Quasiparticle Charge and the Activated Conductance of a Quantum Hall Liquid

Dung-Hai Lee
IBM Research Division, T. J. Watson Research Center, Yorktown Heights, New York 10598

Steven Kivelson
Department of Physics, University of California at Los Angeles, Los Angeles, California 90024-1547

Shou-Cheng Zhang
IBM Research Division, Almaden Research Center, San Jose, California 95120
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We provide a theoretical basis for Clark's proposal that for quantum Hall liquids at magic filling factors, where the longitudinal conductivity is exponentially activated, $\sigma_{xx} = \sigma_{xx}^0 e^{-\Delta/k_B T}$, the prefactor σ_{xx}^0 is proportional to the square of the quasiparticle charge. We also propose that the same experiments uncover a remarkable law of corresponding states.

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When a 2D electron gas is placed in a strong perpendicular magnetic field incompressible quantum Hall liquids (QHL) are formed at a series of rational filling factors [1]. The signatures of these QHL states are (i) $\rho_{xx} \to 0$ as the temperature $T \to 0$ and (ii) ρ_{xy} is quantized with value $\rho_{xy} = S_{xy}^{-1} h/e^2$, where S_{xy} is one of a hierarchy of possible rational numbers. If one fixes the magnetic field so that the filling factor is equal to one of the magic fractions S_{xy}, and studies ρ_{xx} as a function of temperature, one finds experimentally that within a well-defined temperature range $\rho_{xx} \simeq \rho_{xx}^0 e^{-\Delta/k_B T}$. Surprisingly, on careful analysis of the experiments [2], Clark *et al.* demonstrated the amazing fact that ρ_{xx}^0 is the same among pairs of QHL states whose filling fractions have the same numerator. Extending this analysis, Clark defined the conductivity prefactor, σ_{xx}^0, using the formula $\sigma_{xx}^0 \equiv \rho_{xx}^0/[(\rho_{xx}^0)^2 + (S_{xy}^{-1} h/e^2)^2]$ and found that $\sigma_{xx}^0 \simeq e^2/hq^2$, where p/q is the irreducible fraction corresponding to S_{xy}. In an interesting remark, Clark suggested that these experiments effectively measure the quasiparticle charge [3] $e^* = e/q$.

In this Letter, we provide a theoretical basis for Clark's interpretation of the experimental results. In addition, we propose that Clark's experiments are a manifestation of a remarkable "law of corresponding states [4]," which states that *regardless of the electronic state, the conductivity tensor of a 2D electron gas at a magic filling factor can be parametrized by a single conductivity σ_b.* If we represent the quasiparticles as bosons coupled to a statistical gauge field [5], we shall see below that σ_b is the dimensionless conductivity of these bosons.

The QHL's form a hierarchy of states in which a continuous zero-temperature transition can occur between a parent and daughter state [6,7] as a function of the magnetic field strength or as a function of disorder. For example, the $S_{xy} = 0$ (insulating) state is the parent [8] of the $S_{xy} = 1$ QHL, and the $S_{xy} = 1$ state is the parent of the $S_{xy} = \frac{2}{3}$ state. At the transition, the critical conductance, σ_{xx}^c is observed to be finite. We will show that *if certain assumptions are satisfied,* σ_{xx}^0 is related to σ_{xx}^c via the relation $\sigma_{xx}^0 = (1+\theta^2) S_2(0) \sigma_{xx}^c$, where $S_2(0)$ is a universal constant, and θ is the statistical parameter [9] of the quasiholes in the QHL state.

Before we embark on the general analysis, it is instructive to begin by considering a limit of the problem where the relation between the activated conductance and a critical conductance can be simply established. Consider a system of noninteracting electrons in a transverse magnetic field in the presence of a disorder potential. For noninteracting particles the real part of the dc σ_{xx} is given by

$$\sigma_{xx} = -\int \frac{\partial f}{\partial E} \sigma_1(E) , \tag{1a}$$

where f is the Fermi-Dirac (or Bose-Einstein) distribution function, and

$$\sigma_1(E) = \frac{\pi}{A} \sum_{n,m} \delta(E - E_n) \delta(E - E_m) |\langle n | J_x | m \rangle|^2 , \tag{1b}$$

where A is the total area, $|n\rangle$ and $|m\rangle$ are single-particle eigenstates, and $\mathbf{J}$ is the current operator. For fermions at zero temperature, $-\partial f/\partial E = \delta(E - E_F)$, and hence $\sigma_{xx} = \sigma_1(E_F)$. Disorder causes [7] all single-particle eigenstates to be localized except those at a discrete set of critical energies E_c (which in the limit of weak disorder lie near the middle of each Landau band). Thus, $\sigma_1(E) = 0$ for $E \neq E_c$ and $\sigma_1(E_c) = \sigma_1^c$. The zero-temperature transition between two integer quantum Hall liquids, or from an integer quantum Hall liquid to an insulator, occurs when one of the E_c passes through the Fermi energy. At the critical point, $E_F = E_c$, and $\sigma_{xx} = \sigma_{xx}^c = \sigma_1^c$. Therefore σ_1^c is the critical conductance at the zero-temperature transition between QHL's or between a QHL and an insulator. Let us now consider an integer QHL (i.e., the Fermi energy lies in a region of localized states) at finite temperature. At very low temperatures variable-range hopping will dominate the conduction, but at intermediate temperatures activated conduction sets in, where, in addition to thermal activation, temperature introduces an effective system size $L_{\rm eff} \propto T^{-p/2}$.

According to the scaling picture [7], the energy band of delocalized states for a system of size $L_{\rm eff}$ has an energy width Γ where $\Gamma \propto L_{\rm eff}^{-1/\nu_\xi}$ (the current best estimate for ν_ξ is $\nu_\xi \simeq \frac{7}{3}$). Therefore $\sigma_1(E) = \sigma_1^c S_1(E - E_c/\Gamma)$, where $S_1(x)$ is a scaling function satisfying $S_1(0) = 1$ and $S_1(x) \to 0$ as $|x| \to \infty$. The activated conductivity is given by

$$\sigma_{xx} \simeq \sigma_1^c [e^{-(E_{-c} - E_F)/k_B T} + e^{-(E_F - E_{+c})/k_B T}] S_2\left(\frac{k_B T}{\Gamma}\right), \tag{2a}$$

where $E_{\pm c}$ are the energies of the delocalized states in the filled and empty Landau bands. $S_2(y) \equiv \int_{-\infty}^{\infty} dx \times e^{-x} S_1(xy)$ is also a scaling function as long as y is small enough so that the integral converges. At a magic filling, E_F is halfway between Landau bands, and particles and holes are created in pairs; therefore $E_{-c} - E_F = E_F - E_{+c} = \Delta$, and Eq. (2a) reduces to

$$\sigma_{xx} \simeq 2\sigma_1^c e^{-\Delta/k_B T} S_2\left(\frac{k_B T}{\Gamma}\right). \tag{2b}$$

Hence for the temperature range in which $k_B T \ll \Delta$, but $k_B T$ is still large enough so that the variable-range hopping can be ignored, σ_{xx}^0 is given by $\sigma_{xx}^0 = 2\sigma_1^c S_2(0) = 2S_2(0)\sigma_{xx}^c$. If we suppose that σ_{xx}^c is universal, then so is σ_{xx}^0. We shall return to this question at the end of this paper. Inverting the logic, therefore, the universal $\sigma_{xx}^0 \simeq e^2/h$ observed by Clark *et al.* [10] in the integer quantum Hall effect is experimental evidence that σ_{xx}^c is indeed universal.

In the rest of this paper, we shall derive a law of corresponding states which, among other things, allows us to generalize the above results to include the fractional quantum Hall effect.

The law of corresponding states.—Consider a 2D electron gas at a magic filling factor (it may or may not be in the QHL phase). Let the dimensionless Hall conductance, charge, and statistical parameter of the quasiholes *in the QHL phase* be S_{xy}, $e^* = \eta e$, and θ, respectively [9]. We define a new parameter σ_b which will generally depend on temperature and the microscopic details. As shown below, we find that the physical conductivity tensor can be parametrized in terms of σ_b as

$$\sigma_{xx} = \frac{(\eta e)^2}{h} \frac{\sigma_b}{1 + (\sigma_b \theta)^2},$$
$$\sigma_{xy} = \frac{e^2}{h}\left(S_{xy} - \theta\eta^2 \frac{\sigma_b^2}{1 + (\sigma_b\theta)^2}\right). \tag{3}$$

If we allow the σ_b in Eq. (3) to vary between zero and infinity, we obtain a conductivity tensor which interpolates between that of a QHL and its parent state.

Let us first apply Eq. (3) to the $T = 0$ transitions between the $S_{xy} = n$ integer QHL (where $\sigma_b = 0$) and its parent $S_{xy} = n - 1$ QHL (where $\sigma_b = \infty$). According to the scaling theory [7], at the transition the Hall conductivity is $\sigma_{xy}^c = (n - \frac{1}{2})e^2/h$. Set $\theta = \eta = 1$ (for the IQHE, the quasihole carries charge $+e$ and has the statistics of a fermion) and $S_{xy} = n$ and invert Eq. (3) with $\sigma_{xy} = (n - \frac{1}{2})e^2/h$; the result is $\sigma_b^c = 1$. This in turn leads to the prediction that $\sigma_{xx}^c = \frac{1}{2}e^2/h$. If we now *assume* that $\sigma_b^c = 1$ is *universal* we obtain

$$\sigma_{xx}^c = \frac{(\eta e)^2}{h} \frac{1}{1 + \theta^2},$$
$$\sigma_{xy}^c = \frac{e^2}{h}\left(S_{xy} - \theta\eta^2 \frac{1}{1 + \theta^2}\right) \tag{4}$$

for *all* transitions between QHL's and between a QHL and an insulator.

A correspondence also exists at finite temperatures and away from the critical point. As examples, we consider the filling factors at which Clark measured the activated conductivity: $\nu = \frac{2}{3}, \frac{2}{5}, \frac{3}{5}, \frac{3}{7}, \frac{4}{7}, \frac{4}{9}$. At these filling factors the theoretically expected values of the quasihole charge and statistics parameters are $\eta = -\frac{1}{3}, \frac{1}{5}, -\frac{1}{5}, \frac{1}{7}, -\frac{1}{7}, \frac{1}{9}$ and $\theta = -\frac{1}{3}, \frac{3}{5}, -\frac{3}{5}, \frac{5}{7}, -\frac{5}{7}, \frac{7}{9}$, respectively. Substituting these values of η and θ into Eq. (3) and inverting the conductivity tensor we obtain

$$\rho_{xx} = \frac{h}{e^2} \frac{\sigma_b}{\sigma_b^2 + 4}, \quad \rho_{xx} = \frac{h}{e^2} \frac{\sigma_b}{4\sigma_b^2 + 9},$$
$$\rho_{xx} = \frac{h}{e^2} \frac{\sigma_b}{9\sigma_b^2 + 16}, \tag{5}$$

for $\nu = \frac{2}{3}, \frac{2}{5}$, $\nu = \frac{3}{5}, \frac{3}{7}$, and $\nu = \frac{4}{7}, \frac{4}{9}$, respectively. Therefore, pairs of filling factors have the same ρ_{xx} *so long as they have the same numerator and the same* σ_b. In general σ_b depends on sample-specific details such as the strength of the disorder potential, the temperature, the value of magnetic field, etc. However, we shall later show that for finite but low temperatures $\sigma_b \simeq S_2(0) \times e^{-\Delta/k_B T}$, *independent of microscopic details.* As a result, among pairs of filling factors which have the same numerator in Eq. (5), ρ_{xx}^0 is the same. Moreover, σ_{xx}^0, the prefactor of the activated conductivity, is given by

$$\sigma_{xx}^0 = \frac{(e^*)^2}{h} S_2(\theta). \tag{6}$$

Indeed, as Clark proposed, σ_{xx}^0 is proportional to the square of the quasiparticle charge. This result combined with Eq. (4) yields $\sigma_{xx}^0 = (1 + \theta^2) S_2(0) \sigma_{xx}^c$.

Derivation of the law of corresponding states.—To understand the origin of Eq. (3) and the meaning of σ_b, let us consider the following long-wavelength and low-frequency effective action for a 2D electron gas at a magic filling fraction (we have represented the quasiparticles as bosons coupled to a statistical gauge field, and chosen the units so that $e/c = \hbar = 1$):

$$S=\int d^2r\,d\tau\left[-\frac{i}{4\pi}S_{xy}\varepsilon_{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda+\frac{i}{4\pi\theta}\varepsilon_{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda-i(a_\mu-\eta A_\mu)J_\mu+S_m\right].\tag{7a}$$

Here S_{xy}, θ, and ηe are the dimensionless Hall conductance, statistics parameter, and effective charge of quasiholes *in the QHL phase*, respectively. $J_0=\sum_i q_i\delta^2(\mathbf{r}-\mathbf{r}_i)$ and $\mathbf{J}=\sum_i q_i(d\mathbf{r}_i/d\tau)\delta^2(\mathbf{r}-\mathbf{r}_i)$ are the charge density and current of bosons with charge q_i, situated at position $\mathbf{r}_i(\tau)$, a_μ is the statistical gauge field, and A_μ is the physical gauge field. The partition function is given by

$$Z=\sum_{N_\pm}\sum_P\frac{1}{N_+!N_-!}\int_{\mathbf{r}_i(\tau=\beta)=\mathbf{r}_{P(i)}(\tau=0)}D[\mathbf{r}_i(\tau)]D[a_\mu]e^{-S}.\tag{7b}$$

The last term S_m in Eq. (7a) is given by

$$S_m=N_+\Delta_++N_-\Delta_-+\int d^2r\,d\tau\,\tfrac{1}{2}\eta^2J_0VJ_0-\eta\mu(\mathbf{r})J_0+i\eta\mathbf{J}\cdot\mathbf{A}_{\rm ext},\tag{7c}$$

where Δ_+ (Δ_-) is the quasihole (quasielectron) creation energy, N_+ (N_-) is the total number of quasiholes (quasielectrons) (at a magic filling fraction $N_+=N_-$), $\boldsymbol{\partial}\times\mathbf{A}_{\rm ext}=H$ (H is the applied magnetic field), and $\mu(\mathbf{r})$ is the disorder potential. Equations (7) define a problem of massive bosons in a magnetic field ηH, moving according to guiding-center dynamics in a spatially random potential $\eta\mu(\mathbf{r})$, and interacting via $\eta^2V(\mathbf{r})$ with each other. To calculate the total conductivities one has to integrate out J_μ [i.e., perform the sum over $N_\pm$ and the path integral over $\{\mathbf{r}_i(\tau)\}$ and a_μ]. The end result is an effective action which depends on A_μ alone.

To quadratic order, the effect of integrating out J_μ is to produce the following effective action:

$$S'_{\rm eff}=\int d^2r\,d\tau-\frac{i}{4\pi}S_{xy}\varepsilon_{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda+\frac{i}{4\pi\theta}\varepsilon_{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda+\frac{1}{4\pi}f_{0i}\pi_1f_{0i}+\frac{1}{4\pi}f_{12}\pi_2f_{12},\tag{8}$$

where $f_{\mu\nu}\equiv\partial_\mu(a_\nu-\eta A_\nu)-\partial_\nu(a_\mu-\eta A_\mu)$, and π_1,π_2 are nonlocal space-time functions which contain all the information about the boson J_μ-J_ν correlation functions. The σ_b in Eq. (3) is the boson conductivity which is given by

$$\sigma_b\equiv\lim_{\omega\to0}\frac{1}{\omega}\int d^2r\,d\tau\,e^{-i\omega\tau}\langle J_k(\mathbf{r},\tau)J_k(0,0)\rangle=\lim_{\omega\to0}\omega\pi_1(\mathbf{q}=0,\omega).\tag{9}$$

We can now integrate out a_μ to obtain the final effective action for A_μ,

$$S_{\rm eff}=\int d^2r\,d\tau-\frac{i}{4\pi}\left[S_{xy}-\frac{\eta^2}{\theta}+\Pi_3\right]\varepsilon_{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda+\frac{1}{4\pi}F_{0i}\Pi_1F_{0i}+\frac{1}{4\pi}F_{12}\Pi_2F_{12},\tag{10a}$$

where $F_{\mu\nu}\equiv\partial_\mu A_\nu-\partial_\nu A_\mu$, and $\Pi_{1,2,3}$ are related to $\pi_{1,2}$ via

$$\Pi_1=\left(\frac{\eta}{\theta}\right)^2\frac{\pi_1}{D},\quad\Pi_2=\left(\frac{\eta}{\theta}\right)^2\frac{\pi_2}{D},\quad\Pi_3=\left(\frac{\eta}{\theta}\right)^2\frac{1}{\theta D},\tag{10b}$$

where

$$D(\mathbf{q},\omega)\equiv\omega^2\pi_1(\mathbf{q},\omega)^2+q^2\pi_1(\mathbf{q},\omega)\pi_2(\mathbf{q},\omega)+\theta^{-2}.$$

From Eq. (8) the total conductivities can be deduced via

$$\sigma_{xx}=\frac{e^2}{h}\lim_{\omega\to0}\omega\Pi_1(\mathbf{q}=0,\omega),$$
$$\sigma_{xy}=\frac{e^2}{h}\lim_{\omega\to0}\left[S_{xy}-\frac{\eta^2}{\theta}+\Pi_3(\mathbf{q}=0,\omega)\right],\tag{11}$$

and the result is Eq. (3).

We first comment on the zero-temperature transitions between QHL's or between a QHL and an insulator. In our theory, the transition is triggered by the condensation of the boson particles and antiparticles in Eq. (7c). To be more specific, on the QHL side of the transition the strength of the disorder $W=\langle\mu(\mathbf{r})^2\rangle^{1/2}\ll\Delta\equiv\frac{1}{2}(\Delta_-+\Delta_+)$, and the boson conductivity $\sigma_b=0$ at zero temperature. The transition is approached when $W\to W_c=O(\Delta)$ and at the transition $\sigma_b>0$. For $W>W_c$, the Bose particle and antiparticle condense into a superconducting glass state; hence $\sigma_b\to\infty$. At the critical point, requiring Eq. (3) to reproduce the prediction of the scaling theory [7] that $\sigma_{xx}^c=(n-\frac{1}{2})e^2/h$, we obtain $\sigma_b^c=1$, which in turn implies $\sigma_{xx}^c=\frac{1}{2}e^2/h$. This result, combined with Eqs. (1a) and (1b), yields $\sigma_1^c=\frac{1}{2}e^2/h$.

We next verify Clark's conjecture by showing that $\sigma_b\simeq S_2(0)e^{-\Delta/k_BT}$ for $k_BT\ll\Delta$. For this discussion let us consider a weak random potential (i.e., $W\ll\Delta$), and low temperature (i.e., $k_BT\ll\Delta$). Under these conditions, the density of thermally excited boson particle/antiparticle pairs is extremely low. Therefore to a good approximation we can ignore the interaction between the bosons [this is especially true if the interaction $V(\mathbf{r})$ is short range]. In that case, it is meaningful to talk about the single-particle eigenstates of the model defined by (7c), and use Eqs. (1a) and (1b) to compute the low-temperature conductivity. Since aside from the boundary condition on the path integral, the action in Eq. (7c) coincides with the action of the electron guiding centers in the lowest Landau level, we can identify the *single-particle* eigenstates and the matrix elements of current operator in these two problems, so long as H and $\mu(\mathbf{r})$ are the same. Therefore we can identify their $\sigma_1(E)$. In general $\sigma_1(E)$ depends on the microscopic details; however, we shall present arguments below that $\sigma_1^c=\frac{1}{2}e^2/h$ independent of microscopic details. Now let us assume for the moment that $\sigma_1(E_c)$ is indeed universal and see what the consequences are.

Under the free particle approximation, we use Eqs. (1a) and (1b) to compute the boson conductivity σ_b. Similar calculations as in the earlier part of the paper give

$$\sigma_b \simeq S_2\left(\frac{k_BT}{\Gamma}\right)e^{-\Delta/k_BT} \tag{12}$$

for the temperature range where both Γ and $\Delta \gg k_BT$, where $\Delta\equiv\frac{1}{2}(\Delta_+ + \Delta_-)$ is the average creation energy of quasielectron and quasihole.

We now address the question of the universality of $\sigma_1(E_c)$. According to our theory, transitions between QHL's (or between a QHL and an insulator) occur when the bosons in Eq. (7c) condense. In general, to obtain the behavior of an observable at a critical point, a full renormalization-group calculation is necessary. Fortunately, due to current conservation, the boson current operator does not acquire an anomalous dimension upon renormalization [11-13]; therefore the boson conductivity σ_b remains dimensionless at the critical point. *Assuming there are no degenerate irrelevant operators,* standard renormalization-group arguments require that the critical boson conductivity σ_b^c be universal [12]. This fact, combined with Eq. (3) and the fact that $\sigma_{xy}^c=(n-\frac{1}{2})e^2/h$ for the transition between the integer quantum Hall plateaus, implies $\sigma_b^c=1$. This in turn implies that $\sigma_1^c=\frac{1}{2}e^2/h$. Experimentally, the absolute value of σ_{xx} is very hard to determine. There is an ongoing theoretical effort to compute σ_{xx} at the transition in a variety of random potentials [14].

Our results rest on the assumptions (which we have argued are likely to be valid) that (1) at the Chern-Simons boson insulator to superfluid transition, there are no degenerate irrelevant operators, and hence the critical conductance is universal, and (2) at sufficiently low temperature the thermally excited Chern-Simons bosons are approximately independent of each other.

It is important to note that even given the validity of these assumptions, the prefactor conductivity cannot in principle be used to determine the quasiparticle charge with *unlimited* accuracy since at any finite temperature, $S_2(k_BT/\Gamma)$ differs somewhat from $S_2(0)$, while at asymptotically low temperatures there will be a crossover from activated conduction to variable-range hopping.

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