Hall Voltage Sign Reversal in Thin Superconducting Films

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A novel approach to the superconducting Hall effect is developed, based on the opposing drift of the thermally excited quasiparticles. These collide quasielastically with the hydrodynamic superfluid velocity field circulating *far outside the core* of a vortex, thereby transferring momentum to the latter. The predicted Hall angle, by BCS theory, is of the order of $k_B T_c$ divided by the Fermi energy, has sign opposite to that in the normal state, because of the backflow, and disappears at low temperature.

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There is considerable experimental evidence [1] that the Hall voltage reverses its direction when a thin film in a weak perpendicular field is cooled below its superconducting transition temperature. Under these conditions, the magnetic screening coming from the self-field of a vortex can be neglected and the vortices can be treated individually as decoupled entities. As noted by Josephson [2], and discussed in detail by Hagen *et al.* [3], the motion of the individual vortices, in response to the forces acting on them, sets up the Hall voltage that is observed. In this Letter we present a way of thinking about these forces that leads to a straightforward and simple physical explanation of the Hall voltage sign reversal.

Essential to our approach is the Landau picture of the reduction of the effective superfluid mass density in liquid He II resulting from the anisotropic distribution of thermally excited phonons and rotons. The analogous two-fluid picture advanced by Bardeen [4] for a superconductor is to regard $\mathbf{P}$, the momentum density in a superconductor, as composed, similarly as in He II, of two distinct components. To set the stage for our discussion of the Hall effect, it is useful to review briefly Bardeen's two-fluid model. The "superfluid" contribution to $\mathbf{P}$ is $\mathbf{P}_s = \rho \mathbf{v}_s$, where $\mathbf{v}_s$ and ρ are the superfluid velocity (proportional to the gradient of the Cooper pair phase) and the *total* mass density of the charge carriers, respectively. By Galilean covariance E_p, the energy of a thermally excited quasiparticle of momentum $\mathbf{p}$, is shifted by $\mathbf{v}_s \cdot \mathbf{p}$, causing the equilibrium occupation probability to change by

$$\Delta f = \frac{\partial f}{\partial E} \mathbf{v}_s \cdot \mathbf{p} \,. \tag{1}$$

Here $f = (1+\exp\beta E)^{-1}$ is the Fermi function and $k_B T = \beta^{-1}$ is Boltzmann's constant times the temperature. Introducing $N(0)$, the density of states, and taking the angular average over the Fermi surface, we project $\mathbf{p}$, of magnitude $p = p_F$, onto the direction of $\mathbf{v}_s$. This yields, for the normal fluid contribution to $\mathbf{P}$,

$$\begin{aligned}\mathbf{P}_n = -\rho_n \mathbf{v}_s &= 2N(0) \int d\epsilon_p \frac{\partial f}{\partial E} \langle (\mathbf{v}_s \cdot \mathbf{p}) \mathbf{p} \rangle_{\text{ang}} \\ &= \frac{2}{3} N(0) p_F^2 \int d\epsilon_p \frac{\partial f}{\partial E} \mathbf{v}_s \,,\end{aligned} \tag{2}$$

where $\epsilon_p = \pm (E_p^2 - \Delta^2)^{1/2}$ is the normal-state quasiparticle energy and Δ is the BCS energy gap. This equation defines ρ_n, the effective mass density of the normal fluid. Limiting the integration over the interval $\Delta \le E < \infty$ to one branch of the quasiparticle spectrum adds a factor of 2 to Eq. (2) and yields [5], describing the *backflow*,

$$\rho_n = -\frac{4}{3} N(0) p_F^2 \int_\Delta^\infty dE \frac{E}{(E^2-\Delta^2)^{1/2}} \frac{\partial f}{\partial E} \,. \tag{3}$$

Adding together the two, *physically quite different components* yields the total momentum density

$$\mathbf{P} = \mathbf{P}_s + \mathbf{P}_n = (\rho - \rho_n) \mathbf{v}_s \equiv \rho_s \mathbf{v}_s \tag{4}$$

and the current density $\mathbf{J} = (q/m)\mathbf{P} = (q/m)\rho_s \mathbf{v}_s$, where q/m is the charge-to-mass ratio for the charge carriers. For what follows, it is important to appreciate the fact that $\rho_s = \rho - \rho_n$ is a *composite property*, with its separate components, ρ and ρ_n, possessing their own physical realities. (In this respect, the Ginzburg-Landau theory is somewhat misleading.)

Continuing with Bardeen's two-fluid picture, we proceed now to study the Hall effect by calculating the momentum transfer to an idealized zero-core vortex which, for the moment, is regarded as pinned at the origin. As with the momentum density in Eq. (4), the momentum transfer has two distinct components: (1) that from the condensate of the Cooper pairs and (2) that from the backflowing gas of quasiparticle excitations.

Considering the superfluid contribution first, it is convenient to introduce a right-handed coordinate system with unit vectors $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ in the plane of the film and $\hat{\mathbf{z}}$ normal to it. The weak magnetic field in the z direction causes the film to set up the vortex with the local superfluid velocity field $\mathbf{u}(\mathbf{r})$ specified by the quantized circulation vector $\hat{\boldsymbol{\kappa}} = \kappa \hat{\mathbf{z}} = \pm |\kappa| \hat{\mathbf{z}}$, for positive or negative charge carriers, respectively. The superposed uniform momentum density field is ρ times the superfluid velocity $\mathbf{v}_s = v_{sx} \hat{\mathbf{x}} = \pm |v_s| \hat{\mathbf{x}}$. Following Nozières and Vinen [6] and Hillel [7], we compute the force acting on the vortex by considering its hydrodynamic interaction with the superposed momentum density field $\rho \mathbf{v}(\mathbf{r}) = \rho \mathbf{v}_s + \rho \mathbf{u}(\mathbf{r})$, where $\mathbf{u}(\mathbf{r}) = \kappa \hat{\boldsymbol{\phi}} / 2\pi r$. The unit vector $\hat{\boldsymbol{\phi}}$ is in the direction of increasing azimuthal angle $\phi = \tan^{-1}(y/x)$.

The radial distance from the vortex is $r=(x^2+y^2)^{1/2}$. The x-y off-diagonal element of the stress tensor, which describes the rate of flow of y momentum across an element of area normal to the x axis, is $\rho v_x v_y$. The total flow per unit vortex length across a plane at $x=l$ is therefore

$$\rho\int_{-\infty}^{\infty} v_x v_y\,dy=\rho v_{sx}\int_{-\infty}^{\infty} u_y\,dy=\rho v_{sx}\frac{\kappa l}{2\pi}\int_{-\infty}^{\infty}\frac{dy}{l^2+y^2}$$
$$=\tfrac{1}{2}\rho\kappa v_{sx}=\tfrac{1}{2}\rho|\kappa||v_s|\,, \tag{5}$$

a positive definite result (for the given geometry), independent of the sign of the charge carriers. An equal and opposite flow of y momentum takes place across the plane $x=-l$. By momentum conservation, the recoil is $\mathbf{F}_s=-\rho|\kappa||v_s|\hat{\mathbf{y}}$, the Magnus force that is familiar from irrotational potential flow in classical hydrodynamics. It is worth noting that the Lorentz force does not play any role in this derivation.

In Eq. (4), the total net momentum density was obtained only after including the individual momenta of all of the quasiparticles. Similarly, for the Hall effect, in order to find the net force on the vortex, it is necessary to add to $\mathbf{F}_s$ the recoil force $\mathbf{F}_n=F_{nx}\hat{\mathbf{x}}+F_{ny}\hat{\mathbf{y}}$ resulting from the interaction with the quasiparticles passing by the vortex. The same physical mechanism generates both of the Cartesian components of $\mathbf{F}_n$. This unified approach tests the accuracy of our method of calculation of F_{nx}, a kind of *drag force* which is *the new result* in this paper, by, at the same time, producing a result for F_{ny}, the transverse component, that is likely to be granted general acceptance. In fact, what we are about to derive, namely, $F_{ny}=\rho_n|\kappa||v_s|$, will not come as any surprise. Added to $\mathbf{F}_s$, it reduces the net Magnus force to

$$F_{My}=F_{sy}+F_{ny}=-(\rho-\rho_s)|\kappa||v_s|=-\rho_s|\kappa||v_s|\,,$$

as expected. In spite of the noncontroversial nature of this outcome, we, nevertheless, present here a derivation of it. The purpose of this excursion is to prepare the grounds for a similar computation of the longitudinal component, F_{nx}.

Instead of representing a quasiparticle of momentum $\mathbf{p}$ by a plane wave, we use a straight-line trajectory. This quasiclassical approximation is justified by the fact that the velocity field of the vortex $\mathbf{u}(\mathbf{r})$ reaches out to distances r that are orders of magnitude greater than the de Broglie wavelength. It is the same approximation used by Lifschitz and Pitaevskii [8] and by Sonin [8] in their pioneering work on the scattering of rotons by a vortex in superfluid ^{4}He. Galilean covariance requires the local quasiparticle spectrum, at any point along the trajectory, to be shifted by $\mathbf{p}\cdot\mathbf{v}$, leading to the Hamiltonian

$$H=E_p+\mathbf{p}\cdot\mathbf{v}(\mathbf{r})=E_p+p\hat{\mathbf{p}}\cdot\mathbf{v}(\mathbf{r})=\text{const}\,, \tag{6}$$

where energy conservation has been imposed along the trajectory. By the straight-line approximation, the unit vector $\hat{\mathbf{p}}\equiv\mathbf{p}/p$ is a constant, which, for the moment, we take to be in the x-y plane. The force exerted on the quasiparticle is $d\mathbf{p}/dt=-\nabla H=-p\nabla[\hat{\mathbf{p}}\cdot\mathbf{v}(\mathbf{r})]$. The application of this formula to the computation of the net momentum transfer from passing by the vortex is greatly simplified by imagining an infinite family of parallel trajectories, all with the same initial momentum $\mathbf{p}_i$ and with a continuous distribution of impact parameters. With all of the x-y plane filled in this way, Eq. (6) establishes a functional relationship, at any arbitrary point in the plane, between the coordinate $\mathbf{r}$ and the *scalar field p, the magnitude of the momentum.* Differentiating Eq. (6) consequently yields

$$p\nabla[\hat{\mathbf{p}}\cdot\mathbf{v}(\mathbf{r})]+V\nabla p=0\,, \tag{7}$$

where the quasiparticle velocity is $V=dE_p/d_p+\hat{\mathbf{p}}\cdot\mathbf{v}(\mathbf{r})$. Making use of Eq. (7) in the preceding force equation, and dividing by the velocity, gives for the momentum transfer per unit path length traversed,

$$\frac{d\mathbf{p}}{ds}=\frac{dt}{ds}\frac{d\mathbf{p}}{dt}=\frac{1}{V}\frac{d\mathbf{p}}{dt}=\nabla p=\nabla(p-p_F^*)\,, \tag{8}$$

a generalization of an identity [9] that has been derived for the interaction of rotons with a vortex in superfluid ^{4}He. The asterisk indicates a slight shift of the Fermi velocity, of $O(v_s)$.

Summing the momentum transfer over the impact-parameter interval $-b_0\le b\le b_0$ and interchanging the order of integration in the plane yields (see Fig. 2 and accompanying text of Ref. [9])

$$\int_{-b_0}^{b_0} p_y^{sc}(b)\,db=\int_{-b_0}^{b_0} dy\int_{-\infty}^{\infty} dx\,\frac{\partial(p-p_F^*)}{\partial y}$$
$$=\int_{-\infty}^{\infty} dx\,[(p-p_F^*)|_{b_0}-(p-p_F^*)|_{-b_0}]\,.$$

We replace p_F^* by p_i and choose b_0 large enough that $p-p_i$ is very small everywhere along the trajectories $y=b=\pm b_0$, which permits us to write Eq. (6) in the differential form $V_i(p-p_i)+\mathbf{p}_i\cdot\mathbf{v}(\mathbf{r})=0$, or

$$p-p_i=-\frac{\mathbf{v}(\mathbf{r})\cdot\mathbf{p}_i}{V_i}=-\frac{v_x p_i}{V_i}=-\frac{u_x(\mathbf{r})p_i}{V_i}-\frac{v_{sx}p_i}{V_i}\,.$$

The constant last term drops out from the previous equation, leaving, in the limit $b_0\to\infty$,

$$\int_{-\infty}^{\infty} p_y^{sc}(b)\,db=\left(\frac{p_i}{V_i}\right)\lim_{b_0\to\infty}\int_{-\infty}^{\infty} dx\,(u_x|_{b_0}-u_x|_{-b_0})$$
$$=-\left(\frac{p_i}{V_i}\right)\oint dl\cdot\mathbf{u}=-\frac{\kappa p_i}{V_i}\,.$$

To correctly describe the anomalous holelike branch ($p_i<p_F^*$) as well as the normal quasiparticle branch ($p_i>p_F^*$), replace V_i by its absolute value. (This takes the reversal of the x integration into account.) For arbitrary orientation of $\hat{\mathbf{p}}$, p_i is to be replaced by p_x. The velocity $|V_i|$ cancels when multiplying by the differential flux $2N(0)|V_i|\Delta f\,d\epsilon_p$. The ensuing phase-space integration, except for the factor κ, is identical to the total quasiparticle momentum density of Eq. (2).

The outcome of the computation is that the y component of the total force per unit length exerted by the quasiparticles on the vortex is $F_{ny}=-\kappa P_{nx}=\rho_n\kappa v_{sx}$, which, added to $\mathbf{F}_s$, gives for the net Magnus force

$$\mathbf{F}_M=\mathbf{F}_s+F_{ny}\hat{\mathbf{y}}=-(\rho-\rho_n)\kappa v_{sx}\hat{\mathbf{y}}=-\rho_s|\kappa||v_s|\hat{\mathbf{y}},$$

as was to be demonstrated, thereby confirming the reliability of the quasiclassical method of calculating.

We now turn to the backflow drag force, F_{nx}. This results from the shift exerted by the vortex field on the quasiparticle energies. By local Galilean covariance, the shift is proportional to $p\sin\theta$, the magnitude of the projection of $\mathbf{p}$ onto the x-y plane. If the maximum shift along the trajectory, for a given impact parameter b, is less than the amount by which the initial quasiparticle energy exceeds the gap energy, then the velocity will be nonvanishing throughout, and the trajectory will be uninterrupted and practically undeflected. The requirement for no Andreev reflection is expressed by the inequality

$$(\mathbf{u}\cdot\mathbf{p})_{\max}=|\kappa|p\sin\theta/2\pi|b|<\bar{H}_{\mathbf{p}_i}-\Delta, \tag{9}$$

where $\bar{H}_{\mathbf{p}_i}=E_{p_i}+(p_i-p_F)\hat{\mathbf{p}}_i\cdot\mathbf{v}_s$ represents the initial quasiparticle energy [the overbar indicates that the constant $p_F(\hat{\mathbf{p}}_i\cdot\mathbf{v}_s)$ has been subtracted from Eq. (6)]. On the other hand, if this inequality is violated, Andreev reflection will occur. Passing through the minimum in the spectrum, at $p=p_F^*$ (slightly shifted from p_F), the local value of p goes to the final value of p_f on the other branch, which reverses the velocity. This causes the charge carrier to give up an amount of x momentum equal to $(p_i-p_f)\hat{\mathbf{p}}_i\cdot\hat{\mathbf{x}}$, and to return on essentially the same straight-line trajectory along which it initially entered the vortex field. The range of impact parameter for which Andreev reflection occurs is given by

$$0\le|b|\le b_{\mathbf{p}_i}^*\equiv|\kappa|p_F^*\sin\theta/2\pi(\bar{H}_{\mathbf{p}_i}-\Delta). \tag{10}$$

The Andreev reflection establishes a one-to-one mapping between the quasiparticle momenta $p_i=p$ and $p_f=p'$ on the normal and anomalous branches of the spectrum, respectively. By energy conservation, $\bar{H}_{\mathbf{p}'}=\bar{H}_{\mathbf{p}}$, from which it follows that $b_{\mathbf{p}'}^*=b_{\mathbf{p}}^*$. While the reflection $p\to p'$ transfers x momentum of $(p-p')\hat{\mathbf{p}}\cdot\hat{\mathbf{x}}$, the reverse reflection, $p'\to p$, transfers the negative of this. The net transfer is determined by the difference in the initial populations and is proportional to

$$\begin{aligned}(p-p')\hat{\mathbf{p}}\cdot\hat{\mathbf{x}}(\Delta f_{\mathbf{p}}-\Delta f_{\mathbf{p}'})&=(p-p')^2\hat{\mathbf{p}}\cdot\hat{\mathbf{x}}\hat{\mathbf{p}}\cdot\mathbf{v}_s\frac{\partial f}{\partial E}\\&=(p-p')^2\sin^2\theta\cos^2\phi\frac{\partial f}{\partial E}v_{sx}.\end{aligned} \tag{11}$$

The $O(v_s)$ deviation of p_F^* from p_F can now be neglected. The number of such pairs of quasiparticle reflections in the momentum interval dp is proportional to

$$\begin{aligned}2N(0)b_{\mathbf{p}}^*d\epsilon_p&=2N(0)b_{\mathbf{p}}^*v_Fdp\\&=2N(0)b_{\mathbf{p}}^*(dp/|d\bar{H}_{\mathbf{p}}|)|d\bar{H}_{\mathbf{p}}|,\end{aligned}$$

where v_F is the Fermi velocity. For the flux, we multiply by the component of the velocity in the x-y plane, $\sin\theta|d\bar{H}_{\mathbf{p}}/d_p|$, causing the derivative to disappear from the calculation. The resulting product, $2N(0)v_fb_{\mathbf{p}}^*\times\sin\theta|d\bar{H}_{\mathbf{p}}|$, is invariant under the mapping $\mathbf{p}\leftrightarrow\mathbf{p}'$, which permits the replacement of $\bar{H}_{\mathbf{p}}=\bar{H}_{\mathbf{p}'}$ by E_p. Correspondingly, we replace $(p-p')^2$ by

$$4\epsilon_p^2/v_F^2=4(E_p^2-\Delta^2)/v_F^2=(E_p-\Delta)(E_p+\Delta)4/v_F^2.$$

The first factor of this will cancel with the denominator of $b_{\mathbf{p}}^*$. Substituting from Eq. (10) and collecting factors yields the quasiparticle drag force of the form $F_{nx}=-Dv_{sx}$ with the drag coefficient

$$\begin{aligned}D&=-\frac{4N(0)}{\pi v_F}p_F|\kappa|\langle\sin^4\theta\cos^2\phi\rangle_{\rm ang}\int_\Delta^\infty dE(E+\Delta)\frac{\partial f}{\partial E}\\&=\frac{16N(0)}{15\pi v_F}p_Fk_BT|\kappa|g\left(\frac{\Delta}{k_BT}\right)=\frac{4}{5\pi}\rho|\kappa|\frac{k_BT}{\epsilon_F}g\left(\frac{\Delta}{k_BT}\right),\end{aligned} \tag{12}$$

where the final form comes from the degenerate free fermion gas formula $N(0)p_F/v_F=3\rho/4\epsilon_F$, ϵ_F being the Fermi energy. The angle average is $\langle\sin^4\theta\cos^2\phi\rangle=\frac{4}{15}$. The scaling function,

$$g(y)=-\int_y^\infty du(u+y)\frac{d}{du}\frac{1}{e^u+1}=\frac{2y}{e^y+1}+\ln(1+e^{-y}), \tag{13}$$

rises to a maximum value of somewhat less than 1 in the vicinity of $y=0.9$, and falls, for $y\gg1$, as $g(y)\sim(2y+1)e^{-y}$. The resultant force on the vortex $\mathbf{F}=\mathbf{F}_M+F_{nx}\hat{\mathbf{x}}$ is tilted in the x direction (direction of current flow) relative to the Magnus force by the angle β, where

$$\tan\beta=-\frac{F_{nx}}{F_{My}}=\mp\frac{4}{5\pi}\frac{\rho}{\rho_s}\frac{k_BT}{\epsilon_F}g\left(\frac{\Delta}{k_BT}\right), \tag{14}$$

with the $\pm$ sign referring to positive and negative charge carriers, respectively. Nozières and Vinen [6] have proposed a picture of the Hall effect at $T=0$ that involves a force generated *within the vortex core* and proportional to $-\mathbf{v}_s$. It is, thus, similar in form to our quasiparticle drag force $-D\mathbf{v}_s$, which, however, is of a quite different physical nature, depending as it does on the interaction of *thermally excited quasiparticles outside the core*. The work of Bardeen and Stephen [10] is similarly limited to processes occurring inside the core, which is also true of a recent effort [11] to account for the Hall voltage sign reversal. The present work serves to support the observation by Hagen *et al.* [3] that a Nozières-Vinen-type force of the form $-D\mathbf{v}_s$ might provide a phenomenological explanation of the Hall voltage sign reversal.

Hagen *et al.* [1] have presented a detailed phenomenological study of the additional forces that may act on a moving vortex, that are linear in its velocity $\mathbf{v}_v$. If there is a transverse component, the resultant velocity-dependent force will be tilted by an angle γ relative to the direction of $\mathbf{v}_v$. Steady-state equilibrium then requires $\mathbf{v}_v$ to be tilted by the angle $\beta-\gamma$ relative to the Magnus force. The Hall angle is consequently $\alpha=\beta-\gamma$ with $\tan\alpha=(\tan\beta-\tan\gamma)/(1+\tan\beta\tan\gamma)$, equivalent to Eq. (4) of Ref. [1]. In the limit $v_s\to 0$, corresponding to very small current, and consequently very weak driving forces (Magnus and backflow drag), it follows also that $v_v\to 0$. The Hall angle, being determined by the *direction* of $\mathbf{v}_v$, does not vanish in this limit. However, when $\mathbf{v}_v\to 0$, any forces proportional to v_v become negligible in comparison to any residual static pinning forces, no matter how small. Vortex motion will then only occur discontinuously by jumps, or hops, that result from random thermal excitation up and over the potential barriers between pinning sites. In this regime, it is natural to assume that the pinning forces are of purely thermodynamic origin, insensitive to the sign of κ. The hops would consequently be, on the average, in the direction of $\mathbf{F}_M+F_{nx}\hat{\mathbf{x}}$. On this assumption, we can set $\gamma=0$ and equate $\beta=\alpha$ in Eq. (14). In general, the latter is associated with $\mathbf{v}_v$ in terms of the electric field [2,3]:

$$\tan\alpha=\frac{E_y}{E_x}=-\frac{v_{vx}}{v_{vy}}=-\frac{F_{nx}}{F_{My}}=-\frac{D}{\rho_s\kappa}=\mp\frac{D}{\rho_s|\kappa|}\,. \tag{15}$$

Because D is a *positive definite* drag coefficient, on general physical grounds $\tan\alpha$ is *opposite in sign* to that in the normal state, regardless of the sign of the charge carriers. (A change of carrier sign changes κ and, thus, α, below the transition. At the same time, the normal-state value of α changes sign, staying opposite in sign to that of α in the superconducting state.) The point of this paper has been not simply to make this general, and, we believe, compelling qualitative intuitive argument for the sign reversal, but, in addition, to bolster it with the concrete, albeit idealized, computational result shown in Eq. (12).

In summary, we have demonstrated that the hydrodynamic interaction of the preferentially backward-moving entropy-rich gas of quasiparticles with the outer reaches of the vortex velocity field produces some momentum transfer, by Andreev reflection, which can be regarded as a special type of quasielastic scattering. The resulting drag force yields a Hall angle opposite in sign to that in the normal state. The magnitude, $k_BT_c/4\epsilon_F$, compares favorably for the high-T_c cuprate superconductors [12]. For these superconductors, the ratio of mean free path to correlation length (and, thus, core radius) is such as to render the application of our idealized theory not totally unreasonable. It needs emphasizing that the mechanism of the backflow drag force advanced in this Letter does not require any interaction with the vortex core. All of the effect comes from the hydrodynamic coupling with the circulating superfluid. It remains a task for the future to carry out a more realistic computation of the drag coefficient by studying the interactions of the quasiparticles with a vortex of nonvanishing core radius [13], as well as testing the validity of the tacitly assumed adiabatic adjustment of the quasiparticles to the local superfluid velocity.

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[12] The data exhibited by Hagen *et al.* in Fig. 2 of Ref. [1] show a minimum in $\tan\alpha$ of magnitude 4×10^{-3}, at $T=90$ K. If we assume a Fermi energy of the order of 1 eV, our idealized theoretical estimate for the size of the effect, $k_BT/4\epsilon_F$, is of the order of 2×10^{-3}. The apparent discrepancy of a factor of 2 is acceptable in view of the neglect of effective mass and anisotropy. For the "classic" superconductors the predicted effect is much smaller because of the lower temperatures and the higher Fermi energies.

[13] One aspect of this has already been taken into account by the neglect of wide angle scattering.