Sound in Sand

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We have studied the vibrations in a granular material and have found that they are qualitatively different from those found in solids or liquids. In response to an external oscillation, the vibrations experienced by a single particle in the material are exceedingly noisy with a power spectrum $S(f) \propto f^{2.2}$ over more than 5 decades in frequency. The frequency response depends on the detailed packing of the beads, reminiscent of conductance fluctuations in mesoscopic metals.

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The fundamental properties of a system are often governed by its elementary excitations. For a dry, noncohesive, granular material like sand, the excitations of primary interest are the vibrations. As a result of the nature of the contacts between individual grains in such a material, one can expect its vibrational excitations to be quite different from those associated with normal solids or liquids. The contact between two spherical grains, known as a Hertzian contact [1], has a nonlinear stress-strain relation $\Delta \propto P^{2/3}$, where Δ is the deformation of a grain under a pressure P. By modeling sand as a homogeneous medium one derives [2] that the sound speed c varies as $c \propto P^{1/6}$. Since the pressure at a point far away from the container walls [3] is proportional to the depth h, this implies that $c \propto h^{1/6}$. Because sound travels faster at the bottom, acoustic waves launched horizontally inside the sand will bend upward in a "mirage" effect and eventually escape from the top surface. Thus no conventional horizontal sound is allowed. Furthermore, the top surface is singular in the sense that $c=0$ and dc/dh diverges at that point.

Such an analysis may, however, be too simple since real sand is not homogeneous but granular. In sand, each particle has several physical contacts with its neighbors leading to the formation of a three-dimensional force-chain network. Because of this distinctive arching [4] behavior of granular material, the spatial fluctuations of the network can extend to length scales much larger than the size of a single grain. Small amplitude vibrations, which are very sensitive to the degree of contact between particles, travel predominantly along this quasistatic network so that it may not be valid to treat them in the same manner as in a homogeneous system.

Several studies have been made of what happens to a sandpile when it is subjected to large vibrations. Convection due to vibration within a sandpile has been known since Faraday [5]; Evesque and Rajchenbach [6] and Laroche, Duady, and Fauve [7] have recently studied the role of this convection in the instability of a flat free surface of a sandpile. Other studies have shown [8] that vibrations can cause the slope of the free surface of a sandpile to relax logarithmically. Mehta and Barker have also argued [9] that, depending on the vibration intensity, sand can also become more compact as well as more fluid. We report here the nature of the low amplitude vibration of individual particles within a granular medium and show some of the surprising ways that sound propagation in this medium differs from that found in homogeneous consolidated materials.

In our experiments, the granular material, which consisted of spherical glass beads of diameter $d=0.5$ cm, was contained in a rigid box of lateral dimensions 28 cm×28 cm as shown in the inset of Fig. 1(a). The depth of material ranged from 8 to 15 cm. In order to reduce the reflection of the sound waves from the walls, the container was lined with 3-cm-thick Styrofoam sheets. The

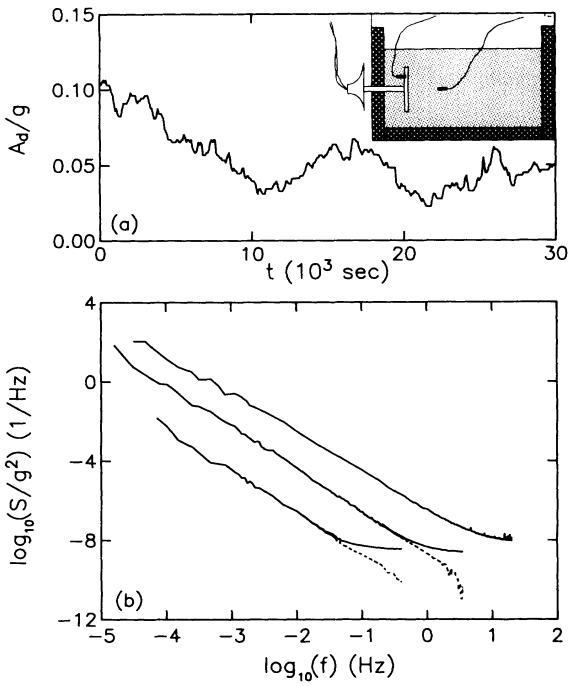

FIG. 1. (a) A typical time trace of A_d, the acceleration amplitude of the detector, for $A_s=1.4g$ ($g=9.8$ m/s^2) and $\nu=4$ kHz. Inset: A schematic diagram of the experimental configuration. (b) The power spectra S of the fluctuations in A_d at a source frequency $\nu=4$ kHz. The three curves correspond to (from bottom to top) source amplitudes A_s of $0.35g$, $1.4g$, and $4.5g$. The dashed curves show the corresponding spectra when the instrumental noise is subtracted.

results we report here are insensitive to the boundary conditions. This was tested by replacing the Styrofoam with other materials. In addition, the apparatus was carefully isolated from external vibrations and temperature fluctuations. In order to have a source with a well-defined amplitude and direction of motion, the vibration was transmitted to the grains by a 7-cm-diam aluminum disk buried in the material which was connected to an external speaker by a horizontal rigid rod, as shown in the illustration. We monitored the acceleration of the disk with an accelerometer attached to its back and controlled its amplitude through an electronic feedback loop. We embedded small detection accelerometers in the same at distances from 2 to 10 cm away from the center of the source. The size of the accelerometers, diameter 0.7 cm and length 1.2 cm, were chosen to be comparable to that of a single bead, in order to measure the motion equivalent to that of a single grain. Since accelerometers are quite massive, they respond only to vibrations transmitted through their contacts with the solid surroundings instead of to the sound pressure in air. They detect the acceleration along their axes, which we aligned to be along the direction of motion of the disk.

We first measured the speed of sound in our granular system by measuring the time delay between the generation of a pulse and its arrival at the detector. We obtained $c=280\pm30$ m/s. This is much slower than the sound speed in glass ($\sim$4000 m/s) but is comparable to the speed in air [10]. We checked that the measured vibrations were those carried by the solid network and not by the interstitial air by measuring the same value when the surrounding air was replaced by vacuum and by helium.

In order to measure the spectrum of vibrations, we drove the source with a single frequency ν: $A_s\sin(2\pi\nu t)$. Here A_s, the acceleration amplitude of the source, was kept constant and small enough so that the total power of all the harmonics was less than 1% of the power at frequency ν. The acceleration of the detector had the form $A_d(t)\sin[2\pi\nu t+\phi(t)]$, where $A_d(t)$ varied greatly in time. We measured the acceleration amplitude $A_d(t)$ and the relative phase $\phi(t)$ by using a lock-in amplifier locked to frequency ν. In Fig. 1(a), we show a typical time trace of the acceleration amplitude of the detector, $A_d(t)$, for a source frequency $\nu=4$ kHz and amplitude $A_s=1.4g$ (where $g=9.8$ m/s^2 is the acceleration of gravity). The fluctuations in $A_d(t)$ occur on many time scales, and for long enough time, they become comparable to the mean value of the vibration amplitude itself, $\delta A_d\sim\langle A_d\rangle$.

The power spectrum of such a time trace reveals the frequency components of the signal. In Fig. 1(b), we show the power spectra $S(f)$ of these fluctuations at three different source amplitudes: $A_s=0.35g$, $1.4g$, and $4.5g$. The spectra show not only that the fluctuations cover a very wide range of frequencies f, but also that there are correlations in the noise over very long periods. For example, in the middle curve in Fig. 1(b), the data for $A_s=1.4g$ can be fitted by a straight line on this log-log plot down to our lowest measurement frequency $f=2\times10^{-5}$ Hz. This corresponds to power-law behavior: $S(f)\propto 1/f^{\alpha}$, with $\alpha=2.2\pm0.05$. These results are robust over a wide range of source frequencies ν (from 500 to 8×10^3 Hz) and amplitudes A_s (from $0.014g$ to $3.4g$). The high-frequency background arises from the electronic noise of the accelerometer. For two of the spectra in the graph, we have subtracted the measured background: The power-law behavior extends over more than 5 decades in frequency. Since α is greater than 1, a low-frequency cutoff must exist, otherwise the integral of such a spectrum would diverge. Measurements of the total variance δA_d after the sand had been thoroughly disturbed allowed us to estimate a lower bound for this cutoff frequency which, for the middle curve in Fig. 1(b), is approximately 2×10^{-6} Hz.

We have tried to elucidate the origin of these time-dependent fluctuations by measuring their dependence on the source amplitude A_s. In Fig. 2(a), we show $\Delta A_d/A_s$, where ΔA_d is the rms amplitude of the fluctuations of A_d measured in the window from 1.6×10^{-3} to 64×10^{-3} Hz, plotted versus the source amplitude A_s. If some external factor were responsible for the fluctuations so that the vibration acted merely as a probing signal, $A_d(t)=A_s\eta(t)$, then $\Delta A_d/A_s=\Delta\eta$ would be independent of the source amplitude. On the other hand, if the vibration itself were acting both as the drive of the fluctuations and as well as the probe, then $\Delta A_d/A_s$ would increase as the source amplitude is increased. As seen in the figure, the fluctuations increase as A_s increases in the low amplitude region indicating that the vibration of the source itself is responsible for the fluctuations of A_d. As the vibration amplitude is increased, the fluctuations saturate, and a plateau regime appears for $0.28g<A_s<3.4g$. Figure 2(b) shows the time average of the detector amplitude, $\langle A_d\rangle$, as a function of A_s. The fit is by a power-law form $\langle A_d\rangle\propto A_s^{\beta}$. We find that $\beta=0.75\pm0.05$ instead of

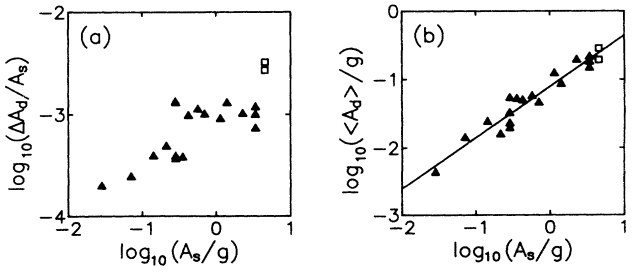

FIG. 2. (a) The rms amplitude of the fluctuations, ΔA_d, normalized by the source amplitude A_s, vs A_s. (b) The time average of the detector amplitude $\langle A_d\rangle$ vs A_s. The line is a power-law fit to the data: $\langle A_d\rangle\propto A_s^{\beta}$, with $\beta=0.75$. The power spectra corresponding to the solid triangles have exponents $\alpha=2.2$, while those for the open squares have $\alpha=2.0$.

the linear response $\beta = 1.0$. At very low amplitude, one expects the response to be linear and the data do suggest a crossover to this behavior below $A_s = 0.1g$. We emphasize that these striking noise spectra and the nonlinear behavior were obtained in the regime where all the higher harmonics of ν contained less than 1% of the total power.

Since the fluctuations are driven by the vibration itself, we checked to see whether a higher vibration amplitude might speed up the structural rearrangement and raise the low-frequency cutoff. The upper curve of Fig. 1(b) shows the power spectrum at a higher source amplitude ($4.5g$). The spectrum still behaves as a power law $S(f) \propto 1/f^{\alpha}$; however, it now has $\alpha = 2.0 \pm 0.05$ instead of $\alpha = 2.2$. The low-frequency cutoff is still beyond the experimental time scale. The data for these high amplitudes have also been shown in Fig. 2. The spectra corresponding to the open squares have exponent $\alpha = 2.0 \pm 0.05$, while those corresponding to the solid triangles have exponent $\alpha = 2.2 \pm 0.05$. The discontinuity between the solid triangles and the open squares in Fig. 2(a) as well as the change in exponent α suggests a transition to a qualitatively different regime. At even higher amplitudes, the material becomes fluidized and the spectra of fluctuations cannot be fitted by a single power law.

We would have needed to lower the source amplitude still farther than we have shown in Fig. 2 in order to be in the linear-response regime. We were prevented from obtaining those results by another intrinsic property of granular material which is unexpected from our experience with more conventional solids or liquids. Once we started the low amplitude measurements, we were immediately struck by the observation that the detector amplitude was extremely sensitive to the temperature fluctuations in the pile. A temperature change of only 0.04 K inside the pile, produced by a change of the ambient temperature or by a local heater, could cause a factor of 3 reversible change in the measured vibration transmission. Even after careful temperature isolation, thermal variations were still the dominant cause of the observed fluctuations at very low vibration amplitudes. It is not yet clear whether it is the temperature or the temperature gradient that causes this effect.

We now turn our attention to the source-frequency (ν) dependence of the vibration. We used a very small value of source acceleration such that the fluctuation with time is negligible during the duration of the measurement. Figure 3 shows the frequency dependence of the vibration transmission, $\eta(\nu) = A_d(\nu)/A_s(\nu)$, when the source and the detector were 6 cm apart. The lower curve in Fig. 3(a), displaced vertically for clarity, was obtained immediately after the one above it. The excellent reproducibility of the two curves demonstrates that the structure is truly due to the variation with frequency rather than due to fluctuations caused by vibrations. The large average attenuation further assures us that the response peaks are not the resonances of the container. In Fig. 3(b), we

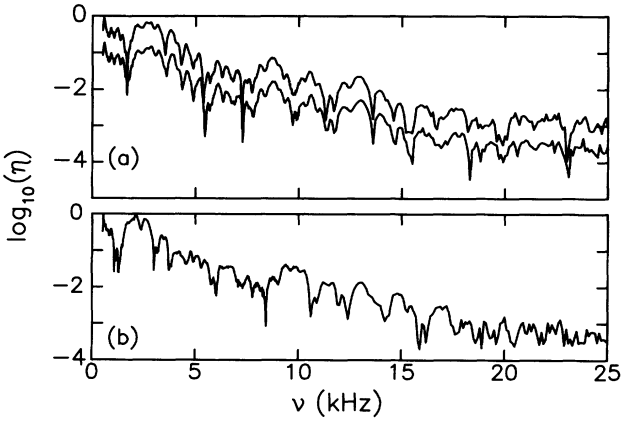

FIG. 3. (a) The frequency response $\eta(\nu)$ of the detector when placed 6 cm away from the source. The lower curve, displaced for clarity, was measured immediately after the upper one in order to show the reproducibility of the structure. (b) The same measurement as in (a) but with a slightly disturbed arrangement of the beads.

show another frequency response curve for the same configuration of detector and source but with a slightly disturbed packing of the beads. Although the response generally decreases as the frequency is increased in both Figs. 3(a) and 3(b), the lack of correlation between their fine structure indicates that $\eta(\nu)$ depends crucially on the placement of all the beads within the pile. Thus the frequency response pattern could be considered as a "fingerprint" of a specific structure.

If the vibrations travel along force chains, it is probable that there will often be several paths connecting the source and the detector; the response function $\eta(\nu)$ will be determined by the interference of the waves traveling along different routes. This interference will be sensitive to the precise packing of the beads and should be sample specific. A typical frequency scale for the response to change significantly is $\Delta\nu \approx c/\Delta L_{\text{eff}}$, where c is the speed of sound and ΔL_{eff} is the difference in length between two paths. If we estimate ΔL_{eff} as the direct distance between the source and detector, we find that $\Delta\nu \approx 5\times10^{3}$ Hz, which is too large to fit the data in Fig. 3: $\Delta\nu \approx 5\times10^{2}$ Hz. It is unclear how to provide a better estimate for $\Delta\nu$. Although the interference in this medium is reminiscent of universal conductance fluctuations in a mesoscopic system [11], the phenomenon reported here has a new aspect: The mean free path is much less than the distance traveled by the sound in one period.

We believe the temporal fluctuation in the response is a consequence of the structural relaxation of the sandpile under the very vibration that the sand is transporting. Loosely packed glass beads are in a metastable state. While subjected to vibration, they undergo rearrangement as was seen in high amplitude studies [5–8] where the vibration intensity plays the role of an effective tem-

perature [12]. Thus the vibration inevitably raises the effective temperature of the granular material from zero to some finite value. It is nevertheless a surprise that the relaxation happened under an apparently very small vibration amplitude which is less than 220 Å (for $\nu = 4 \times 10^3$ Hz and $A_s = 1.4g$) in terms of the displacement of the source and where no visible motion of the beads was observed.

As distinct from what happens in other media, vibration in granular materials can only propagate from one bead to another along the contact between them. Thus not only the diameter d of a bead but also the deformation at the contact between beads is of fundamental importance for understanding the propagation of sound in these materials. If a bead is moved by a displacement δ, defined as the average deformation of a bead under its own weight, the response can change by a significant amount. The deformation of a bead at a depth h will scale as $\delta(h/d)^{2/3}$. For the beads we used, we estimate that $\delta \approx 70$ Å so that it is plausible that the small vibrations we applied (< 220 Å) could induce large fluctuations in the transmission properties of the force chains. Likewise, since the thermal expansion of a single grain [10] is roughly 400 Å/K, the temperature dependence of the transmission might also be related to a similar origin. However, this argument does not indicate the origin of the correlations and power-law behavior in the noise.

The excitations in granular materials have unexpected properties which are not found in other condensed systems. We have focused on the spectrum of vibrations that a single particle in such a material experiences in response to an externally driven monochromatic vibration. Even for low amplitude excitation, the vibrations experienced by the detector are exceedingly noisy with a power spectrum that obeys a power law over more than 5 decades in frequency: $S(f) \propto 1/f^{\alpha}$, with $\alpha \approx 2.2$. This behavior crosses over the $\alpha \approx 2.0$ at higher amplitudes. These fluctuations are due to the rearrangements which occur during vibration. It is suggestive that a random walk, as well as the high-frequency tail of a random Lorentzian process, can give rise to a $1/f^2$ power spectrum. It is not clear, however, what determines the low-frequency cutoffs in models based on these analogies nor why the power law is greater than 2 at low amplitudes. A second surprising feature that we discovered is the extreme sensitivity of the sound propagation to the temperature within the material. Finally the response of the medium has a very complicated but reproducible spectrum as a function of the driving frequency. This observation suggests that vibration propagation in sand might be related to wave interference phenomena in random media such as conductance fluctuations in mesoscopic metals and localization of classical waves.

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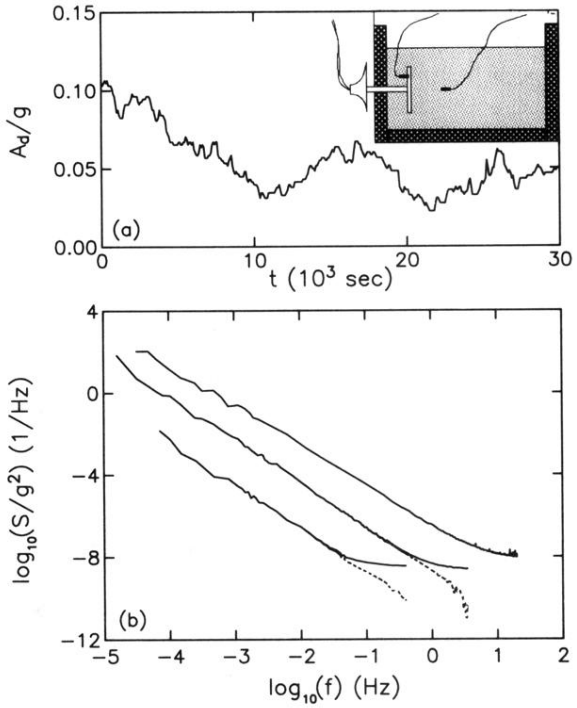

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