A, $mc^2\gamma_0 \sim 1240$ MeV, $\varepsilon_n \sim 2\times10^{-6}\pi$ m, $\lambda_w \sim 2$ cm, $B \sim 0.66$ T, and plasma density $n_p \sim 1.5\times10^{13}$ cm^{-3}, we find extremely high gain, $L_G/2 \sim 2.1$ m (without conditioning $L_G/2 \sim 26$ m). However, $mc^2\Delta\gamma_c \sim 17$ MeV and the corresponding conditioner would be several hundred meters long [17].

The broader conclusion from this kind of analysis is that conventional microwave linacs and focusing lattices are not optimally designed as FEL drivers. We have in some sense demonstrated this "by construction," albeit a simple construction; we are optimistic that more sophisticated conditioner designs will make feasible more compact conditioners, and ultimately high-gain FEL operation in the x-ray regime.

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