

Determination of g_R/g_L in Left-Right Symmetric Models at Hadron Colliders

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Recently proposed rare decays $Z' \rightarrow f_1 \bar{f}_2 V$ (with $V=W$ or Z and $f_{1,2}$ ordinary fermions) and the forward-backward asymmetry are found to be sensitive diagnostic tests for the gauge coupling ratio g_R/g_L in left-right symmetric models ($SU_{2L} \times SU_{2R} \times U_{1(B-L)}$). At projected luminosities of the CERN Large Hadron Collider and the Superconducting Super Collider, g_R/g_L can be determined statistically to $\sim 1\%$ for $M_{Z'} \approx 1$ TeV. The absence of related decays $W'^{\pm} \rightarrow f_1 \bar{f}_2 W^{\pm}$ would be a clean signature that W' couples to right-handed currents. Once g_R/g_L is determined, the ratio $M_{Z'}/M_{W'}$ and the decays $Z' \rightarrow W^+ W^-$ ($W'^{\pm} \rightarrow W^{\pm} Z$) provide probes for the symmetry breaking sector.

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Left-right symmetric models [1] encompass a class of theories with the extended gauge structure $SU_{2L} \times SU_{2R} \times U_{1(B-L)}$, and g_L, g_R, g_{B-L} as the respective gauge couplings. They are the simplest extended gauge groups which introduce charged current interactions for the right-handed fermions. In a class of such models parity is a Lagrangian symmetry, which is spontaneously broken at a lower scale. In addition, the left-right gauge symmetry is a part of grand unified gauge groups such as $SO(10)$ and E_6 and of a class of heterotic superstring vacua.

The left-right symmetry is spontaneously broken to the standard electroweak gauge group $SU_{2L} \times U_{1Y}$ at a high scale with a constraint on the hypercharge $Y = T_{3R} + Y_{B-L}/2$. At this stage three gauge bosons, W'^{\pm} and Z' , acquire a mass. For the models with $g_L = g_R$ and equal magnitude of the left-handed and the right-handed quark-mixing matrix elements, the bound on the mass of the heavy charged W' is $M_{W'} > 1.4$ TeV, based on the $K_S - K_L$ mass difference [2], and the $W - W'$ mixing angle is $|\theta_+| < 0.003$, from universality [3]. For general left-right symmetric models these bounds are much weaker [4]: $g_L M_{W'}/g_R > 300$ GeV and $g_R |\theta_+|/g_L < 0.013$. Analyses of the Z pole, weak neutral current, and collider data put lower bounds [5-8] on the Z' mass in the range of 400-1000 GeV and on the $Z - Z'$ mixing angle $|\theta_N| < 0.008$.

After the discovery of the Z' and/or W'^{\pm} by their leptonic decays ($Z' \rightarrow l^+ l^-$, $W' \rightarrow l \nu$, where ν is a "right-handed" neutrino, which could be heavy or light), the next goal would be to determine the symmetry-breaking structure and the g_R/g_L ratio. The first candidate for such a test is the $M_{Z'}/M_{W'}$ mass ratio, which can be written as [9]

$$\frac{M_{Z'}^2}{M_{W'}^2} = \frac{\kappa^2 \cot^2 \theta_W}{\kappa^2 \cot^2 \theta_W - 1} \frac{1}{\rho_R}, \quad (1)$$

where $\kappa \equiv g_R/g_L$, $\sin \theta_W = e/g_L$, and

$$\rho_R = \frac{\sum_i (t_{iR}^2 + t_{iR} - t_{iR}^2) |\langle \phi_i \rangle|^2}{\sum_i 2t_{iR}^2 |\langle \phi_i \rangle|^2},$$

with t_{iR} and t_{3iR} being SU_{2R} quantum numbers of the Higgs fields ϕ_i . The parameter ρ_R efficiently parametrizes the Higgs sector and has a limited range for a large class of models.

The parameter κ is often assumed to be unity. However, $\kappa \neq 1$ in models [10] with D parity broken at a large scale ($> 10^9$ GeV); for a typical class of such models $\kappa < 1$. Note also that the constraint

$$\frac{1}{e^2} = \frac{1}{g_L^2} \left(1 + \frac{1}{\kappa^2} \right) + \frac{1}{g_{B-L}^2} \quad (2)$$

and $e^2/g_L^2 = \sin^2 \theta_W \sim 0.23$ imply that κ must satisfy $\kappa^2 \cot^2 \theta_W - 1 \geq 0$, and thus $\kappa \geq 0.55$ for the theory to be consistent with experiment.

Once κ is determined from experiments discussed below, the ratio in Eq. (1) yields information about the Higgs sector of the theory. For example, for the Higgs sector with $\phi \sim (2, 2, 0)$, $\Delta_L = (3, 1, 2)$, $\Delta_R = (1, 3, 2)$, and $\langle \Delta_R \rangle \gg \langle \phi \rangle \gg \langle \Delta_L \rangle$ (see the first model in Table I), one obtains $\rho_R = \frac{1}{2}$ and thus for $\kappa = 1$ (0.7), $M_{Z'}/M_{W'} = 1.7$ (2.3). Other mutually exclusive values for the mass ratio can be obtained for other Higgs sectors (see Table I).

Other probes of the symmetry breaking structure are the decays [11,12] $Z' \rightarrow W^+ W^-$ and $W'^{\pm} \rightarrow W^{\pm} Z$, which are diagnostic tests for the $Z - Z'$ mixing angle θ_N and the $W - W'$ mixing angles θ_+ , respectively. Although such decays are suppressed by $\theta_{N,+}^2$ ($\propto M_{Z',W}^4/M_{Z',W'}^4$), the longitudinal components of gauge bosons give an enhancement $\propto M_{Z',W'}^4/M_{Z',W}^4$, thus yielding sizable decay rates:

$$\Gamma_{Z'WW} = \frac{\alpha \theta_N^2}{48 \tan^2 \theta_W} M_{Z'} \left(\frac{M_{Z'}}{M_W} \right)^4, \quad (3)$$

$$\Gamma_{W'ZW} = \frac{\alpha |\theta_+|^2}{48 \sin^2 \theta_W} M_{W'} \left(\frac{M_{W'}}{M_W} \right)^4, \quad (4)$$

where

$$\theta_N = \frac{-\sin \theta_W (1 + \kappa^2 \cot^2 \theta_W \sigma_3)}{(\kappa^2 \cot^2 \theta_W - 1)^{1/2} (M_{Z'}^2/M_{W'}^2)}$$

and $|\theta_+| = \kappa|\sigma_+|(M_W^2/M_{Z'}^2)$. Parameters

$$\sigma_3 \equiv \frac{\sum_i t_{i3L}t_{i3R}|\langle\phi_i\rangle|^2}{\sum_i t_{i3L}^2|\langle\phi_i\rangle|^2}$$

and

$$\sigma_+ = \frac{\sum_i \langle\phi_i|T_-T_+|\phi_i\rangle}{\sum_i (t_{iL}^2 + t_{iL} - t_{iL}^2)|\langle\phi_i\rangle|^2}$$

again efficiently parametrize a Higgs sector and have a limited range for a large class of theories. (See Table I for the value of $\sigma_{3,+}$ parameters for three typical Higgs sectors.) Here $t_{i3(L,R)}$ are the $SU_{2(L,R)}$ quantum numbers of the Higgs field ϕ_i , $T_{\pm(L,R)}$ are the raising and lowering operators of $SU_{2(L,R)}$, and $\alpha = e^2/4\pi$. Once κ is determined, the mixing angles $\theta_{N,+}$ can then yield information on the Higgs sector. For example, for the first model in Table I, one has $\sigma_3 \approx -1$ and $0 \leq |\sigma_+| \leq 1$. Thus, for $\kappa=1$ (0.7), $C_N \equiv \theta_N/(M_Z^2/M_{Z'}^2) \approx 0.73$ (0.38) and $C_+ \equiv |\theta_+|/(M_W^2/M_{Z'}^2) \leq 1$ (0.7). Note that $\sigma_3 \approx -1$ has a fixed value for the first and the third models of Table I and therefore C_N is fixed for these two models for a given κ . For the second model $0 \geq \sigma_3 \geq -1$, and thus for $\kappa=1$ (0.7), one has $-0.31 \leq C_N \leq 0.73$ ($-0.60 \leq C_N \leq 0.38$). On the other hand, $0 \leq |\sigma_+| \leq 1$, and thus $0 \leq |C_+| \leq \kappa$ for all three models.

Another important set of experiments are those that would provide a diagnostic study of the gauge coupling ratio $\kappa = g_R/g_L$. One possibility would be the measurement of σ_B . However, while the production cross section σ could be calculated for a given set of $V' \equiv (W'^{\pm}, Z')$ couplings to within a few percent, the theoretical branching ratio, $B \equiv \Gamma(V' \rightarrow f\bar{f})/\Gamma(V' \rightarrow \sum_i f_i\bar{f}_i)$ is model dependent because it depends on whether exotic fermions contribute to the V' width, which could easily change σ_B by a factor of 2. Thus, σ_B is not a reliable way to measure κ . However, once κ is known, σ_B would be a useful indirect probe for the existence of exotic fermions.

In this Letter we describe two important probes for studying the gauge coupling ratio κ . Both rely on the fact that the heavy Z' coupling depends on κ , and both are independent of absolute production rates or branching

ratios. The first test consists of the recently proposed [13] rare decays $Z' \rightarrow f_1\bar{f}_2V$, with $V=W$ or Z and $f_{1,2}$ the ordinary fermions. For $V=Z$ the combination $g_1^2g_2^2(\hat{g}_{L1}^2\hat{g}_{L2}^2 + \hat{g}_{R1}^2\hat{g}_{R2}^2)$ is measured, while for $V=W$ such decays project out \hat{g}_{L2} couplings. Here, g_1 and g_2 are the gauge coupling constants of the V and Z' , respectively, while $\hat{g}_{(L,R)1}$ and $\hat{g}_{(L,R)2}$ correspond to the (left,right) charges of the V and Z' , respectively. Although such decays are suppressed by a factor $\alpha/2\pi$ compared to $Z' \rightarrow f\bar{f}$, they have a logarithmic enhancement proportional to $\ln^2(M_{Z'}^2/M_W^2)$, due to collinear and infrared singularities of QED [14]. The second probe for gauge couplings is the forward-backward asymmetry A_{FB} for the process $pp \rightarrow Z' \rightarrow e^+e^-$ or $\mu^+\mu^-$.

In the left-right symmetric model the neutral current gauge interaction term can be written as

$$-L_{NC} = eJ_{em}^\mu A_\mu + g_1J_1^\mu Z_{1\mu} + g_2J_2^\mu Z_{2\mu}. \tag{5}$$

Here $g_1 = g_L/\cos\theta_W$ and $g_2 = g_1(5\sin^2\theta_W/3)^{1/2}$. The photon A , the $SU_2 \times U_{1Y}$ boson Z_1 , and the additional Z_2 gauge boson in the weak eigenstate basis are related to the neutral gauge bosons W_{3L} , W_{3R} , and B in the $SU_{2L} \times SU_{2R} \times U_{1(B-L)}$ basis by

$$A = e(W_{3L}/g_L + W_{3R}/g_R + B/g_{B-L}),$$

$$Z_1 = e[W_{3L}/g_Y - (g_Y/g_L)(W_{3R}/g_R + B/g_{B-L})],$$

and

$$Z_2 = g_Y(W_{3R}/g_{B-L} - B/g_R).$$

Here, $1/g_Y^2 \equiv 1/g_R^2 + 1/g_{B-L}^2$. The currents are defined as

$$J_j^\mu = \frac{1}{2} \sum_i \bar{\psi}_i \gamma^\mu [\hat{g}_{Vj}^i - \hat{g}_{Aj}^i \gamma_5] \psi_i, \quad j=1,2, \tag{6}$$

where the sum runs over fermions and the $\hat{g}_{(V,A)j}^i$ correspond to the vector and axial-vector couplings of Z_j to the i th flavor. The corresponding chiral couplings are $\hat{g}_{(L,R)j}^i = \frac{1}{2}(\hat{g}_{Vj}^i \pm \hat{g}_{Aj}^i)$. With the above definition of the gauge bosons one can show [15] that $J_2^\mu = \sqrt{3/5}[\beta J_{3R}^\mu - (1/2\beta)J_{B-L}^\mu]$, where $\beta = (\kappa^2 \cot^2\theta_W - 1)^{1/2}$, and thus $\hat{g}_{(L,R)2}^i$ can be readily obtained from the $SU_{2L} \times SU_{2R}$

TABLE I. The Higgs multiplets [beside the Higgs field $\phi \sim (2,2,0)$], the hierarchy of their vacuum expectation values (VEV's), the values for the ρ_R parameter and $\sigma_{3,+}$ parameters, and the values for $M_{Z'}/M_W$ (for $\kappa=1,0.7$). The two diagonal vacuum expectation values of the Higgs field ϕ are denoted k and k' .

Higgs sector	VEV's	ρ_R	σ_3	σ_+	$M_{Z'}/M_W$
$\Delta_L \sim (3,1,2)$	$\langle\Delta_R\rangle \gg \langle\phi\rangle \gg \langle\Delta_L\rangle$	$\frac{1}{2}$	-1	$\frac{2k^*k'}{ k ^2 + k' ^2}$	1.7, 2.3
$\Delta_R \sim (1,3,2)$					
$\delta_L \sim (2,1,1)$	$\langle\delta_R\rangle \gg \{\langle\phi\rangle, \langle\delta_L\rangle\}$	1	$\frac{- k ^2 - k' ^2}{ k ^2 + k' ^2 + \langle\delta_L\rangle ^2}$	$\frac{2k^*k'}{ k ^2 + k' ^2 + \langle\delta_L\rangle ^2}$	1.2, 1.6
$\delta_R \sim (1,2,1)$					
$\chi_L \sim (3,1,0)$	$\langle\chi_R\rangle \gg \langle\phi\rangle \gg \langle\chi_L\rangle$	∞	-1	$\frac{2k^*k'}{ k ^2 + k' ^2}$	0, 0
$\chi_R \sim (1,3,0)$					

$\times U_{1(B-L)}$ quantum numbers of the i th fermion.

Because of existing limits one can safely neglect Z_1 - Z_2 mixing, and thus the weak eigenstates Z_1 and Z_2 can be identified with the mass eigenstates Z and Z' , respectively. The decay rate for the process $Z' \rightarrow f\bar{f}Z$ can be written for $\mu = M_Z^2/M_{Z'}^2 \ll 1$ as [13]

$$\Gamma_{f\bar{f}Z} = \frac{M_{Z'} C_{f\bar{f}Z}}{192\pi^3} \{(\ln\mu)^2 + 3\ln\mu + 5 - \frac{1}{3}\pi^2 + O(\mu)\}, \quad (7)$$

which clearly indicates the collinear and infrared singularity. Here $C_{f\bar{f}Z} = g_1^2 g_2^2 (\hat{g}_{L1}^2 \hat{g}_{L2}^2 + \hat{g}_{R1}^2 \hat{g}_{R2}^2)$. The decay rate for $Z' \rightarrow f_1 \bar{f}_2 W$ has the same form as Eq. (7) with the corresponding changes in the gauge couplings (i.e., $\hat{g}_{L1} = \cos\theta_W/\sqrt{2}$, $\hat{g}_{R1} = 0$), and M_Z replaced by M_W . Decay rates $W' \rightarrow f_1 \bar{f}_2 V$ with $V = Z$ or W can be studied in an analogous manner.

Rare decays $Z' \rightarrow f_1 \bar{f}_2 V$ should be compared with the basic process $Z' \rightarrow f\bar{f}$. In particular, the ratio of branching ratios $r_{12V} = B(Z' \rightarrow f_1 \bar{f}_2 V)/B(Z' \rightarrow l\bar{l})$ can be determined cleanly. The l in the denominator refers to the sum over leptons e and μ .

We first evaluate $\sigma B(Z' \rightarrow e^+ e^-)$ for the CERN Large Hadron Collider (LHC) and the Superconducting Super Collider (SSC) for different values of $M_{Z'}$ and κ . We assume the branching ratios B from decays into 16-plets. This is just an example for estimating statistical errors; the actual values for r 's are independent of the exotic decay modes. The number of events for $Z' \rightarrow e^+ e^-$ (or $\mu^+ \mu^-$) for a one-year run (10^7 s) at the expected LHC luminosity $\mathcal{L} = 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ is 2.8×10^4 , 170, and 4 for $\kappa = 1$ and $M_{Z'} = 1, 3,$ and 5 TeV, respectively. The corresponding numbers for the SSC with $\mathcal{L} = 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$ are 1.1×10^4 , 220, and 22. For the projected luminosities and $M_{Z'} = 1$ TeV, LHC is better by a factor of 2, while the SSC is better for higher values of $M_{Z'}$.

We studied the following leptonic ratios:

$$r_{llZ} \equiv B(Z' \rightarrow l^+ l^- Z)/B(Z' \rightarrow l^+ l^-),$$

$$r_{l\nu W} \equiv B(Z' \rightarrow l^\mp \nu_l W^\pm)/B(Z' \rightarrow l^+ l^-),$$

$$r_{\nu\nu Z} \equiv B(Z' \rightarrow \nu_l \bar{\nu}_l Z)/B(Z' \rightarrow l^+ l^-),$$

where l refers to the summation over e and μ and ν_l in $r_{\nu\nu Z}$ refers to the summation over $\nu_e, \nu_\mu,$ and ν_τ . The above ratios can be measured by either leptonic or hadronic W or Z decays. Ratios where $f_{1,2}$ are hadrons, i.e., $r_{\text{had}Z}$ and $r_{\text{had}W}$, are defined analogously. We do not include the t quark in the definition of $r_{\text{had}(W,Z)}$.

The detection of leptonic and semileptonic decays for r 's has been discussed in Ref. [13] for a class of extended gauge structures, including the left-right symmetric model with $\kappa = 1$. Here we would like to explore r 's as a function of κ . In Fig. 1 the r 's are plotted as a function of κ along with typical error bars. It turns out that the r 's are sensitive functions of κ for $0.8 < \kappa < 1.1$ (with the exception of r_{llZ} which is insensitive due to $\hat{g}_{L1}^2 \sim \hat{g}_{R1}^2$ for $l = e, \mu$). In particular, $r_{l\nu W}$ and $r_{\nu\nu Z}$ would yield in-

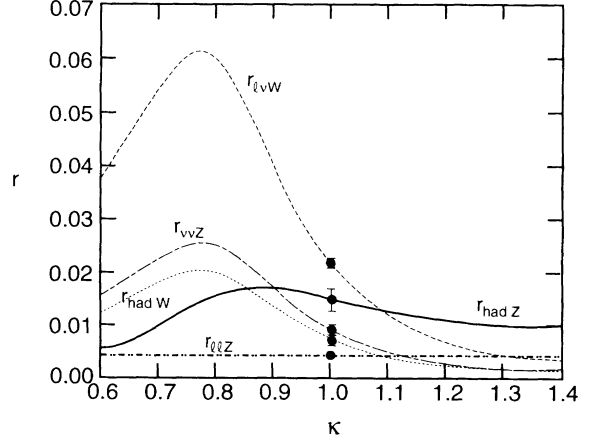


FIG. 1. Ratios r as a function of $\kappa = g_R/g_L$. The error bars are for the LHC with $M_{Z'} = 1$ TeV. For r_{llZ} , $r_{l\nu W}$, and $r_{\nu\nu Z}$, the error bars include both leptonic and hadronic decays of W and Z , while the error bars for $r_{\text{had}W}$ and $r_{\text{had}Z}$ include only leptonic decays.

dependent determinations of κ , each with a statistical precision of $< 1\%$ for $M_{Z'} = 1$ TeV. Also, the comparison of r 's [13] for a general E_6 gauge boson with the range of r 's in the left-right symmetric models indicates that r 's can provide a useful tool to distinguish between the two types of models.

Backgrounds associated with the above ratios have been discussed in Ref. [13] for the left-right symmetric model with $\kappa = 1$ and a class of E_6 models. The same arguments apply to the case with $\kappa \neq 1$. Purely leptonic decays $Z' \rightarrow Z l^+ l^-$ with $Z \rightarrow l^+ l^-$ are clean events with good statistics. On the other hand, $Z' \rightarrow Z \nu_l \bar{\nu}_l$ has a serious background [13] $pp \rightarrow ZZ$, with $Z \rightarrow \nu_l \bar{\nu}_l$. The most promising signal is for $Z' \rightarrow W^\pm e^\mp \nu_e$ with $W \rightarrow \text{hadrons}$. The background from $pp \rightarrow W^+ W^-$ can be cleanly eliminated at a loss of only $\sim 4\%$ of the signal by requiring the transverse mass $m_{Tl\nu_l}$ of the $l\nu_l$ system to be larger than 90 GeV. These events also have a comparable rate with $Z' \rightarrow W^+ W^-$ events, which probe the symmetry breaking scheme. The latter can again be eliminated by an $m_{T\nu_l}$ cut.

Semileptonic decay modes $Z' \rightarrow V l_1 \bar{l}_2$ with V decaying hadronically have a completely different kinematic distribution [13] from the QCD background [16,17] $pp \rightarrow V + \text{jet} + \text{jet}$ where V decays leptonically. However, the kinematics of semihadronic decays $Z' \rightarrow V + \text{hadrons}$ with $V \rightarrow \bar{l}_1 l_2$ suffer from the strong QCD background $pp \rightarrow V + \text{jet} + \text{jet}$ where V decays leptonically.

The forward-backward asymmetry [18,19] A_{FB} for $pp \rightarrow Z' \rightarrow e^+ e^-$ or $\mu^+ \mu^-$ serves as another diagnostic test for Z' gauge couplings. Let θ be the e^- angle with respect to the beam in the Z' rest frame and y be the Z' rapidity. Then $A_{FB}(y) = [F(y) - B(y)]/[F(y) + B(y)]$, with $F(y) \pm B(y) = [\int_0^1 \pm \int_{-1}^0] d\cos\theta d^2\sigma/dy d\cos\theta$. For pp colliders $A_{FB}(y) = -A_{FB}(-y)$. The integrated forward-backward asymmetry is then defined as

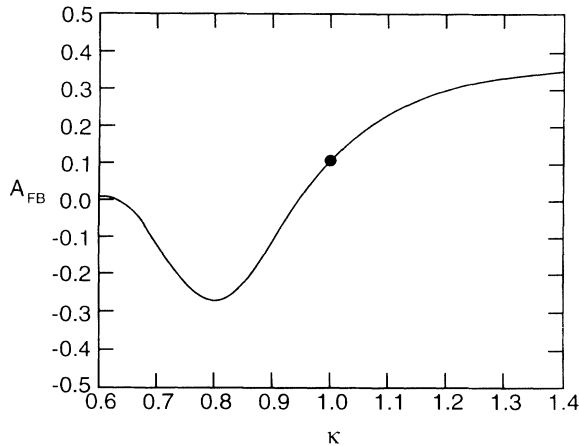


FIG. 2. A_{FB} at the LHC with $M_{Z'}=1$ TeV, plotted as a function of $\kappa = g_R/g_L$.

$$A_{FB} = \frac{\int_{y_{\min}^0}^{y_{\max}^0} [F(y) - B(y)] dy}{\int_{y_{\min}^0}^{y_{\max}^0} [F(y) + B(y)] dy} \quad (8)$$

The statistical error is $\Delta A_{FB} = [(1 - A_{FB}^2)/N]^{1/2}$, where N is the total number of events.

In Fig. 2 A_{FB} is plotted as a function of κ as well as a typical error bar. Again A_{FB} is a sensitive function of κ in the physically interesting region $0.8 < \kappa < 1.1$, and it serves as an excellent complement to the rare decays. For $M_{Z'}=1$ TeV it would yield an independent determination of κ with a statistical precision better than 1%.

The forward-backward asymmetry A_{FB} for the process $pp \rightarrow W' \rightarrow e\nu$ or $\mu\nu$ yields information about the $[(\hat{g}_{L2}^q)^2 - (\hat{g}_{R2}^q)^2] \times [(\hat{g}_{L2}^f)^2 - (\hat{g}_{R2}^f)^2]$ combination of the W' gauge couplings, and thus it cannot distinguish between pure left-handed or pure right-handed coupling of a W' . On the other hand, the rare decay $W' \rightarrow f\bar{f}W$ (for fermions f much lighter than $M_{W'}$) directly probes the $(\hat{g}_{L2}^f)^2$, and thus it should be highly suppressed (by the square of the W - W' mixing angle or by $m_f^2/M_{W'}^2$) in the left-right symmetric models. The absence of such events would thus provide a good check that the coupling of W' is right handed. ($W' \rightarrow f\bar{f}W$ events can in principle be distinguished from $W' \rightarrow WZ$ events by appropriate kinematic cuts.)

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