Packet Spreading, Stabilization, and Localization in Superstrong Fields

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We examine the stabilization regime for an atom interacting with a superintense laser field. We investigate the transition from the ground-state wave function to the formation of a stable localized wave function. The quantum mechanical wave-packet spreading plays a key role in determining the final degree of stabilization. We compare simple analytical predictions with exact numerical calculations and stress the importance of the laser pulse shape for the final ground-state probability.

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Recently it has been discovered theoretically [1–8] and experimentally [9] that ionization of atoms can be suppressed in superstrong fields. This phenomenon has been called stabilization and it is characterized by a decreasing ionization probability with increasing laser intensity. Basically two different theoretical frameworks have been developed which predict this rather counterintuitive behavior. One of them is restricted to atomic systems with a Rydberg series of energy levels [4–6]. The second type of stabilization was already predicted in 1984 by Gavrila and co-workers for large laser frequencies [1]. In their theory a transformation to an accelerated coordinate frame [Kramers-Henneberger (KH) transformation] [10] which oscillates with a binding-free electron in the field has been proven to be very helpful. In this frame, stabilization was shown to be due to an effective excitation of bound eigenstates of a time-averaged (KH) Hamiltonian whose decay rate is a decreasing function of laser intensity. This mechanism of stabilization was investigated in exact numerical calculations by Su, Eberly, and Javanainen (SEJ) [2] for one spatial dimension and by Kulander, Shafer, and Krause [3] for three dimensions. Spatially the excitation of the KH bound states becomes manifest in form of a localized wave function which has a characteristic two-peak (dichotomous) [1] or multipeak (polychotomous) [7] structure. For a sufficiently high laser intensity the low-lying KH bound states have their main spatial support far away from the center of the atomic potential and their overlap with the bare ground-state wave function can be vanishingly small for superstrong fields.

In this Letter we will investigate the transition from the ground-state wave function to the quasistationary (KH) regime characterized by a fully developed localized (polychotomous) wave function. Up to a characteristic time the main decay mechanism of the ground state is connected with the quantum mechanical spreading [11] of the electron wave packet, which is initially given by the ground state and becomes free from the interaction with the atomic potential in a superstrong laser field. The atomic binding potential plays surprisingly almost no role for the electron's dynamics; only at later times does the presence of the atom become quite important and lead to the formation of the already mentioned localized (polychotomous) wave function. The transition between these two regimes becomes clearly apparent in the time dependence of the wave function as well as the final ground-state probability after the end of the pulse. A fully analytically soluble model leads to well interpretable results which are confirmed by exact numerical calculations.

We have solved numerically the time-dependent Schrödinger equation for an electron initially in the ground state of a one-dimensional short-range potential $V(x)$ [12] under the action of a superstrong laser field $\mathcal{E}(t)\sin(\omega t)$. The envelope $\mathcal{E}(t)$ was linearly turned on and off over two optical cycles ($\omega=0.0628$ a.u.) with a steady plateau value of $\mathcal{E}=0.64$ a.u. This pulse shape has the desired feature of a very rapid turn-on without leading to any drift motion which could deplete the ground state in the plateau region. The special importance of the appropriate turn-off will be discussed below.

A field strength of $\mathcal{E}=0.64$ a.u. is certainly in the stabilization regime because (as we will show later) a weaker field ($\mathcal{E}=0.1$ a.u., e.g.) would completely photodetach the atom within a few optical cycles, whereas for $\mathcal{E}=0.64$ a.u., e.g., an appreciable amount of population can survive in the ground state.

In Fig. 1 we monitor two snapshots of the time-dependent electron's spatial distribution $|\Psi(x,t)|^2$ in the field's plateau region after (a) 4 and (b) 40 optical periods. In the superstrong field the laser force is much larger than the binding force such that the laser field simply pulls the electron out of the atom in the direction of the field. However, it is important to stress that this process is quite different from the usual one-photon or multiphoton detachment [13] which is based on the inelastic absorption of one or several photons. The center of the wave packet merely oscillates like a classical particle between its two turning points $\pm\mathcal{E}/\omega^2$ [which value is ≈ 160 a.u. for our parameters and much larger than the initial width ($\Delta x_0=5.1$ a.u.) of the wave function] while its spatial width $\Delta x(t)$ is increasing. To stress the irrelevance of the atomic potential $V(x)$ for short times,

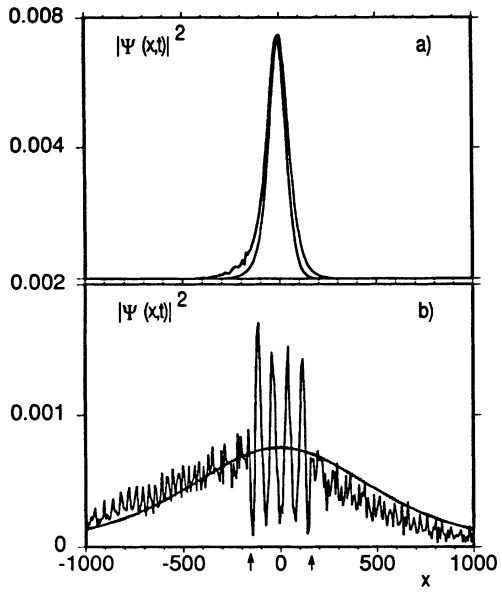

FIG. 1. The spatial distribution of the wave function $|\Psi(x,t)|^2$ after (a) 4 and (b) 40 optical cycles. For comparison the smooth curve corresponds to the binding-free time evolution of the ground-state wave function. The classical turning points at $\pm\mathcal{E}/\omega^2$ are marked by the arrows ($\mathcal{E}=0.64$ a.u., $\omega=0.0628$ a.u., linear turn-on over 2 optical cycles).

the second (smooth) curve in Fig. 1 corresponds to the free development of a wave function $\tilde{\Psi}$ in the absence of the binding potential. The agreement between the two wave functions with and without the atomic potential remains good until a characteristic time t^*, when the spatial width $\Delta x(t)$ becomes of the order of the maximum excursion $\mathcal{E}/\omega^2$. If we approximate the ground state by a Gaussian wave function this width is given by

$$\Delta x(t)=(4\Delta x_0^2+t^2)^{1/2}/2\Delta x_0 . \tag{1}$$

The characteristic time t^* is determined by the requirement $\Delta x(t^*)=\mathcal{E}/\omega^2$ and corresponds for our parameters ($\mathcal{E}=0.64$ a.u., $\omega=0.0628$ a.u.) to roughly 16 optical cycles. The good agreement [14] between the electron wave function with and without the atomic potential is intuitively anticipated. In the superstrong field the electron's oscillatory velocity is extremely large ($\approx\mathcal{E}/\omega$) when it passes over the (narrow) atomic potential. Correspondingly the effective interaction is vanishingly small and the presence of the atomic potential is expected to be rather irrelevant.

The second time regime [Fig. 1(b)] is entered after time t^* when the wave packet has spread over the whole oscillation interval $\pm\mathcal{E}/\omega^2$. Now the electron wave packet is so broad that it can overlap with the atomic potential even when the center of the wave packet is far away at one of its turning points. Around these instants of time the wave packet is at rest and can therefore efficiently interact with the potential to form the localized (polychotomous) shape of the wave packet as predicted by the KH theory. This illustrates how spreading can contribute to the population of the spatially localized KH states. Note that the width of this trapped portion in Fig. 1(b) agrees very well with $\mathcal{E}/\omega^2$.

The same transition from the spreading to the quasistationary KH-SEJ regime can be observed if we compute the final ground-state probability $|\langle\psi_g|\Psi(T)\rangle|^2$ as a function of the pulse length T [Fig. 2(a)]. For comparison we show in Fig. 2(b) the corresponding projection for a wave function $\tilde{\Psi}(T)$ whose time evolution was computed without the atomic potential. Both curves are almost indistinguishable up to pulse durations $T<t^*$, where t^* is the already described time based on the spatial properties of the wave packet. In the spreading regime the maximum amplitudes of this projection decrease inversely proportionally to time [15] due to the wave-packet spreading, which plays the key role for the irreversible decay of the ground state.

The fact that up to the characteristic time t^* the dynamics is not influenced by the atomic potential at all enables us to describe the full quantum evolution even analytically in the framework of a simple model. The corresponding Schrödinger equation without the atomic potential $V(x)$ describes the Volkov evolution of the wave packet initially coinciding with the ground-state wave function ψ_g. Such an equation can be solved analytically [16] with ψ_g modeled by a Gaussian function [17] to give the atomic-force-free wave function $\tilde{\Psi}$. The residual ground-state probability w_g after the pulse is turned off is determined as

$$w_g(T)=|\langle\psi_g|\tilde{\Psi}(T)\rangle|^2 = \frac{2(\Delta x_0)^2}{[4(\Delta x_0)^4+T^2]^{1/2}}\exp[-E] , \tag{2a}$$

where the positive exponent E is given by

$$E=+\frac{2(\Delta x_0)^2[x(T)]^2}{4(\Delta x_0)^4+T^2}-\frac{2T(\Delta x_0)^2x(T)\dot{x}(T)}{4(\Delta x_0)^4+T^2}+\frac{(\Delta x_0)^2[2(\Delta x_0)^8+9T^2(\Delta x_0)^4+T^4][\dot{x}(T)]^2}{[4(\Delta x_0)^4+T^2][(\Delta x_0)^4+T^2]} . \tag{2b}$$

Note that only classical parameters like the excursion $x(T)$ and the velocity $\dot{x}(T)$ [$x(0)=\dot{x}(0)=0$] of a classical binding-free electron in the laser field enter this expression which is applicable for any pulse shape $\mathcal{E}(t)$. It shows that in a superstrong field the residual ground-state probability w_g is not exponentially small only if $\dot{x}(T)=0$ for any T, e.g., if there is no drift after the end of the pulse. This condition is automatically fulfilled for our chosen trapezoidal pulse shape $\mathcal{E}(t)$ and for almost all realizable laser pulses [18]. Under this condition $w_g(T)$ is maximal at those pulse dura-

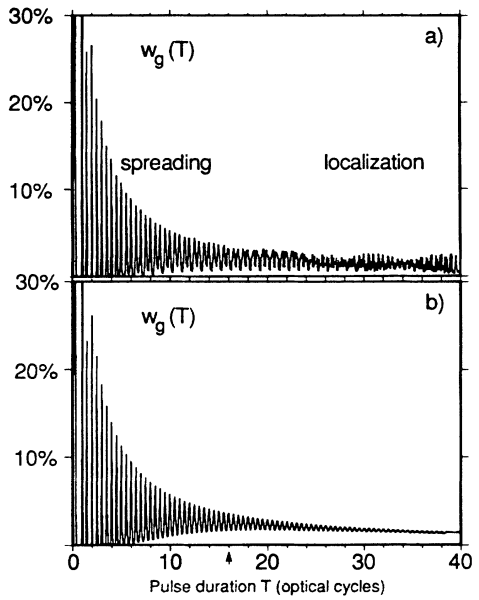

FIG. 2. The final ground-state probability $w_g(T)$ as a function of the pulse duration T. (a) Numerical simulation with the atomic potential $V(x)$ fully taken into account. (b) Same as (a), but the analytical prediction [Eq. (2)]. The arrow marks time $t^*(\mathcal{E}=0.64$ a.u., $\omega=0.0628$ a.u., linear turn-on and turn-off over 2 optical cycles).

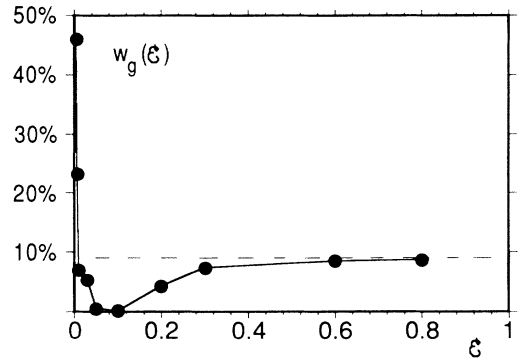

FIG. 3. The final ground-state probability $w_g(\mathcal{E})$ after a pulse duration of 6 optical cycles as a function of the laser field amplitude $\mathcal{E}$. The dashed line indicates the maximum possible degree of stabilization [Eq. (2)] ($\omega=0.0628$ a.u., linear turn-on and turn-off over 2 optical cycles).

tions T at which the final electron displacement $x(T)$ is zero.

In case of an *abruptly* turned-off laser field of the same strength we find (in agreement with our analytical formula) a maximum ground-state population of only less than 2% which stresses again the importance of an appropriate pulse turn-off. An abruptly turned-off laser pulse cannot automatically fulfill both requirements $\dot{x}(T)=0$ and $x(T)=0$ at the end of the pulse.

We would like to stress that the spreading regime corresponds to the asymptotic behavior in a superstrong field in which the maximum remaining population in the ground state does not depend on the field strength $\mathcal{E}$ at all. This result is illustrated in Fig. 3 where this probability is plotted for various laser field amplitudes $\mathcal{E}$ after a smooth pulse of 6 optical cycles. The weak field regime ($\mathcal{E}\leq 0.1$ a.u.) is characterized by a decreasing ground-state population. In the strong field regime ($\mathcal{E}\approx 0.1$ a.u.) the pulse can completely photodetach the electron. The growing part of the curve ($\mathcal{E}\geq 0.1$ a.u.) corresponds to the effect of stabilization for our short-term potential. The maximal degree of stabilization is determined by spreading and is indicated in the figure by the dashed line. This degree depends only on the laser pulse duration [Eq. (2)] and cannot be increased by any means because there is no way to exclude spreading in a superstrong field.

Our analysis has been presented for one spatial dimension to facilitate a quantitative comparison with an exact numerical calculation. A generalization of our final formula [Eq. (2)] to three dimensions is straightforward and would indicate a more rapid decay due to spreading [19]. Our study has been performed for a short-range potential with only one bound state. However, we would expect that the basic features discussed above occur similarly in the ground state of a long-range potential. In case of a long-range potential it is not clear, however, how much of the spread wave packet would contribute to irreversible ionization and which portion could be trapped by higher-lying bound states.

We have shown that the occurrence of stabilization can be characterized by two almost counterintuitive stages in time. During the first few optical cycles of the laser pulse the electron is "effectively" decoupled from the interaction with the atomic potential and therefore cannot gain energy irreversibly. In this regime spreading is basically the only mechanism for decay. However, at a later time when the width of the spread wave packet has exceeded the classical free-electron quiver amplitude, this decay is interrupted by the atomic potential. In this regime the presence of the potential is crucially important for the formation of the quasistable polychotomous wave function and its long time behavior. The KH-SEJ picture is shown to be applicable only under the condition that the field is not too strong [$\mathcal{E}/\omega^2 < \Delta x(t)$] and the pulse duration is not too short. A laser pulse shape is most suitable for trapping a maximum amount of population in the ground state if a classical free electron would have both a zero position and zero velocity at the end of such a pulse. The spreading mechanism determines the maximum degree of stabilization, or trapping of ground-state population, achievable in a superstrong field.

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