# Crossover Phenomenon for Hopping Conduction in Strong Magnetic Fields

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We find that strong magnetic fields reduce the effectiveness of the Coulomb gap in determining the variable-range-hopping (VRH) resistivity $\rho$. We propose a crossover phenomenon characterized by a temperature $T_c$, with the Coulomb gap affecting the VRH resistivity only below $T_c$. The value of $T_c$ decreases as the magnetic field increases. Our theoretical model explains the observed crossover from a $\ln\rho(H=0)\propto T^{-1/2}$ law to a $\ln\rho(H)\propto[T_0(H)/T]^{1/3}$ law with $T_0\propto H$.

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The resistivity of an Anderson insulator at low temperatures follows Mott's variable-range-hopping (VRH) law [1]

$$\rho=\rho_0\exp[(T_0/T)^p]\,, \qquad (1)$$

with $p=\frac{1}{4}$ in a three-dimensional system. Pollak [2] and Efros and Shklovskii [3] have predicted that long-range electron-electron interactions reduce the density of states (DOS) at the Fermi energy, forming the so-called Coulomb gap (CG) [3,4]. The existence of the CG manifests itself in a different VRH law [4] in which $p=\frac{1}{2}$ in Eq. (1). Experimental evidence for the CG was previously demonstrated on different materials [4]. The appearance of the CG was demonstrated [5,6] by applying a magnetic field to a material in the metallic state and driving it into the insulating state. The Mott VRH law with $p=\frac{1}{4}$ is first obtained [6] near the metal-insulator transition. As the magnetic field is further increased, the electron states become more localized and a CG is formed. In this regime, the resistivity changes [6] to the $\ln\rho\propto T^{-1/2}$ law.

In this Letter, we describe an experiment in which the $\ln\rho\propto T^{-1/2}$ is observed at zero magnetic field. We then applied a strong magnetic field and found, surprisingly, that $p=\frac{1}{3}$ and $T_0\propto H$, which is predicted theoretically *only* for a *nonzero* DOS. Our experiment clearly demonstrates that the Coulomb gap becomes ineffective in determining the VRH resistivity in strong magnetic fields. We present a theory in which the Coulomb gap affects the VRH resistivity only below a temperature $T_c$. As the magnetic field increases, $T_c$ is reduced. This diminishes the role of the CG which will show up only at extremely low temperatures. Our calculated "phase diagram" of $T_c$ vs $H$ is consistent with the present data.

Our interpretation and theory of the CG relies on the fact that the single-particle DOS vanishes at the Fermi energy. Effects of many-body electron-electron interactions as discussed, for example, by Knotek and Pollak [2] are ignored. These effects were recently reexamined by Mochena and Pollak [2].

Our Ge:As samples were prepared from single crystals of germanium, doped by the shallow donor impurity arsenide. In order to measure the resistivity of a semiconductor at very low temperatures, the concentration of impurities $n$ must be very close to the Mott concentration $n_c$ at which a metal-insulator transition takes place. The two samples studied in the present work have an impurity concentration of $n=3.2\times10^{17}$ cm$^{-3}$, which is close to the critical concentration of $n_c=3.5\times10^{17}$ cm$^{-3}$ for Ge:As. Since $n=0.9n_c$, it allows us to prolong the measurements of the resistance down to 30 mK. Because we are quite near the metal-insulator transition, the control upon the level of doping must be carried out to high accuracy. Highly uniform dopant concentrations can be obtained through neutron transmutation doping (NTD) in which three of the five stable isotopes of Ge are transmuted by nuclear reactions with thermal neutrons:

$$\begin{aligned}&{}^{70}\mathrm{Ge}(n,\gamma)^{71}\mathrm{Ge}\to{}^{71}\mathrm{Ge},\ \text{shallow acceptor}\,,\\&{}^{74}\mathrm{Ge}(n,\gamma)^{75}\mathrm{Ge}\to{}^{75}\mathrm{As},\ \text{shallow donor}\,,\\&{}^{76}\mathrm{Ge}(n,\gamma)^{77}\mathrm{Ge}\to{}^{77}\mathrm{As}\to{}^{77}\mathrm{Se},\ \text{deep donor}\,.\end{aligned} \qquad (2)$$

The ratio of the concentration of the different dopants is determined by the thermal-neutron cross section and the abundance of the dopant production of isotopes. For natural Ge, the NTD technique leads [7] to creation of a $p$-type material with 40% compensation. In order to obtain $n$-type materials with small compensation, a specially grown crystal of Ge was used, enriched artificially in the isotope $^{74}$Ge. As a result, a series of samples of $n$-Ge:As was obtained [8]. Fused contacts to $n$-Ge were prepared by alloying In+2% As and Ge at 300 °C in vacuum $10^{-5}$ torr during 1 min. Ohmic behavior of contacts was checked at each measured temperature.

We have measured $\rho(T,H)$ and deduced the magnetic-field dependence of $p$ and $T_0$ by using a four-probe dc method and a two-probe ac bridge. The temperature was measured by GR-200 germanium and RF-80 rhodium-iron resistance temperature sensors. The four-probe dc

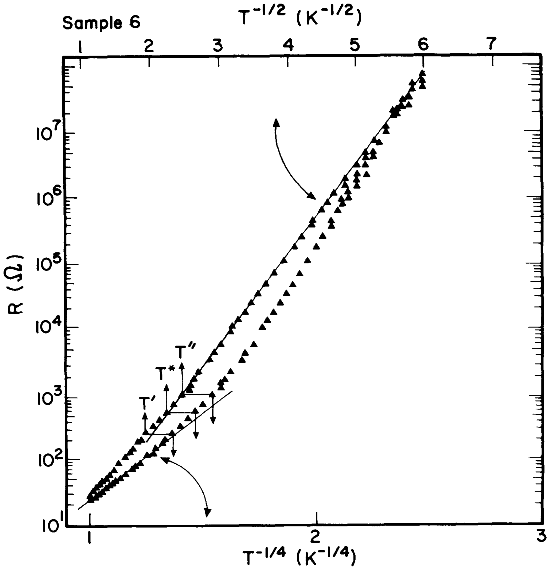


FIG. 1. The temperature dependence of the resistance as a function of $T^{-1/2}$ (upper scale) and as a function of $T^{-1/4}$ (lower scale) at zero magnetic field.

method was used down to 150–200 mK. The measuring current did not exceed 1 nA; the power levels were $P \leq 10^{-11}$ W. At lower temperatures the two-probe ac bridge was used in order to reduce the power level. At some temperatures resistance measurements were made by both dc and ac methods to assure their equivalence.

In Fig. 1, we plot $\rho(T,H=0)$ as a function of $T^{-1/2}$ (upper scale). We see that for $T<0.17$ K, the data clearly support the existence of a Coulomb gap. In order to determine the best exponent, the value $p$ was varied and then fitted to minimize the least-squares deviation $\chi^2 = \sum[\ln(\rho_{\rm exp}) - \ln(\rho_{\rm calc})]^2$. The result of such an analysis for the two samples studied is shown in Fig. 2. It is clear that at zero magnetic field there is a sharp minimum at $p=0.54$ (which is close to the theoretical value $p=\frac{1}{2}$). Upon increasing $H$, the minimum is shifted to lower values and at $H \geq 30$ kOe, $p$ lies between 0.3 and 0.35, and is thus close to the theoretical value $p=\frac{1}{3}$.

In Fig. 3, we plot $\ln\rho(T,H)$ as a function of $T^{-1/3}$ for magnetic fields $H \geq 30$ kOe and show that it follows a straight line (the deviations for lower resistivities at the low-temperature end are connected with self-heating effects, which were more active at strong magnetic fields, probably because of an additional noise induction). From the slopes of the curves we obtain the values of $T_0$ (we indicate it as $T_{1/3}$) which is proportional to $H$ (see inset of Fig. 3). This is very surprising because it contradicts current theories [4,9] which are summarized in the table below. The exponent $p$ is given for the case of $g(\epsilon_F) \neq 0$ and for the CG case for different values of $x$ which determines the form of the exponential falloff of the localized

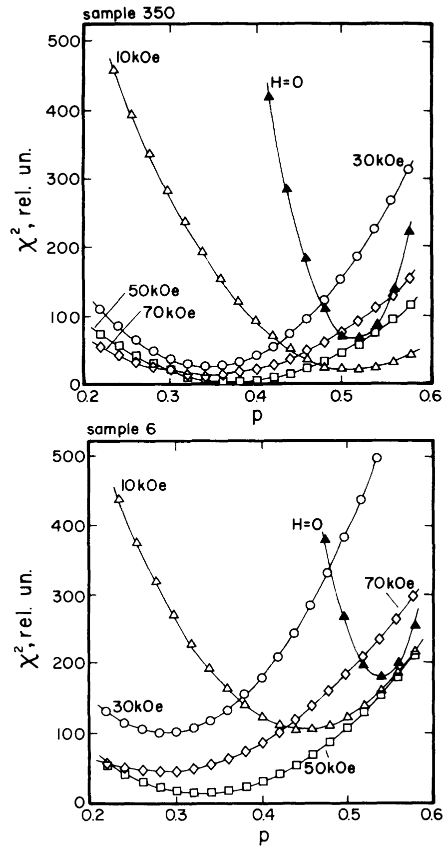


FIG. 2. The least-squares deviation $\chi^2$ as a function of $p$ for different magnetic fields for the two samples studied.

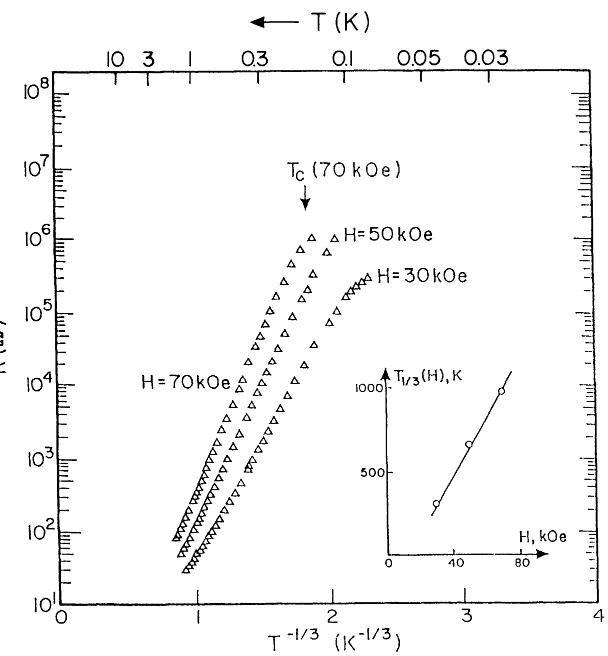


FIG. 3. The temperature dependence of the resistance as a function of $T^{-1/3}$ at strong magnetic fields. Inset: The dependence of the slope $T_{1/3}$ on magnetic field.

wave function $\phi(r) \sim \exp[-(r/b)^x]$:

| | $x$ | $g(\epsilon_F)\neq 0$ | $g(\epsilon_F)=0$ |
|---|---|---|---|
| $H=0$ | 1 | $p=\frac{1}{4}$ | $p=\frac{1}{2}$ |
| Strong $H$ | 2 | $p=\frac{1}{3}$, $T_0 \propto H$ | $p=\frac{3}{5}$, $T_0 \propto H^{1/3}$ |

In a magnetic field the radial part of the wave function perpendicular to the direction of the magnetic field is given by $\phi(r) \propto \exp[-(r/b)^x]$, where $b$ is a function of magnetic field and disorder [4]. For strong magnetic fields [4,9], $x=2$.

Our experiment is in the strong-magnetic-field regime and according to the table, we expect $p$ to *increase* from $p=\frac{1}{2}$ to $p=\frac{3}{5}$ with a *weak* magnetic-field dependence of $T_0$, only as $H^{1/3}$. In contrast, we find a *decrease* in $p$ from $p=\frac{1}{2}$ to $p=\frac{1}{3}$ with a *strong* magnetic-field dependence of $T_0$, being proportional to $H$. Such behavior was predicted theoretically by Shklovskii and Efros [see their Eqs. (9.2.8) and (9.2.9) in [4]] for the case of $g(\epsilon_F)\neq 0$, and relies on the fact that the radial part of the wave function falls off as $\phi \sim \exp[-(r/b)^2]$. This was recently confirmed [9] for a two-dimensional system. Our experiment indicates that in a strong magnetic field the effectiveness of the CG is reduced and indeed we find $p=\frac{1}{3}$ and $T_0 \propto H$. If, however, one assumes [4] that $x=1$ in a strong magnetic field, it follows that $p=\frac{1}{2}$ and $T_0 \propto H^{1/2}$ in the case of a CG and $p=\frac{1}{4}$ and $T_0 \propto H^{1/4}$ for $g(\epsilon_F)\neq 0$. Our experiment clearly disagrees with both these possibilities and confirms that in a strong magnetic field $x=2$.

We now turn to a theoretical treatment of the effectiveness of the Coulomb gap in a strong magnetic field. The CG is effective only at low temperatures $k_B T < \epsilon_c$, where $\epsilon_c$ is the energy at which the density of states turns into its unperturbed value of $g(\epsilon_F)$. At relatively high $T$, the width of the "optimal band" $\Delta(T)$ of localized levels which are involved in the hopping conductivity is much larger than $\epsilon_c$ and one may neglect the influence of the CG on the conductivity. This means that with decreasing temperature the crossover from Mott's VRH law with $p=\frac{1}{4}$ to the CG law with $p=\frac{1}{2}$ must be observed. This can be characterized by a "crossover temperature" $T_c$ for which the CG is effective only for $T<T_c$. One can determine $T_c$ from the equation $\Delta(T_c)=\epsilon_c$. Using $\Delta(T)=(k_B T)^{3/4}[g(\epsilon_F)\xi^3]^{1/4}$ and $\epsilon_c=(e^2/\kappa)^{3/2} g(\epsilon_F)^{1/2}$, where $\xi$ is the localization length and $\kappa$ the dielectric constant, we get

$$k_B T_c = e^4 g(\epsilon_F)\xi/\kappa^2 . \quad (3)$$

For $T>T_c$, we must recover Mott's VRH law with $p=\frac{1}{4}$.

An equivalent approach is to consider the average hopping distance $R(T)$ by following similar arguments as presented in Ref. [4] [Chap. 9, see, for example, Eqs. (9.1.7) and (9.1.8)]. In this approach $R(T)$ increases as $T$ decreases. Consequently, the CG is effective only for $R(T) > R_c$, where $R_c = e^2/\kappa\epsilon_c$ because it corresponds to $k_B T < \epsilon_c$. Using $R(T)=\xi(T_0/T)^p$ and equating $R(T_c)$ with $R_c$ we also get (3). The observation of a crossover effect at zero magnetic field was found by many authors [10].

Let us again discuss Fig. 1, in which we also plot $\rho(T,H=0)$ as a function of $T^{-1/4}$ (lower scale). It is seen that from 1 K down to $T'=0.31$ K, the $T^{-1/4}$ law is observed. This interval of temperatures and the changes in resistance are too small to use the same procedure for the determination of index $p$ as was used for the low-temperature part of the curve.

However, if we assume that there is real "$T^{-1/4}$ law," we can estimate $T_c$ and compare it with experiment. Thus, the experimental "transition interval" from a $T^{-1/4}$ law to a $T^{-1/2}$ law is between $T'=0.31$ K and $T''=0.17$ K with a midpoint of $T_c=\frac{1}{2}(T'+T'')=0.24$ K which we indicate in Fig. 1 as $T^*$. We now determine $T_c$ independently from Eq. (3) by determining $T_0$ in different regions. Theoretically, $T_0$ depends on $p$ and we denote it by $T_p$. Thus, $T_{1/4}$ and $T_{1/2}$ are the slopes of the $T^{-1/4}$ and $T^{-1/2}$ laws, respectively. For Mott's VRH law, $T_{1/4}=21/g(\epsilon_F)\xi^3 k_B$, and in the Efros and Shklovskii CG regime [4], $T_{1/2}=2.8e^2/\kappa\xi k_B$. Equation (3) may be rewritten in terms of the measured temperatures $T_{1/4}$ and $T_{1/2}$ as

$$T_c = 2.7 T_{1/2}^2 / T_{1/4} . \quad (4)$$

From Fig. 1 we obtain $T_{1/4}=1400 \pm 350$ K and $T_{1/2}=10.5 \pm 1.5$ K which leads to $T_c=0.21 \pm 0.04$ K, in very good agreement with the experimental crossover temperature. We now turn to hopping in a strong magnetic field. Here, $\phi(r)$ falls off more rapidly and $x$ changes [4,9] from unity to $x=2$. This reduces the overlap integrals as the magnetic field increases and results in a shorter hopping distance $R(T,H)$, making the CG less effective. Since $R_c$ is an intrinsic property of the gap, it is independent of the magnetic field. This leads to a strong-magnetic-field dependence of $T_c$, since it is determined from the condition $R(T_c,H)=R_c$. The limiting functional dependences of $T_c$ on $H$, as determined from the above condition, are given for weak magnetic fields by

$$T_c(H \to 0) = T_c(0)[1 - 2.4\times 10^{-3}(\xi/l_H)^4 (T_{1/4}/T_{1/2})] \quad (5a)$$

and for strong magnetic fields ($l_H < \xi$) by

$$k_B T_c(H) = \left(\frac{g(\epsilon_F)}{2.1}\right)^{5/4} \left(\frac{3.17e^2}{\kappa}\right)^{9/4} l_H a_H^{1/2} , \quad (5b)$$

where $l_H=(c\hbar/eH)^{1/2}$ is the magnetic length and $a_H$ the decay length of the electron wave function in the mag-

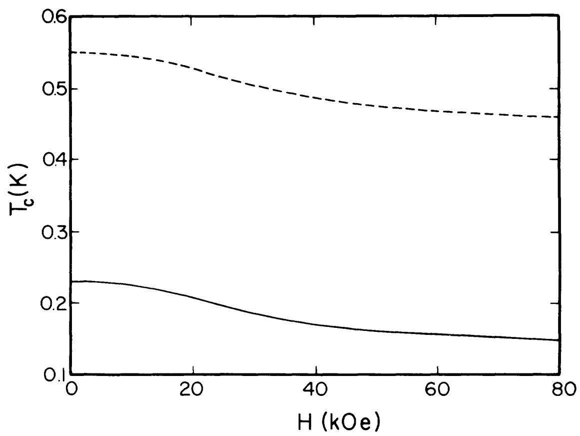


FIG. 4. The calculated dependence of crossover temperature $T_c$ on magnetic field at different scaling parameters. Solid line: $n/n_c=0.90$, $\xi=17.9$ nm; dashed line: $n/n_c=0.78$, $\xi=12.5$ nm.

netic-field direction.

In Fig. 4, we plot $T_c$ as a function of magnetic field. It should be noted that $T_c$ as a function of $H$ depends strongly on the concentration $n_c-n$. This sensitivity arises from the fact that the dielectric constant $\kappa$ diverges at the transition [11]. Using the scaling relation [12,13], it follows that $\kappa \propto \xi^2$ and $T_c(0) \propto 1/\xi^3$.

For strong magnetic fields $T_c(H)$ is much more sensitive to $n_c-n$, since from Eq. (5b) we get $T_c(H) \propto 1/\xi^{9/2}$. We also plot in Fig. 4 (dashed curve) $T_c$ as a function of $H$ for a larger $n_c-n$ in which $\xi$ is smaller by only 30%. It is seen that $T_c(0)$ increases by more than a factor of 2 and $T_c(H)$ by more than a factor of 3. *Thus, the* $T_c(H)$ *diagram is easier to observe for smaller* $n_c-n$. In the present experiment, $n=0.9n_c$, which yields an observable crossover temperature $T_c^*(0)=0.24$ K. However, $T_c(H)$ shifts rapidly to lower temperatures in a magnetic field. For $H=70$ kOe, we get $T_c(H=70 \text{ kOe})=0.15$ K. *All* our data points at $H=70$ kOe (see Fig. 3) are *above* 0.15 K and therefore correspond to the regime of VRH in the *absence* of a CG. This explains why the experimental results, which yield $p=\frac{1}{3}$ and $T_{1/3} \propto H$, are consistent with the theory of VRH in a strong magnetic field with a nonzero DOS at the Fermi energy.

In summary, we have demonstrated the existence of a crossover phenomenon in VRH in strong magnetic fields. For $H=0$, we observed a crossover from the Mott law $\ln\rho \propto T^{-1/4}$ to a $\ln\rho \propto T^{-1/2}$ law at $T_c=0.24$ K, in agreement with our calculations. As the magnetic field increases, we predict that $T_c$ decreases, making the Coulomb gap effective only at extremely low temperatures.

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