Comment on "Does Quantum Mechanics Violate the Bell Inequalities?"

In a recent Letter, Santos [1] studied the tests of Bell's inequalities performed by measuring the polarization correlation of optical photon pairs. He considered 0-1-0 atomic cascades and one-channel polarizers [2,3] and argued that measurements of the correlation of photon pairs [4] do not show violation of Bell's inequalities. We would like to show an approach by which a correction in his analysis makes it equivalent to that of Clauser and Shimony [2,3]. Our conclusion is therefore that measurements of polarization correlations of photon pairs clearly show violation of Bell's inequalities.

Santos claims that Eqs. (7) and (8) of his Letter "are correct quantum probabilities obtained by means of the standard quantum rules for the calculation of expectation values." He does not define explicitly the wave function $|\Psi\rangle$ and the operators U_1A_1 and U_2B_2 of these equations. We have therefore made the following simple (though tedious) exercise. We define the function Ψ' according to the analysis made by Shimony [3] (especially in the Appendix), and it is given explicitly by

$$\frac{16\pi}{\sqrt{3}}\Psi'=\begin{pmatrix}-(C_\varphi^2C_\vartheta+S_\varphi^2)+iC_\varphi S_\varphi(1-C_\vartheta)\\ C_\varphi S_\varphi(1-C_\vartheta)-i(C_\varphi^2+S_\varphi^2C_\vartheta)\end{pmatrix}_1\begin{pmatrix}-(C_\varphi^2C_\vartheta-S_\varphi^2)+iC_\varphi S_\varphi(1+C_\vartheta)\\ -C_\varphi S_\varphi(1+C_\vartheta)-i(C_\varphi^2-S_\varphi^2C_\vartheta)\end{pmatrix}_2$$
$$+2\begin{pmatrix}C_\varphi S_\vartheta\\ S_\varphi S_\vartheta\end{pmatrix}_1\begin{pmatrix}C_\varphi S_\vartheta\\ S_\varphi S_\vartheta\end{pmatrix}_2+\begin{pmatrix}(C_\varphi^2C_\vartheta+S_\varphi^2)+iC_\varphi S_\varphi(1-C_\vartheta)\\ -C_\varphi S_\varphi(1-C_\vartheta)-i(C_\varphi^2+S_\varphi^2C_\vartheta)\end{pmatrix}_1\begin{pmatrix}(C_\varphi^2C_\vartheta-S_\varphi^2)+iC_\varphi S_\varphi(1+C_\vartheta)\\ C_\varphi S_\varphi(1+C_\vartheta)-i(C_\varphi^2-S_\varphi^2C_\vartheta)\end{pmatrix}_2. \tag{1}$$

Here we used a short notation where $C_\varphi=\cos\varphi$, $C_\vartheta=\cos\vartheta$, and $S_\varphi=\sin\varphi$. ϑ and φ are the spherical angles and the subscripts 1 and 2 refer to the first and second photons, respectively. The operator Q is defined by [2,3]

$$Q(\gamma)=\begin{pmatrix}\cos^2\gamma & \cos\gamma\sin\gamma\\ \cos\gamma\sin\gamma & \sin^2\gamma\end{pmatrix}, \tag{2}$$

where γ is the polarization direction. By straightforward calculations we get

$$p(\gamma_1,\gamma_2)=\langle\Psi'|Q_1(\gamma_1)Q_2(\gamma_2)|\Psi'\rangle$$
$$=(\Omega/8\pi)^2\alpha(\vartheta_0)[1+F(\vartheta_0)\cos2(\gamma_1-\gamma_2)], \tag{3}$$

where the integration is made over the aperture ϑ_0 of the two photons and Ω is defined as $\Omega=2\pi(1-\cos\vartheta_0)$. This result is exactly equal to Eq. (8) in the Letter by Santos with equivalent expressions for α and F [F is given in relation to Eq. (8) of Shimony [3]].

We now claim that Eq. (7) in the Letter by Santos should be calculated by the same wave function Ψ'. By a straightforward calculation we get

$$p(\gamma_1)=p(\gamma_2)=\langle\Psi'|Q_1(\gamma_1)|\Psi'\rangle$$
$$=\langle\Psi'|Q_2(\gamma_2)|\Psi'\rangle=2(\Omega/8\pi)^2\alpha(\vartheta_0). \tag{4}$$

By comparing our Eqs. (3) and (4) we find that the common factor $(\Omega/8\pi)^2\alpha(\vartheta_0)$ is canceled out in Bell's inequality. Santos's claim that there is a basic difference between his Eqs. (2) and (7) is therefore not justified if we take into account our correction. In other words, by correcting Eq. (7) of Santos and replacing it by Eq. (4) of our Comment we have eliminated any difference between the analysis of Santos and that of Shimony and Clauser [2,3].

Santos claims that the "two-particle state defined above, represented by the Hilbert-space vector $|\psi\rangle$, has not been shown to be a physically realizable state." It should be noticed that Eq. (7) of [1] corresponds to measurements of the polarization of one photon without any correlation with the second photon, while Eq. (8) corresponds to correlations in the measurement of polarization, taking into account correlation in direction. Santos described a quantum mechanical system prepared so that it does not violate Bell's inequalities. However, we agree with Clauser and Shimony [2,3] that by choosing the two opposite lenses with equal apertures Ω, a quantum mechanical system has been prepared in a state represented by Ψ' as given in our Eq. (1). This quantum mechanical system indeed violates Bell's inequality.

Y. Ben-Aryeh and A. Postan
Department of Physics
Technion–Israel Institute of Technology
Haifa, 32000 Israel

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[1] E. Santos, Phys. Rev. Lett. **66**, 1388 (1991).
[2] J. F. Clauser and A. Shimony, Rep. Prog. Phys. **41**, 1881 (1978).
[3] A. Shimony, in *Foundations of Quantum Mechanics*, Proceedings of the International School of Physics "Enrico Fermi," Course IL, edited by B. d'Espagnat (Academic, New York, 1971).
[4] A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. **47**, 46 (1981); **49**, 91 (1982); A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982).