Equilibrium and Perturbed Fluxes and Turbulence Levels in a Tokamak: Implications for Models

K. W. Gentle, B. Richards, M. E. Austin,[a] R. V. Bravenec, D. L. Brower,[b] R. F. Gandy,[a] W. L. Li,[b] P. E. Phillips, D. W. Ross, W. L. Rowan, P. M. Schoch,[c] P. M. Valanju, and A. J. Wootton

Fusion Research Center, University of Texas, Austin, Texas 78712

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Modulation of the gas feed to a tokamak induces a density modulation and also a temperature modulation. Although the density change is fractionally small and the response is linear, the perturbed particle flux can exceed the equilibrium flux. The particle flux Γ thus cannot be a simple homogeneous function of ∇n. The ratio of thermal to particle flux is quite different for the equilibrium and perturbations, precluding a simple linkage of particle and thermal transport. The modulation of the amplitude of density turbulence does not correlate with any of the modulated fluxes.

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Understanding the processes of anomalous transport is a central problem in tokamak physics. The transport is presumed to be driven by some form (or forms) of turbulence, yet the specific mechanisms remain to be established [1,2]. Transport is classically described by a transport matrix, but the complexity and possible nonlinearities of the processes have led to descriptions purely in terms of fluxes, which are to be determined as functions of (local) parameters [2]. The fluxes are often direct inferences from turbulence theories, whereas extraction of a transport matrix can be problematic or ambiguous when the paradigm—a near-equilibrium system driven weakly by a set of thermodynamic forces—is inaccurate. Furthermore, the fluxes may be calculated either from first principles, in which case they depend only upon local equilibrium parameters, or by using observed turbulence characteristics in expressions for the fluxes. The latter approach can make the inferences less sensitive to the accuracy of the nonlinear calculation of turbulence levels, often the least accurate aspect of the theory. Perturbative experiments have proven to be a powerful technique for elucidating transport, especially particle transport [3-5].

Recent experiments using density modulation in TEXT have provided a good characterization of both particle and thermal fluxes. The comparison of perturbed with equilibrium flux is particularly informative, for much indeterminacy in an expression for one flux vanishes for a comparison. The results may be interpreted as placing important constraints on possible forms of the expression for the particle flux and the relation between particle and thermal fluxes. The electrostatic turbulence is also measured in these experiments and confirms the complexity of the relation between turbulence, local parameters, and fluxes.

By modulation of the gas feed, nearly sinusoidal oscillations of the density may be induced. Line-integrated measurements of the density are made with a multichannel interferometer at seven chords across the minor radius. The associated temperature variations are measured by electron cyclotron emission at five positions. The turbulence is obtained from a heavy ion beam probe as the rms value of density fluctuations, denoted $\tilde{n}$, integrated over all frequencies (above 5 kHz to include only turbulence) and wave numbers within the range of the instrument ($0<k<2\ \mathrm{cm}^{-1}$; $k\rho_s<0.25$), and which seems to include the spectral peak. This $\tilde{n}$ is a representative value of the turbulence level, although the inferred dispersion is not that of simple drift waves [6], and no model for the nature of modes constituting it is presumed here. The amplitude and phase of the modulation of each parameter are accurately obtained from Fourier analysis of an interval of six to eight periods of the 30-Hz driving frequency.

For the nearly circular flux surfaces of TEXT, the particle flux obeys the simple conservation equation $\partial n/\partial t = -(1/r)\partial(r\Gamma)/\partial r + S$, where $S(r,t)$ is the source rate from ionization. As a linear equation, it can be written for the equilibrium (n_0, Γ_0, etc.) and the time-dependent components (Δn, $\Delta\Gamma$, etc.), proportional to $e^{i\omega t}$ here. The solutions are

$$\Gamma_0(r) = \frac{1}{r}\int_0^r r' S_0(r')\,dr' , \tag{1a}$$

$$\Delta\Gamma(r) = \frac{1}{r}\int_0^r r'[-i\omega\Delta n(r') + \Delta S(r')]\,dr' . \tag{1b}$$

The radial shape of $S(r)$ is determined from a neutral-particle code solution for the particular plasma profiles in each case, and the multiplicative constant for the equilibrium is fixed from the measured particle confinement time. The functions $n_0(r)$ and $\Delta n(r)$ are actually obtained from an elaborate transport analysis [4,6], which also adjusts the magnitude of ΔS to match boundary conditions. The relevant criterion here is that n_0 and Δn are physically reasonable and fit the observed chord integrals to an accuracy of no worse than 2%. The consequent error in the integrals of Eq. (1) is of similar magnitude.

The results for the quantity $|\Delta\Gamma(r)|/\Gamma_0(r)$ are shown in Fig. 1(a) for low density and various amplitudes of the density perturbation, labeled by the value of $\Delta n/n_0$ at the center. Note that the perturbed fluxes are linearly proportional to the density perturbation amplitude, but are numerically large, not a small fraction of the equilibrium values. This effect is even more dramatic at the higher

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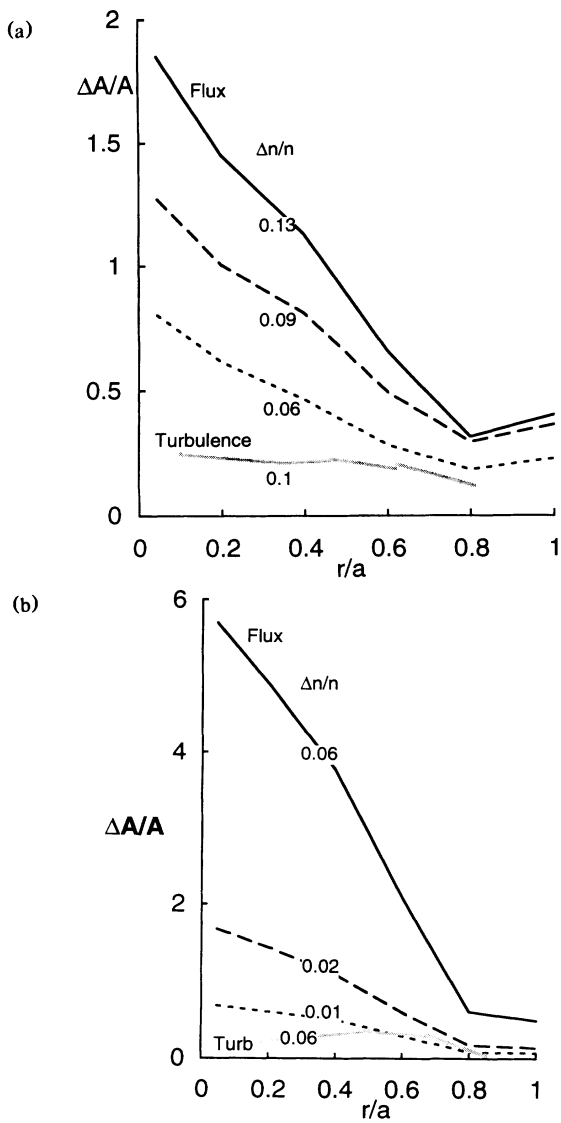

FIG. 1. Ratio of perturbed to equilibrium particle flux ($|\Delta\Gamma|/\Gamma_0$) as a function of radius for various amplitudes of central density modulation $\Delta n/n_0$, and the fractional modulation of the turbulence level. (a) For $\bar{n}=1.8\times10^{19}$ m^{-3}, $I_p=200$ kA, $B_T=2.0$ T. (b) For higher $\bar{n}=3.2\times10^{19}$ m^{-3}.

density of Fig. 1(b). The modulation of the turbulence level ($\Delta\tilde{n}/\tilde{n}_0$) is also shown.

These results virtually exclude any simple homogeneous factorable model for the flux, e.g.,

$$\Gamma_a = C_a n^{a_1} (\nabla n)^{a_2} T^{a_3} (\nabla T)^{a_4} \cdots \tag{2a}$$

or

$$\Gamma_\beta = C_\beta n^{\beta_1} (\tilde{n}/n)^{\beta_2} \cdots , \tag{2b}$$

in terms of local plasma parameters or local turbulence levels. Such forms are often used for convenience and can arise naturally as the result of turbulence calculations. These include resistivity-gradient-driven turbulence [7] and many simpler forms of drift-wave turbulence [1,8]. More generally, the results also exclude forms with a factor $a+b\eta_e$ with $a,b>0$. Such forms include ion pressure-gradient-driven turbulence [9] and most forms of drift-wave turbulence for the range of collisionalities in TEXT [10,11]. The nonlinearity arising from the density dependence in an expression like Eq. (2a), for example, has been mooted as the principal cause of inward convective pinch velocities observed in density perturbation experiments [5].

Fluxes of the form of Eq. (2) are not consistent with a linear response which is comparable with the equilibrium value. A Taylor expansion of these equations implies that $\Delta\Gamma/\Gamma_0 \sim a_1(\Delta n/n_0) + \cdots$. Since the perturbations to all local quantities $\Delta n/n_0$, $\Delta(\nabla n)/\nabla n_0$, etc., are of order 10%, including the turbulence levels as shown in Fig. 1, at least some of the a_i,β_i would have to be quite large to produce the large perturbed fluxes observed. However, if an exponent is large enough to account for $\Delta\Gamma$, the Taylor series would not be accurate and nonlinearities with perturbation amplitude should be apparent in Fig. 1. Such large exponents ($a_i > 10$) would also lead to great differences between equilibrium and perturbed or relaxation behavior, strongly nonsinusoidal wave forms for perturbations, and difficulties in calculating reasonable self-consistent equilibrium profiles.

The most obvious alternative is to replace the simple form of Eq. (2) by a more complex transport model of the sort illustrated in Fig. 2, for which $\Gamma(\nabla n)$ has a significant intercept. The solid and dashed curves represent plausible choices. These forms clearly permit a small fractional change in gradient to cause a large fractional change in flux, and hence $\Delta\Gamma/\Gamma_0$ can be large without requiring any nonlinearities. Such a form for the flux leads naturally to the transport representation customarily used in the analysis of particle transport experiments, $\Gamma = -D\nabla n + Vn$, where V includes all off-diagonal transport processes. The coefficients D and V would depend on local parameters, possibly including gradients, as exemplified by the dashed curve in Fig. 2. The separation of D and V for the equilibrium may not be unique, and the values describing a linear perturbation about equilib-

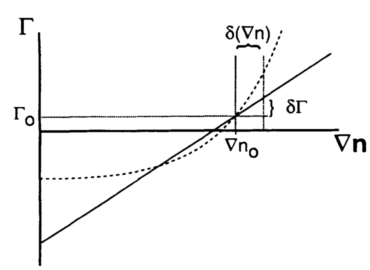

FIG. 2. Sketch of Γ vs ∇n necessary to explain the experimental observations. The solid line is the simplest form suitable, but alternate forms like the dashed curve through the same equilibrium point are also possible.

rium, for example, the D representing the slope of the flux curve in Fig. 2 near the equilibrium point, can differ from those for the equilibrium, for which D would typically be the slope of the straight solid line. Nevertheless, the experiments clearly indicate a form like that of Fig. 2 rather than the simpler Eqs. (2). The rejection of Eq. (2b) is particularly significant. The common working hypothesis that the flux may be inferred from $\tilde{n}^2$ with a simple expression like Eq. (2b) is inappropriate. That is also an implication of Fig. 1: The large fractional perturbation to the flux is not associated with a large fractional perturbation in the turbulence level.

The connection between the observed electrostatic turbulence and transport cannot be uniquely ascertained from these experiments, but three general types of relation are possible: (a) The fluxes are driven entirely by electrostatic turbulence:

$$\Gamma=\int d\omega\,\tilde{n}(\omega)\tilde{\phi}(\omega)|\gamma(\omega)|k_\theta(\omega)\sin[\alpha(\omega)]/B_T\,. \quad (3)$$

The reduction of this general form [2] to Eq. (2b) assumes that the ω dependencies are sufficiently smooth that the mean value theorem can represent the integral with typical values of $\tilde{n}$, the coherence γ, poloidal wave number k_θ, phase angle α, etc., which in turn are smooth functions of the local plasma parameters, and the potential fluctuations $\tilde{\phi}$ are similarly proportional to $\tilde{n}$. On the contrary if there are strong frequency dependencies or functional sensitivities in some terms, e.g., the phase α, such a reduction is impossible. Instead, an expression of the form [11] $\Gamma=\Gamma_0(a+b\eta_e)$ but with $b<0$ can result, which does behave like Fig. 2. (It has proved difficult to obtain this form in the collisionality regimes of this experiment and thus there is no specific prediction of the relation between turbulence and flux.) Merely measuring the turbulence *level* would certainly be insufficient to determine the flux in these analyses. (b) Only part of the flux (e.g., the diffusive portion) is driven by the observed turbulence. The remaining flux is driven by other processes, for example, turbulence at shorter wavelengths. (The most obvious possibility, that of having the inward convection supplied by the neoclassical pinch effect, can be rejected as being much too small [4].) (c) The observed turbulence is an incidental effect not causally related to transport. Since the electrostatic turbulence observed in the interior here extends continuously to the edge, where it becomes the established cause of particle transport [2,12], the former hypotheses are preferred. Furthermore, since the edge has steep gradients and is dominated by diffusion, the association of this turbulence with interior diffusion is a natural conjecture. (A separation of the fluxes into diffusive and convective components suggests that this conjecture is quantitatively admissible, but it leaves open the origin of the equally important convective flux.)

The complexity of particle transport is matched by thermal transport. The simplest turbulence theories corresponding to Eq. (2) imply an energy flux

$$Q=\alpha(kT_e+kT_i)\Gamma\,, \quad (4)$$

with α variously $\frac{3}{2}$ to $\frac{5}{2}$ or higher. Many drift-wave models have $\alpha<\frac{5}{2}$, but an asymptotic limit of dissipative trapped electron (DTE) turbulence [13] has $\alpha\sim 5$ for the electron component. This is another instance in which a quite complicated theory for the particle flux nevertheless reduces to the form of Eq. (2b) for the conditions of the experiment. Likewise η_i modes in regimes for which the electron channel dominates [9] have $\alpha\sim 2$. The difficulty with the hypothesis of Eq. (4) is shown in Fig. 3(a). The energy flux is obtained by analogy with Eq. (1a) assuming the Ohmic input power follows the electron temperature profile with Spitzer resistivity and uniform $Z_{\rm eff}$, corrected for radiation losses. A particle convention process is inadequate to account for thermal transport. For the lowest density in Fig. 3(a), a high value of α in this parameter regime of DTE has permitted a reasonable match [13] between Γ and Q_e for $0.2<r/a<0.5$, but the energy flux becomes quite inconsistent with the particle flux at higher densities. Even in the edge, where in some cases electrostatic turbulence can account for the energy flux [12], temperature fluctuations are important contributors to the thermal transport. The temperature fluctuations can be regarded either as increasing the coefficient α or as introducing thermal conduction independent of particle convection. In the interior, there seems to

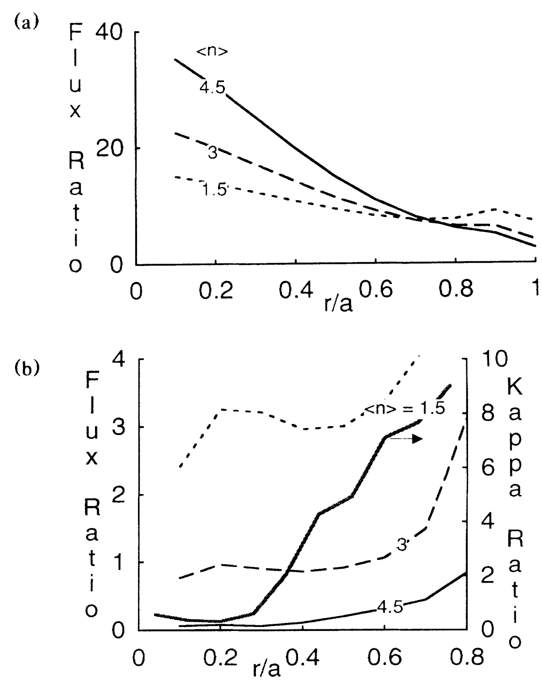

FIG. 3. Energy fluxes for conditions of Fig. 1. (a) Equilibrium ratios $Q/kT_e\Gamma$. (b) Perturbed flux ratios $|\Delta Q_e|/kT_e|\Delta\Gamma|$ and thermal conductivity (kappa) ratio of perturbation to equilibrium for low density (halftone line).

be no simple relation between particle and thermal fluxes. The difficulty of establishing causal relations between turbulence and the multiple transport processes are thus compounded.

The perturbation experiments confirm the separation of particle and thermal transport. Temperature variations are associated with the density perturbations, but through processes more complicated than that of the convective term in energy balance. The perturbed (electron) energy flux can be approximated by

$$\Delta Q_e(r) = \frac{1}{r} \int_0^r r'[\Delta q - \tfrac{3}{2} i\omega(\Delta n T + n\Delta T)]dr' , \qquad (5)$$

where Δq includes all sources and sinks (perturbations to Ohmic input, radiation, and ion coupling) and the source in the particle equation has been neglected. In the plasma interior, the current density remains constant on the time scale of these perturbations, and the only significant source term is $\Delta q = -\frac{3}{2}(\Delta T/T_0)q_0$ from the modulation of conductivity. This ansatz is used to calculate ΔQ_e in the interior, the only region for which reliable values of ΔT are available.

If the energy flux is described by Eq. (4), $\Delta Q_e = \alpha(k\Delta T_e\Gamma + kT_e\Delta\Gamma) \approx \alpha kT_e\Delta\Gamma$, since $\Delta\Gamma/\Gamma$ greatly exceeds $\Delta T/T$. This relation is examined in Fig. 3(b). Although the values are not grossly outside the range of $\frac{3}{2}$ to $\frac{5}{2}$, the results are not well described by any constant, and the ratio differs greatly from the equilibrium result of Fig. 3(a).

A common alternative to Eq. (4) is a transport model with a thermal conductivity $\kappa = Q_c/(dT/dr)$, where Q_c is the conducted energy flux, $Q - \frac{5}{2}kT\Gamma$. This κ has been computed for both the equilibrium and perturbation, and the ratio is plotted in Fig. 3(b). The ratio is not 1 nor even a constant. Furthermore, the relative phases of ΔQ_e (total or conducted), $\Delta\Gamma$, Δn, ΔT_e, and $\Delta\tilde{n}$ are different and vary differently with radius, confirming their independence and the genuine complexity of transport. The simpler models will not suffice and more complicated forms like a $Q = Q_0(c + d/\eta_e)$ analog of the particle flux [11] must be considered.

The clear and strong conclusion from these experiments is that there are at least three separate transport processes in the tokamak plasma, conventionally described as particle diffusion and convection and thermal diffusion. Simple relations like Eqs. (2) and (4) are excluded. In some sense, a transport matrix with independent D and χ and a large off-diagonal term, at least for particle transport, is required. An important consequence of this complexity, and a direct implication of these experiments, is that the turbulence level, by itself, is not necessarily a good measure of any specific transport process or matrix element. Efforts to establish the causal relations between turbulence and transport will require much more care and detail.

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(a)Permanent address: Auburn University, Auburn, AL 36830.

(b)Permanent address: University of California, Los Angeles, CA 90024.

(c)Permanent address: Rensselaer Polytechnic Institute, Troy, NY 12180.

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