Ordered Capillary-Wave States: Quasicrystals, Hexagons, and Radial Waves

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We report the observation of ordered capillary-wave states generated by three and four standing plane waves. The pattern produced by the four standing plane waves forms the first quasicrystal observed in a fluid dynamical system. The states are observed at aspect ratios above 45, and for amplitudes below the amplitude at which a square-symmetric state is formed. We also report the observation of a capillary-wave state generated by three radial waves.

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Parametrical excitations of surface waves by an oscillating gravitational-force field were observed by Faraday in 1831 [1]. Today, it is well known that when the forcing amplitude A exceeds a critical value A_c, waves are formed on the surface with a frequency f_s that is half the value of the forcing frequency f, $f_s=f/2$. The wavelength λ is for depth $h \gg \lambda$ determined by the dispersion relation [2] $(2\pi f_s)^2 = gk(1+k^2a^2)$, where g is the acceleration of gravity, $k \equiv 2\pi/\lambda$, and a is a capillary length [3]. When the capillary (last) term dominates, we speak of capillary waves. A number of instabilities and patterns produced by capillary waves have now been explored, including recent studies of mode interactions and spatiotemporal chaos [4–6]. These phenomena are especially suitable to study in capillary waves, where the aspect ratio $R \equiv L/\lambda$ (L denoting the cell diameter) can be tuned.

In this Letter, we report the observation of ordered surface states generated by three and four pairs of oppositely moving plane waves. The patterns are observed at aspect ratios above 45, and for amplitudes below the amplitude at which the square-symmetric surface state is formed. While the pattern produced by three standing waves is a honeycomb lattice, the pattern produced by four standing waves has no discrete translational symmetry, thus it forms a quasicrystal. We also report the finding of a wide regime where an ordered pattern formed by three radial waves is observed.

The cell used is a cylindrical container (glass bottom and Plexiglas wall), 8.4 cm in diameter and 2 cm deep, filled with ethanol to a depth of ~1 cm. To excite the capillary waves, the cell is vertically forced by a vibration exciter driven by a frequency generator. The surface patterns are visualized by projection of a white-light beam through the cell onto a top plate of ground glass (Fig. 1). The patterns thus formed are sampled by a charge-coupled-device video camera and transferred to a computer. Several forcing frequencies between 300 and 500 Hz were chosen, and for each frequency the amplitude was varied. The phase diagram obtained is shown in Fig. 2. The forcing amplitude is given in units of A_c. The amplitude was measured by a magnetic detector to an ac-

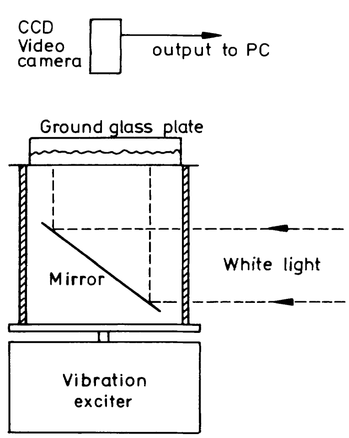

FIG. 1. Schematic illustration of the experimental setup.

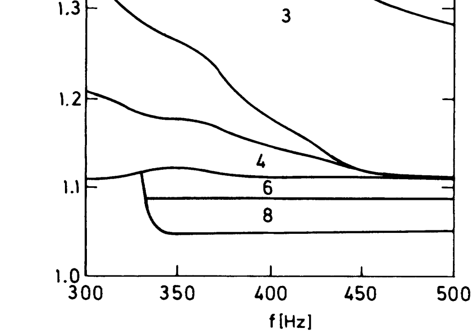

FIG. 2. Phase diagram for capillary waves in a cylindrical cell, vibrated vertically at frequencies between 300 and 500 Hz. The forcing amplitude A is given in units of A_c. Regions, in which surface states generated by eight, six, four, and three waves are observed, are indicated by the number of waves involved.

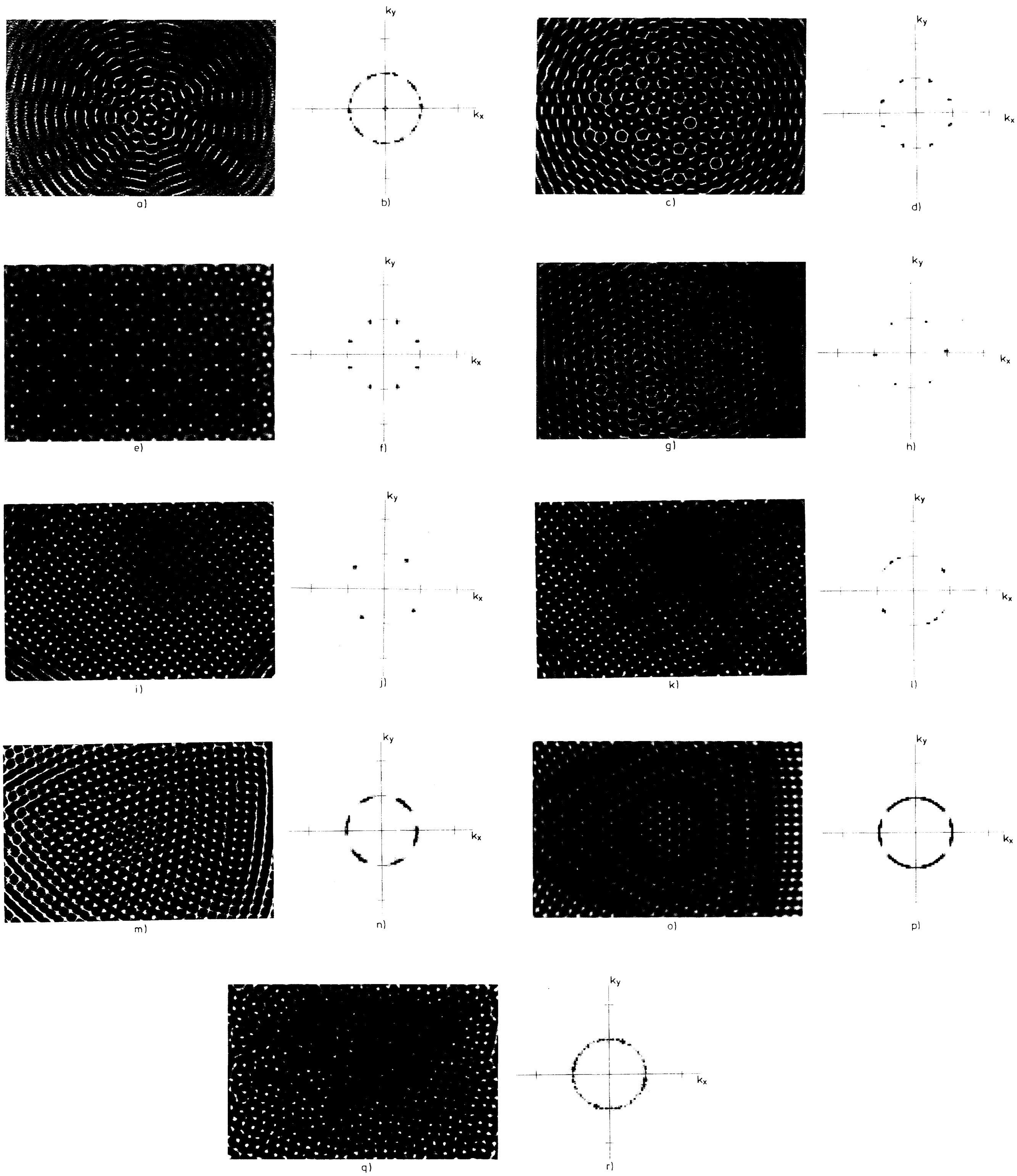

FIG. 3. Images of capillary wave patterns [(a),(c),(g),(i),(k),(m),(q)] and their power spectra [(b),(d),(h),(j),(l),(n),(r)] obtained at a forcing frequency $f=380$ Hz, and for increasing amplitudes A above A_c. The region shown is about 50% of the cell. (a),(b) $A=1.03A_c$. (c),(d) $A=1.08A_c$. (e),(f) Computer-generated images obtained by adding four sine waves of equal amplitude. (g),(h) $A=1.10A_c$. (i),(j) $A=1.14A_c$. (k),(l) $A=1.17A_c$. (m),(n) $A=1.30A_c$. (o),(p) Computer-generated images obtained by adding three radial sine waves of equal amplitude. (q),(r) $A=1.50A_c$.

curacy of 0.2%. We find that the value of A_c agrees with the theoretical prediction [7] $A_c=(2\nu/\lambda f_s)[1+(b/R)\ln(\nu/\pi\epsilon^2 f_s)]$, where $\nu=0.015$ $cm^2 s^{-1}$ is the viscosity, $b \simeq 0.4$, and $\epsilon \simeq 10$ Å is the slip length. The boundary correction (second term) to A_c is (14-20)% of the bulk value in the frequency range considered. Above A_c, the observed wavelength agrees with the dispersion-relation value.

Figure 3 shows a series of images obtained for $f=380$ Hz when increasing the amplitude above A_c. The region shown is about 50% of the cell. To remove intensity variations due to the light source, the sampled images are divided by corresponding background images obtained below A_c. Also shown are the corresponding two-dimensional power spectra. For frequencies between 340 and 440 Hz, we observed the scenario described below for $f=380$ Hz.

For $f=380$ Hz waves with frequency 190 Hz and wavelength $\lambda=1.72$ mm are formed at A_c. Above A_c, a central disordered region is formed [Figs. 3(a) and 3(b)], surrounded by modulated radial waves that manifest the basic modes [4] for the circular geometry. The size of the disordered region grows rapidly with amplitude, indicating an increasing number of dominant modes (Bessel functions). At approximately 5% above A_c, a transition from disorder to order occurs: A stable surface pattern generated by four standing plane waves with wave vectors separated by $\pi/4$ is formed [Figs. 3(c) and 3(d)]. The pattern produced is a quasicrystal—for comparison, Figs. 3(e) and 3(f) show the computer-generated images obtained by adding four sine waves of equal amplitude.

Increasing the amplitude further, to about 9% above A_c, a stable hexagonal pattern generated by three standing plane waves with wave vectors separated by $\pi/3$ is formed [Figs. 3(g) and 3(h)] [8]. We emphasize that the hexagonal pattern as well as the quasicrystalline pattern is extremely stable in the central region, while defects [5] are observed near the cell wall. Below 330 Hz the two patterns are not observed. As a partial explanation for this lower limit, we notice that for low aspect ratios ($R<R_c$) the surface state is dominated by a few strong modes at the forcing amplitude at which the ordered states should have formed. At higher aspect ratios ($R>R_c$) more modes will contribute with correspondingly weaker strengths—weak enough to allow the ordered states to form. We find $R_c=45$ for the 8.4-cm-wide cell. For a cell with diameter 5.75 cm, the ordered pattern is first observed at 520 Hz, corresponding to an aspect ratio $R_c=42$.

When the amplitude is increased to about 11% above A_c, a square pattern of two orthogonal standing plane waves is formed [Figs. 3(i) and 3(j)]. The frequency range for the square pattern extends below 330 Hz, and its observation has been reported by several authors [1,5,6]. Here, the pattern is observed in an amplitude interval that decreases with increasing frequency, vanishing at 460 Hz. The square-symmetric state is more susceptible to defects—at the borders shown in Fig. 2 intermittent states [5] are observed. Above 460 Hz, mode interaction between the square-symmetric state and the quasicrystal state is observed (at $A \simeq 1.11A_c$), producing long periodic states with period about 30 s, in analogy with those found at low frequencies in Ref. [4].

For frequencies below 440 Hz, a defect-mediated [5] breakdown of the square pattern (at 16% above A_c for $f=380$ Hz) is observed [Figs. 3(k) and 3(l)]. However, increasing the amplitude further (to 20% above A_c for $f=380$ Hz), a new ordered state arises [Figs. 3(m) and 3(n)], generated by three radial waves with centers equidistantly positioned outside the cell. The radial form gives an angular scatter in the power spectrum—for comparison, Figs. 3(o) and 3(p) show the computer-generated images obtained by adding three radial sine waves of equal amplitude. The stable pattern formed is observed in a wide amplitude interval of size 20% of A_c. The three radial waves are abruptly destroyed at the upper border, leaving a disordered state of irregularly moving waves [Figs. 3(q) and 3(r)]. For frequencies between 440 and 460 Hz, the radial wave pattern is formed as the square pattern breaks down. Above 460 Hz a transition from the honeycomb lattice to the radial wave pattern is observed.

In summary, we have presented a phase diagram and a sequence of images obtained for capillary waves at frequencies between 300 and 500 Hz. We have observed the formation of ordered surface states generated by four, three, and two standing plane waves with wave vectors equally separated in angle, and a "triangular" state generated by three radial waves [9]. In particular, the four standing plane waves form the first quasicrystal observed in a fluid dynamical system. Below an aspect ratio of $R_c \simeq 45$, only the square-symmetric and the triangular states are observed. Defects are observed at the cell wall, but do not affect the central region, except at the borders for the square-symmetric state, where intermittent patterns are formed. Above 460 Hz, periodic mode interaction between the quasicrystal state and the square-symmetric state is observed at $A \simeq 1.11A_c$.

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[8] The hexagonal pattern has previously only been observed at low frequencies (Ref. [6]).

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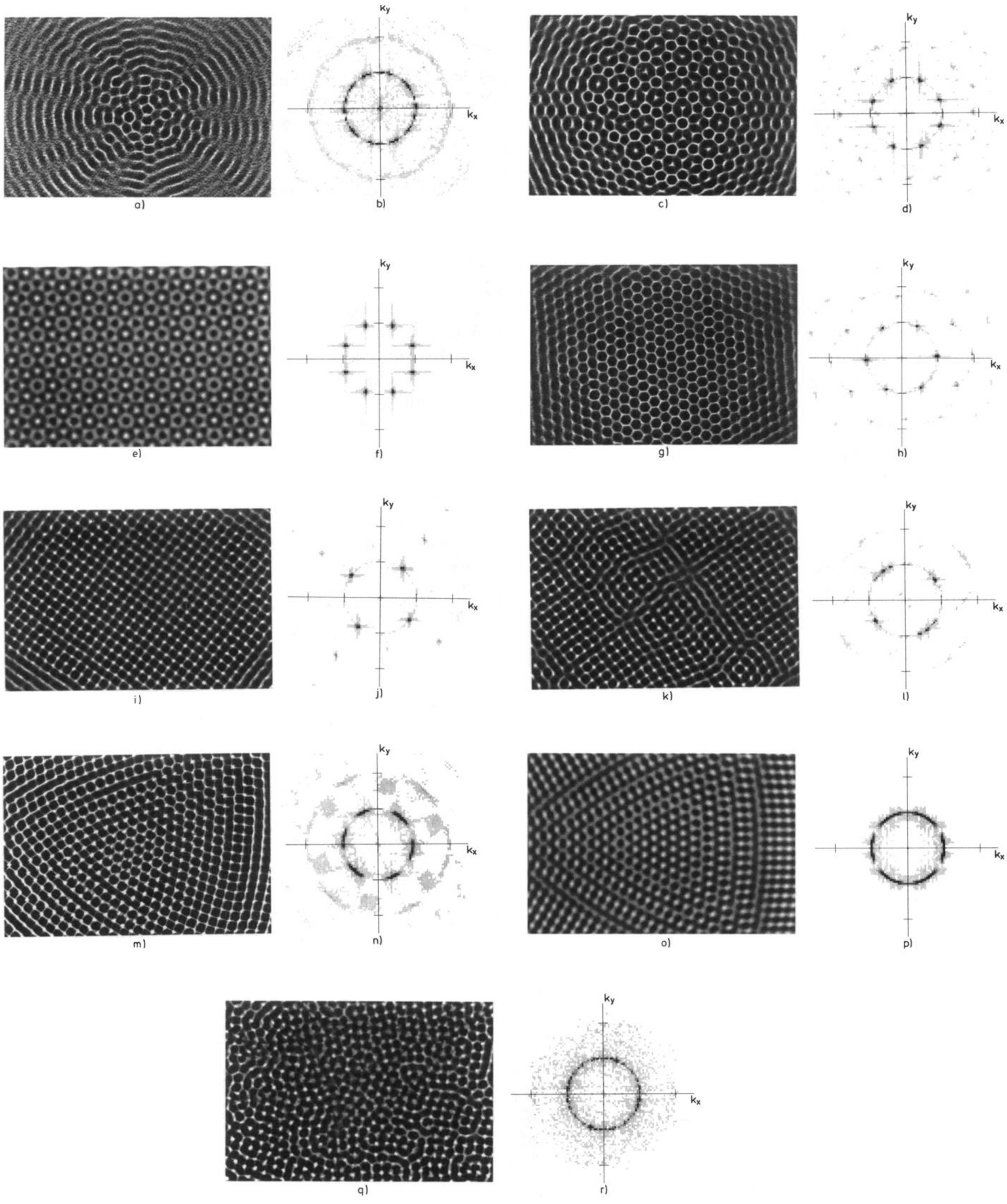

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