Effect of Singular Interaction Terms on Two-Dimensional Fermi Liquids

P. C. E. Stamp
Physics Department, University of British Columbia, 6224 Agricultural Road, Vancouver, British Columbia, Canada V6T 1Z1
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The structure of perturbation theory for an interacting two-dimensional Fermi system is analyzed, in the case where singular terms exist in the quasiparticle interactions. These give a hierarchy of singular terms in the one-particle self-energy. The dominant terms are self-consistently summed, to give a quasiparticle pole vanishing as $Z(\omega) \sim (\omega/\omega_0)^{\rho\delta_0^2/\hbar^2 N(0)\omega_0}$ near the Fermi energy (signaling a breakdown of Fermi-liquid theory), and a form for the scattering amplitude which is consistent with the assumed form for the quasiparticle interaction.

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Until recently it was almost universally assumed that the structure of perturbation theory for two-dimensional interacting fermions was essentially benign, so that Fermi liquids would exist in much the same way as in three dimensions. The remarkable recent claim of Anderson [1], that this is not so, has thus generated enormous controversy, and there are now a large number of papers in print [2] which deny Anderson's claim, for one reason or another. Anderson makes three main assertions, viz., (i) that the quasiparticle interaction function has a term of the form

$$\Delta f_{pp'} \sim \left(\frac{\delta_0}{\pi}\right) \frac{\mathbf{p}' \cdot (\mathbf{p}-\mathbf{p}')}{|\mathbf{p}-\mathbf{p}'|^2} \qquad (1)$$

which is singular as $\mathbf{p} \to \mathbf{p}'$ (forward scattering); (ii) that this leads to a breakdown of Fermi-liquid theory, with a quasiparticle pole $Z(\omega)$ vanishing as $\omega^{(\delta_0/\pi)^2}$, as the quasiparticle $\omega = (\mathbf{p}-\mathbf{p}_F)\cdot \mathbf{v}_F$ goes to zero; (iii) that the actual state of the system will be that of a two-dimensional "tomographic" Luttinger liquid.

Most arguments against these assertions [2] rely on the apparent lack of any singular behavior in low-density expansions of the quasiparticle properties.

The present paper is mostly devoted to the second assertion noted above. However, for completeness I briefly outline why terms like (1) might exist in two dimensions—this question has been discussed in more detail elsewhere [1,3,4]. The main body of the paper is then devoted to deriving the effects of these terms on the quasiparticle properties. As we shall see, they are rather subtle.

(i) Existence of singular interactions.—Anderson has given a rather unconventional argument for the existence of terms like (1) in the quasiparticle interaction function. Although a form similar to (1) is found already in second-order perturbation theory in the crossed channel (by generalizing [3] the techniques of Ref. [5]), this form has momentum-space restrictions which prevent any drastic consequences from ensuing. In fact, provided one uses the "Pauli-restricted" density of states in the Lippmann-Schwinger equation, to give a wave function [1,4]

$$\Psi(\mathbf{r}) \sim U \sum_{\mathbf{Q}} \frac{(1-f_{\mathbf{k}+\mathbf{Q}})(1-f_{\mathbf{k}-\mathbf{Q}})}{E-(E_{\mathbf{k}+\mathbf{Q}}+E_{\mathbf{k}-\mathbf{Q}})} \varphi_{\mathbf{Q}}(\mathbf{r}) \qquad (2)$$

then we *cannot* get a form like (1). However, it is argued by Anderson that in fixing boundary conditions and phase shifts, we should not enforce the Pauli restrictions. This immediately implies that the derivation of (1) is beyond a purely perturbative argument (starting from a short-range U). However, a nonperturbative derivation must still be possible, and later in this paper we shall see that a form like (1) is not necessarily in contradiction with the exact results of Fabrizio, Parola, and Tosatti [2]. To do this we must *derive systematically* the nonperturbative consequences of a singular $\Delta f_{pp'}$. Thus let us *assume* that a (dimensionally correct) term

$$\Delta f_{\mathbf{pp}'} = \frac{1}{N(0)} \left(\frac{\delta_0}{\pi}\right) \frac{\mathbf{p}' \cdot (\mathbf{p}-\mathbf{p}')}{|\mathbf{p}-\mathbf{p}'|^2} \equiv g_{\mathbf{pp}'} \qquad (3)$$

exists in the quasiparticle interaction energy, and explore its consequences. [In (3), $N(0) = m^*/\pi\hbar^2$ is the density of states, and δ_0 is a scattering phase shift.]

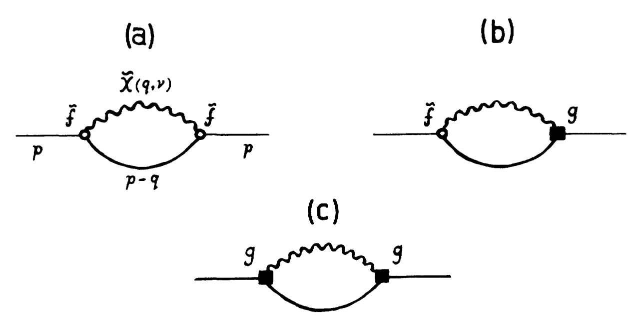

FIG. 1. (a) The leading contributions to the low-energy behavior of $\Sigma(p,\omega)$ in two dimensions, ignoring all singular contributions. $\tilde{\chi}(q,\nu)$ is the fully renormalized dynamic susceptibility (again ignoring singular terms). (b) The contribution to $\Sigma(p,\omega)$ which is first order in $g_{pp'}$ (indicated by a square vertex). (c) The most divergent second-order contribution (in $g_{pp'}$) to $\Sigma(p,\omega)$.

(ii) Low-order divergences.—The interaction $\Delta f_{\mathbf{pp}'}$ leads to drastic changes to the quasiparticle spectrum. To discuss this systematically, let us first split up the total interaction function $f_{\mathbf{pp}'}$ into its regular and singular parts, $f_{\mathbf{pp}'} = \tilde{f}_{\mathbf{pp}'} + g_{\mathbf{pp}'}$, and incorporate all renormalizations from $\tilde{f}_{\mathbf{pp}'}$ into our basic vertices (as well as in the fluctuation propagators and internal fermion lines). If we ignore $g_{\mathbf{pp}'}$ terms, this then yields a two-dimensional Fermi-liquid theory; for example, the diagram of Fig. 1(a) gives the lowest-order contribution to the regular part of the self-energy as [6]

$$\tilde{\Sigma}(p,\omega+i\delta) \sim -\left\{ \left(\frac{\tilde{m}^*}{m} - 1 \right)\omega - i\frac{\pi(\tilde{A}_0)^2}{8v_F^2 N(0)} \left[\omega^2 + 2\omega^2 \ln\left| \frac{\omega_0}{\omega} \right| \right] \right\}. \tag{4}$$

Here $\tilde{A}_0$ is the relevant Landau scattering parameter [7]. Higher-order regular contributions (which yield $\omega^3 \ln\omega$ contributions [8] in three dimensions) merely renormalize m^* here, as well as adding another ω^2 term to $\mathrm{Im}\tilde{\Sigma}$. Note that (4) is exact [6] at low ω, if we ignore contributions from $g_{\mathbf{pp}'}$; all the $(\mathbf{q},\nu)$ dependence of the four-point vertex $\tilde{\Gamma}$ has been incorporated into the fully renormalized dynamic susceptibility $\tilde{\chi}(q,\nu)$.

However, as soon as we add $g_{pp'}$ vertices things change. The lowest possible contribution is shown in Fig. 1(b), and it gives

$$\mathrm{Im}[\Delta^{(1)}\Sigma(p,\omega+i\delta)] = 2\frac{p_F}{v_F}\tilde{A}_0 \left(\frac{\delta_0}{\pi} \right) |\omega| . \tag{5}$$

Notice that this term has the same form as the "marginal-Fermi-liquid theory" (MFLT) [9]. However, it is only the first of many terms. The next contribution is shown in Fig. 1(c), and has a logarithmic divergence:

$$\mathrm{Im}[\Delta^{(2)}\Sigma(p,\omega+i\delta)] = \int d\nu \sum_{\mathbf{q}} (g_{\mathbf{p},\mathbf{p}-\mathbf{q}} g_{\mathbf{p}-\mathbf{q},\mathbf{p}})[1-\eta_{p-q}]\mathrm{Im}\tilde{\chi}(q,\nu)\delta(\nu-(\omega-\epsilon_{p-q}))$$
$$\sim \frac{p_F^2}{8\hbar^2 N(0)} \left(\frac{\delta_0}{\pi} \right)^2 \ln\left| \frac{\omega}{\omega_0} \right| , \tag{6}$$

where $\omega_0 \sim q_c v_F$ is produced by an upper momentum cutoff q_c in the fluctuation propagator $\tilde{\chi}(q,\nu)$; $\eta_p \equiv \eta(\epsilon_p)$ is the Fermi function.

Both $\Delta^{(1)}\Sigma(p,\omega)$ and $\Delta^{(2)}\Sigma(p,\omega)$ indicate an apparent breakdown of Fermi-liquid theory, although $\Delta^{(2)}\Sigma$ is more serious. Clearly a proper understanding of their effects requires that we incorporate $g_{\mathbf{pp}'}$ to all orders in $\Sigma(p,\omega)$; as we shall see, logarithmic divergences come from every *even* order in $g_{\mathbf{pp}'}$.

(iii) Self-consistent summation.—The problem we are faced with is somewhat analogous to that considered by Nozières *et al.* [10] and Anderson [11], in the Kondo and x-ray edge problems—we must *self-consistently* sum all divergent contributions to $\Sigma(p,\omega)$. However, there are two crucial differences—here the quasiparticle is indistinguishable from its fermionic brethren, and moreover it can recoil (subject to Pauli restrictions). Anderson has argued [1] that recoil is prevented by Pauli restrictions to such an extent that the problem maps onto the simple "orthogonality catastrophe" impurity problem [11], leading to a quasiparticle pole having the form specified in the introduction, i.e., $Z(\omega) \sim (\omega/\omega_0)^{(\delta_0/\pi)^2}$. We shall see here that this form is not quite right; in fact, the correct form coming from the *dominant divergent contributions* to $\Sigma(p,\omega)$ is

$$Z(\omega) \sim (\omega/\omega_0)^{\rho\delta_0^2/\hbar^2\tilde{N}(0)\omega_0}$$
$$\equiv \exp\{[\rho\delta_0^2/\hbar^2\tilde{N}(0)\omega_0]\ln|\omega/\omega_0|\} , \tag{7}$$

where $\rho = p_F^2/2\pi\hbar^2$ is the number density of the system [12]. There are also *subdominant* terms, which we will come to presently. Formula (7) implies a breakdown of Fermi-liquid theory.

I now sketch briefly the derivation of (7). Having already incorporated "Fermi-liquid" renormalizations into all graphs for $\Sigma(p,\omega+i\delta)$, we may now systematically add the singular contributions $g_{\mathbf{pp}'}$. Thus for the contribution of nth order in $g_{pp'}$, we add n $g_{pp'}$ vertices to all graphs for $\Sigma(p,\omega+i\delta)$, in all topologically distinct ways.

It is not then immediately obvious how to extract any information—nevertheless it can be done using the following theorem: The dominant divergent contributions to $\mathrm{Im}\Sigma(p,\omega)$ in this theory are given by the sum of the "maximally crossed" *even-order* (in $g_{pp'}$) reduced graphs [13] for $\Sigma(p,\omega)$.

Examples of such maximally crossed reduced graphs are shown in Fig. 2; they are defined by the property that the line cutting the relevant self-energy graph (and thereby specifying a reduced graph) must cross every fluctuation propagator $\tilde{\chi}(\mathbf{q},\nu)$ in the graph. This theorem can be verified by explicit calculation for orders up to fourth order in $g_{\mathbf{pp}'}$, and thereafter proved by induction [14].

It is then relatively easy to calculate the contribution to $Z(\omega)$ coming from these terms, since the maximally crossed reduced graphs can be factorized with a few changes of variable; one then finds that

$$\Delta_{\max}^{(2n)}\Sigma(p,\omega)=\frac{\Delta^{(2)}\Sigma(p,\omega)}{n!}\left\{-\int d\nu\sum_{\mathbf{q}}\frac{g_{\mathbf{p},\mathbf{p}-\mathbf{q}}g_{\mathbf{p}-\mathbf{q},\mathbf{p}}}{(\omega-\epsilon_{\mathbf{p}-\mathbf{q}})^2}[1-\eta_{p-q}]\mathrm{Im}\tilde{\chi}(q,\nu)\right\}^n . \tag{8}$$

Notice that the term in curly brackets is just $-\partial/\partial\omega\,\mathrm{Re}[\Delta^{(2)}\Sigma(p,\omega)]$. Now one might naively try and extract $Z(\omega)$ from this formula just by differentiating with respect to ω. This would be incorrect, however, because we require self-consistency—we must extract only the leading divergencies from $\partial/\partial\omega[\Sigma(p,\omega)]$, and these are not given by a straightforward differentiation of (8). Here we call on some well-known Ward identities. In particular [15], one has $[1-\partial\Sigma(\mathbf{p},\omega)/\partial\omega]=\Lambda^{\infty}(\mathbf{p},\omega)$, where $\Lambda^{\infty}(\mathbf{p},\omega)$ is the three-point vertex $\Lambda^{Q}(\mathbf{p},\omega)$ [which describes the interaction with an external field carrying four-momentum $Q\equiv(\mathbf{q},\nu)$], in the limit $Q\to 0$, $\nu/q\to\infty$ [Fig. 2(c)]. Again, only maximally crossed graphs contribute, and we get

$$\frac{\partial}{\partial\omega}[\mathrm{Re}\Sigma(p,\omega)]=-\sum_{n=1}^{\infty}\frac{1}{n!}\left\{-\int d\nu\sum_{\mathbf{q}}\frac{g_{\mathbf{p},\mathbf{p}-\mathbf{q}}g_{\mathbf{p}-\mathbf{q},\mathbf{p}}}{(\omega-\epsilon_{\mathbf{p}-\mathbf{q}})^2}[1-\eta_{p-q}]\mathrm{Im}\tilde{\chi}(q,\nu)\right\}^n . \tag{9}$$

Now the quantity in the curly brackets can be evaluated to give $\partial\Sigma/\partial\omega=-[\rho\delta_0^2/\hbar^2\omega_0N(0)]\ln|\omega/\omega_0|$ (plus other less divergent terms). Then, since $\partial/\partial\omega[\mathrm{Re}\Sigma(\omega)]=1-Z^{-1}(\omega)$, Eq. (7) immediately follows upon summing the series in (9).

One may go on to consider the leading singular contributions to the complete four-point vertex $\Gamma_{\mathbf{pp}'}(k,\epsilon)$, and to the thermodynamic potential Ω. The most important question here is again one of self-consistency. Thus, we may ask, is the form derived for $\Gamma_{pp'}(k,\epsilon)$ consistent with our initial form for $f_{\mathbf{pp}'}$ (including $g_{\mathbf{pp}'}$)? One may fairly quickly demonstrate that it is. Summing again the dominant (maximally crossed) contributions to $\Gamma_{pp'}(k,\epsilon)$, we find that it is renormalized by a factor $Z_{\mathbf{p}}^{-1}Z_{\mathbf{p}'}^{-1}$, coming from vertex corrections similar to those in Fig. 2(c). But a consistent calculation of $f_{pp'}$ requires $f_{pp'}\sim Z_{\mathbf{p}}Z_{\mathbf{p}'}\Gamma_{pp'}(k,\epsilon)|_{k/\epsilon\to 0}^{k,\epsilon\to 0}$ so that we just get back $g_{\mathbf{pp}'}$. Thus we have established that our theory is *self-consistent*.

We must still address the problem of the subdominant singular contributions to $Z(\omega)$. We have already noted the first such term [Eq. (5)], and it is clear that this is only the first in a whole series [16]. However, not all subdominant terms have this MFLT form—at higher order in $g_{\mathbf{pp}'}$, both $|\omega|$ and $\ln|\omega/\omega_0|$ terms will be mixed together, and one cannot rule out the appearance of other kinds of divergence.

Given the complexity of these subdominant terms, it has not yet been possible to sum them (note that this cannot be done simply using renormalization-group methods, because of the highly nontrivial angular dependence of $g_{pp'}$). Thus it is possible that the formula $Z(\omega)$ may break down at very low ω, if a new energy scale is generated by the subdominant terms—this question is presently under investigation.

(iv) Comments and conclusions.—At this point two questions naturally arise: (a) Why do the usual techniques of low-density expansion [2] fail to recover these results; and (b) can one further deduce the existence of a tomographic Luttinger liquid?

It is clear from the calculations presented above that ladder sums will not show the singular terms in their entirety—in fact, the basic ladder sum in Fig. 1(a), contains no singular terms at all. Thus the usual low-density expansion technique of Galitskiĭ [17] fails in the presence of singular interactions like (3). The zero-density results of Fabrizio, Parola, and Tosatti [2] will presumably yield (2) on functional differentiation, but again, it is then necessary to consistently deal with singular terms to all orders once this singular interaction has been obtained [12]. However, we may now observe that the results given here do not necessarily contradict the zero-density results, because the factor of density ρ in the exponent of (7) now makes the low-density expansion of (7) well-behaved [this is not the case if $Z(\omega)=(\omega/\omega_0)^{(\delta_0/\pi)^2}$, of course]. Thus apparently one objection to the hypothesis of singular interactions has been removed.

The second question is more difficult, since the theory

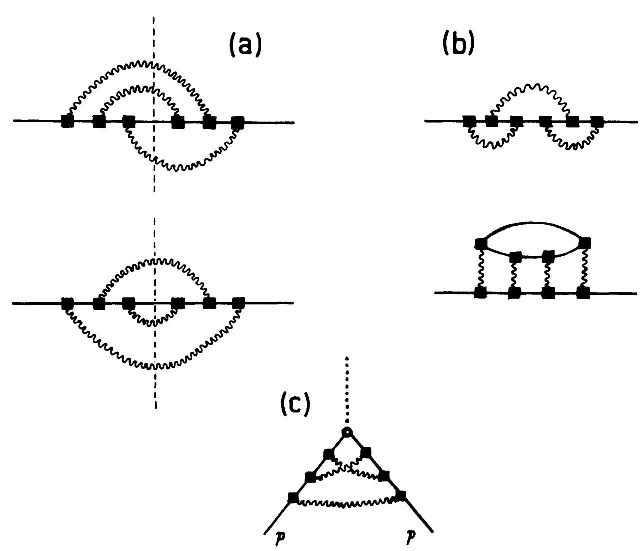

FIG. 2. (a) Examples of some sixth-order (in $g_{pp'}$) maximally crossed reduced graphs; the dashed line shows the reduction, made to cut *all* fluctuation lines. (b) Some graphs that are *not* maximally crossed. (c) A corresponding maximally crossed graph for $\partial/\partial\omega[\mathrm{Re}\Sigma(p,\omega)]$, at sixth order.

presented here merely indicates a breakdown of Fermi-liquid theory—it does not show what will be its replacement. The situation is somewhat analogous to superconductivity theory just before BCS—the instability to Cooper pair formation was known, but it was necessary to find a replacement ground state. Nor is it clear that this will be a Luttinger liquid, since $g_{p\mathbf{p}'}$ still couples fermions moving in different directions. Nevertheless, the conclusions one must draw from Eqs. (2) and (7) are dramatic enough, for they indicate that a completely new microscopic theory will be required to treat two-dimensional fermion systems with singular interactions. Such a theory will have to "build in" the hierarchy of singular terms described here, much as the Cooper pair correlations were built into the BCS state. The present paper has given a systematic way of doing this.

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Note added.—The theorem in this paper, and the results such as (7) which depend on it, have now been justified by a quite different (nonperturbative) method, using the eikonal approximation [18].

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