Experimental Study of the Geometrical Effects in the Localization of Deformation

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We have studied the localization of deformation in a 2D packing model. The samples consist of regular packings of equal parallel cylinders (drinking straws). The local stress-strain characteristics, at the contact between two straws, shows a softening part, responsible for the localization of the deformation. We have analyzed the roughness of the localization band, i.e., the width W of the localized zone versus its length L. Our results demonstrate the self-affine character of the localized zone: We find a power law, $W \sim L^{\zeta}$ with $\zeta = 0.73 \pm 0.07$, which is consistent with recent work on the "weak" disorder model.

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The deformation process in solids is often heterogeneous, and one of the most interesting cases is that of localization: The deformation is essentially concentrated in a narrow zone. This effect is generally due to heterogeneities in the solid or to a nonlinear behavior with a softening character. This localization can take various forms: breakdown, plastic instabilities, shear band observed in soil mechanics [1], etc.

One can try to explain this behavior with the help of bifurcation theory, by considering that the constitutive equation describing a homogeneous deformation may accept an alternative nonhomogeneous solution leading to a localized deformation [2]. Another approach has been developed recently in statistical physics for the fracture of disordered media: The disorder is explicitly taken into account in a discrete lattice [3]. The disorder is often a "disorder" in the "local" characteristics: For example, in *the fuse network model* it is due to fluctuations in the breaking voltage [4]. In a real material, the disorder is difficult to characterize, and it is difficult to make an experimental study of its effects on the localization of the deformation.

The honeycomb two-dimensional cellular solids show a peculiar mechanical behavior [5]: Under an increasing compression—for example, uniaxial—their deformation is first elastic, by (elastic) bending of the cellular walls, and then the deformation becomes localized, by an "elastic buckling" of these walls. The same behavior is observed if the honeycomb lattice is replaced by a regular packing (Schneebeli model) of hollow cylinders (drinking straws, for example). This material appears to be adapted to a study of geometrical effects on localization, with three types of disorder generally observed. (i) Disorder of contact: Even cylinders formally identical present slight defects responsible for an, even weak, heterogeneity in the contacts [6]. (ii) Geometrical disorder: Mixing of cylinders with different diameters yields amorphous structures [7]. (iii) Disorder of composition [8]: By mixing the straws with hard cylinders, it is possible to create a disorder of composition.

We present here the first part of a systematic study of this model, related to contact disorder: Only the behavior of ordered arrays of identical cylinders (same geometry, same material) under uniaxial compression is considered.

We have studied "Schneebeli" models, consisting of regular packings of equal cylindrical straws with horizontal parallel axes. The samples are made with 58 horizontal rows of alternatively 55 and 54 cylinders: The cylinders have a 3 cm length, sufficient to avoid an overall buckling of the samples under the compression, and a 3 mm diameter. The samples are built in a rigid U-shaped frame. To try to avoid edge effects, i.e., nonsymmetrical distribution (no sixfold axes) of the contact stresses on the straws touching the walls, and friction on the walls, the walls of the frame and of the upper piston have been Teflon coated and the straws in contact with them have been replaced by Teflon cylinders with the same diameter and the same length. The samples are placed in a testing machine: A vertical displacement Δh is imposed on the upper plate, at a very low speed (0.2 mm/min). As verified experimentally, the observed phenomena can then be considered as rate independent. The compressive vertical force is measured. During the experiment, photographs of the sample are made in order to allow a geometrical analysis of the strain within the sample.

Figure 1 shows (a), in the upper right corner, the local stress-strain characteristics (at the contact between two straws). At low stress, the (elastic) deformation is due to the bending of the wall of the straw. Then, we observe a maximum of the stress (at "controlled" displacement), followed by a softening zone corresponding to a negative slope. Finally, the slope becomes positive. The main part of the figure shows (b) the global characteristics, relative to the packing, which is very comparable to the local one. We first observe a nonlinear elastic behavior and then a short zone with a negative slope, followed by a plateau which extends over a large range of deformation. The two regimes are also observed in the case of breaking with "controlled" displacement (type I).

Figure 2 shows two photographs of the sample. In the first (a), clusters of "crushed" straws appear whose number and size increase with the strain. These clusters coalesce in a very tortuous band (b), which extends across the sample. The number of straws in this band be-

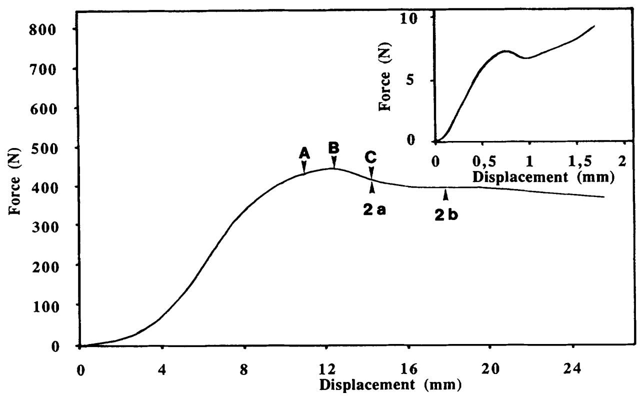

FIG. 1. Typical force-deformation curve of a Schneebeli model of hollow cylinders. Inset: The local characteristics for two straws in contact. The points $2a$,$2b$ give the positions at which the photographs of Fig. 2 have been taken.

comes larger and larger when the displacement increases, without any change in its geometry; on the contrary, we observe that some of the compressed straws out of the band relax (i.e., become normal) at the beginning of the plateau. At all stages of the process, the deformation is invariant along the axis of the straws: The geometry of the band is quite identical at the rear of the sample.

As can be seen in Fig. 2, the position of the band does not arise from the top corners of the sample, as was the case without Teflon cylinders along the walls. However, the band is generally localized in the upper half of the sample; this reveals a gradient of deformation, due to the friction at the walls [9].

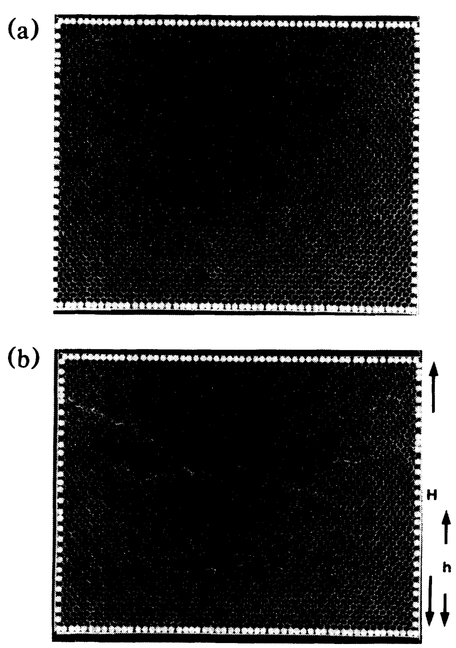

FIG. 2. Two photographs showing (a) the growing of the clusters in number and size and (b) the coalescing of the clusters in a band. The white section areas are Teflon cylinders.

If one suppresses the stress just at the beginning of the plateau, the sample relaxes and recovers its initial height: The test is not destructive, and seems to be reversible. At the second application of the pressure, the geometry of the band is exactly the same, because the structure of the sample is invariant (quenched disorder).

One of the most striking aspects of the localization is the complex geometry of the band, although our experimental setup is extremely ordered. We have previously shown [6] that even in ordered packings of cylinders small geometrical defects are present (at the scale of the deformation, the cylinders are not rigorously the same, and they are never perfectly ordered). These defects create fluctuations in the geometry and in the mechanics of the contacts between grains: This is the disorder of contacts, which determines many aspects of the nonlinear behavior of granular media [10]. This disorder, albeit weak because the geometrical defects are small compared to the large deformation of the system, is present in our samples.

A central problem in the experimental study of the localization of the deformation in mechanics is a precise

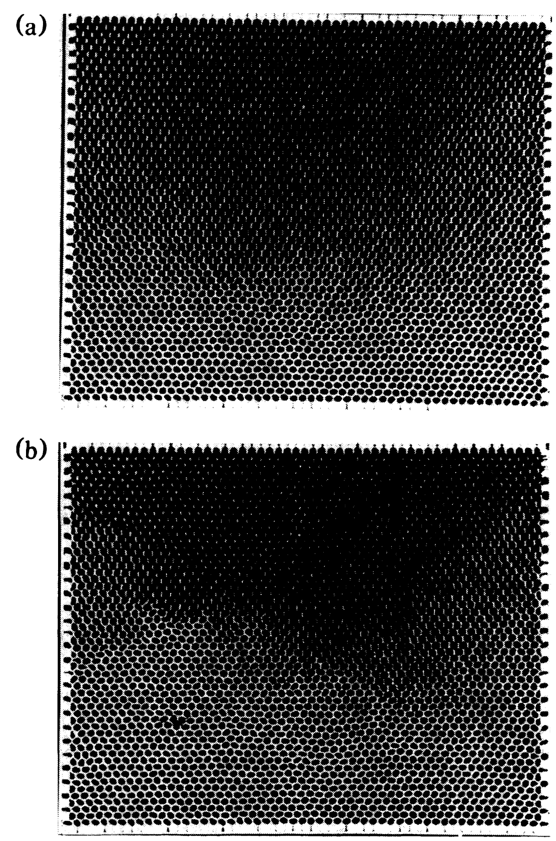

FIG. 3. Two superpositions by the Moiré technique corresponding to pairs of points (a) A-B and (b) B-C in the global characteristics. The second one shows a clear separation of the medium into two parts: The boundary line corresponds to the location of the band.

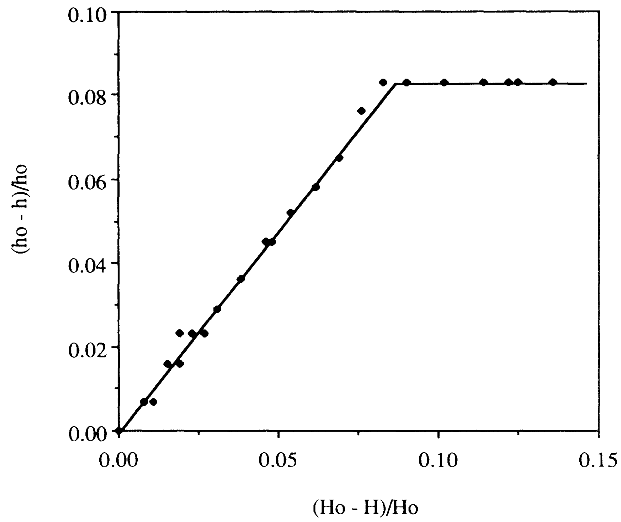

FIG. 4. Variation of the normalized height h of the n lowest rows below the band (see Fig. 2) vs the variation of the total height H of the sample. This height is also normalized. The intersection of the two lines gives the point where the localization is fully realized.

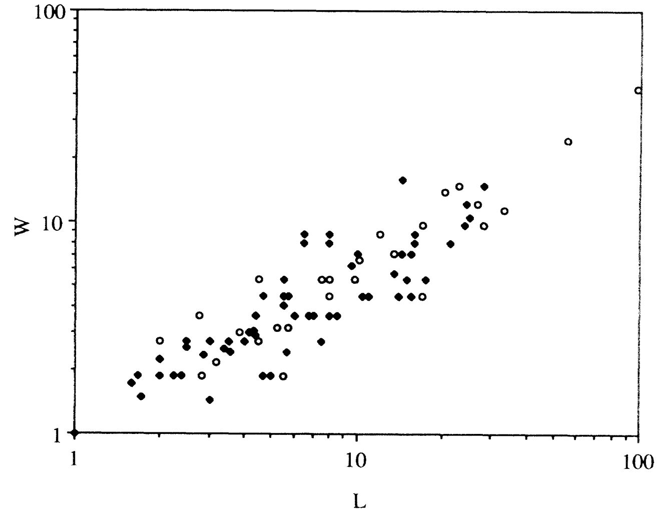

FIG. 5. A log-log plot of the width W of the clusters vs their length L for (●) five small samples and (○) one large sample.

analysis of the evolution of the sample geometry, and of its relation to the macroscopic stress-strain characteristics [11]: For example, in what part of the characteristics does the localization start when taking into account the fact that the observation of large clusters of crushed straws is no more than a consequence of the localization of the deformation? We have used two techniques to study this. The first is the Moiré one: By superposing two successive transparent photographs of our samples, we can detect very small heterogeneities in the dynamics of the deformation. Figure 3 shows two of these superpositions, corresponding to the pairs of points A-B and B-C of the macroscopic characteristics: In the region A-B, the deformation is no longer homogeneous and the localization has already started; in the region B-C, the band divides the sample into two parts, and the localization is fully realized. The second technique allows us to determine precisely the point where this occurs. Observing photographs of Figs. 2 and 3, one can see that the deformation remains homogeneous in the lower part of the sample, under the band. Using the fact that the band is localized in the upper half, we have studied the normalized variations of the height h of the n lowest rows below the band (where the deformation is supposed to be homogeneous) versus the variation of the total height H—normalized too—of the sample: One of the obtained curves is given in Fig. 4. In the "homogeneous elastic" zone, the curve is a straight line with slope 1, and abruptly, at a point which corresponds to the maximum of the macroscopic characteristics, the curve becomes another straight—but horizontal—line: The lower part ignores completely the deformation imposed on the top of the sample.

In this study, the final state of the localization is defined by the spreading of the band across the total width of the sample. But if one wants to study the statistical geometry of the band, it is necessary to define a criterion to decide if a site is crushed or not: A straw is considered as crushed if, in the photographs, one cannot inscribe within a section of the straw a circle with a diameter $\frac{2}{3}$ of the initial diameter.

Our experimental system can be seen as a model for different types of localization of the deformation in materials. So, the central question now is to verify if this assumption is correct. Some recent papers have analyzed the roughness of crack interfaces [12], characterized by a power law $W \sim L^{\zeta}$, where L is the length of the crack and W its width. For "weak" disorder, a universal value $\zeta = 0.7 \pm 0.07$ was found, not far from the universal value $\frac{2}{3}$ obtained for the random directed polymer problem. We have analyzed the geometry of the clusters when their size increases with the deformation. To improve the statistics, measurements have been made on five different samples. During the growth phase, up to the final "one-band" phase, the geometry of the clusters is the same for all the samples; the relation between the width W and the length L of the clusters, measured by marking the crushed straws in the clusters and giving them their initial size, can be fitted by a power law with an exponent $\zeta = 0.73 \pm 0.07$, compatible with the value obtained numerically for the roughness of crack interfaces [12]. The same study on a larger sample (140 rows of alternatively 125 and 124 cylinders) give comparable results: The best fit gives $\zeta = 0.75$. Figure 5 shows a log-log plot of the measurements.

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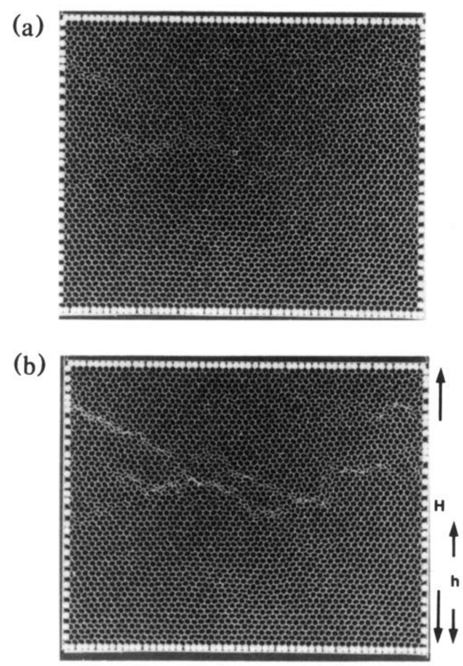

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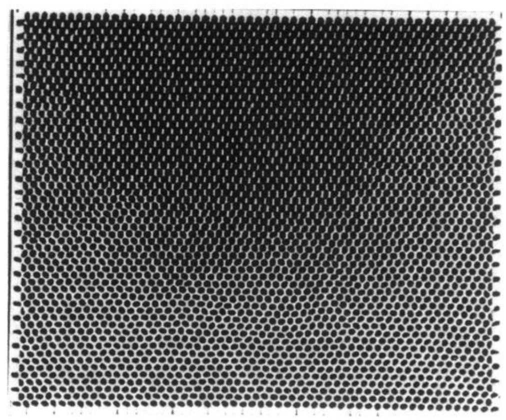
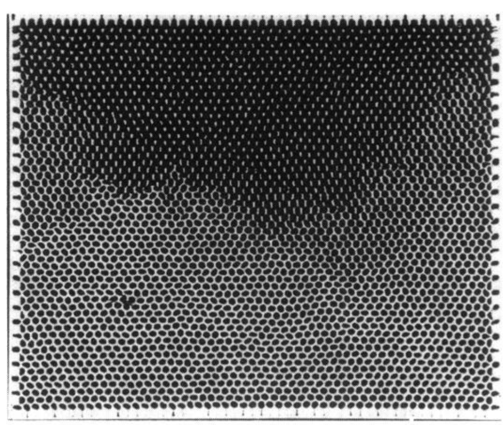
(a)



(b)



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