Pseudospin Symmetry in Nuclear Physics

C. Bahri and J. P. Draayer
Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803

S. A. Moszkowski
Department of Physics, University of California, Los Angeles, Los Angeles, California 90024
(Received 12 November 1991)

The origin and consequences of pseudospin symmetry in nuclear physics, which is exact for an oscillator potential with one-body orbit-orbit (v_{ll}) and spin-orbit (v_{ls}) interaction strengths in the ratio $\mu \equiv 2v_{ll}/v_{ls} = 0.5$, are considered. Specifically, the $v_{ls} \approx 4v_{ll}$ condition is consistent with relativistic mean-field results and a pseudo LS coupling scheme. When deformation dominates, pseudospin extends to pseudo SU(3), which is applicable to superdeformation.

PACS numbers: 21.60.Fw, 21.60.Cs, 21.60.Ev

(1) Introduction.—The three-dimensional isotropic harmonic oscillator (H_0) augmented with the one-body spin-orbit ($l \cdot s$) and orbit-orbit (l^2) interactions,

$$H = H_0 + v_{ls} l \cdot s + v_{ll} l^2 , \tag{1}$$

is a good approximation for the nuclear single-particle Hamiltonian. The l^2 term pushes high angular momentum states down ($v_{ll} < 0$) relative to those with lower l values while the $l \cdot s$ term (coupling spatial and spin degrees of freedom) is required to achieve shell closures ($v_{ls} < 0$) at the magic numbers. Unfortunately, v_{ls} is so large that the $l \cdot s$ term destroys the oscillator SU(3) symmetry for all but light ($A \lesssim 28$) nuclei, rendering it of little value in attempts at unraveling the structure of heavier systems.

This paper shows that this situation gives way to a much more favorable one, because for medium and heavy ($A \gtrsim 100$) nuclei, $v_{ls} \approx 4v_{ll}$ or the Nilsson parameter $\mu \equiv 2v_{ll}/v_{ls} \approx 0.5$. As a consequence, the level splitting generated by the $l \cdot s$ and l^2 interactions can be duplicated by a pseudo-oscillator Hamiltonian plus a pseudo l^2 term, with (at most) a small symmetry-breaking residual pseudo $l \cdot s$ interaction [1-3]. Since common residual interactions are pseudospin scalar operators, a many-particle pseudo LS-coupled shell-model scheme can be employed, and the basis truncated to leading pseudospin symmetries, without losing important physics. In addition, the pseudo LS scheme extends to pseudo SU(3) when deformation dominates [4].

(2) Spherical Nilsson scheme.—In the single-particle picture, the pseudospin concept means a division of the total particle angular momentum into *pseudo* ($\mathbf{j} = \tilde{\mathbf{l}} + \tilde{\mathbf{s}}$) rather than *normal* ($\mathbf{j} = \mathbf{l} + \mathbf{s}$) orbital and spin parts, so $\tilde{l} \pm \frac{1}{2} = l \mp \frac{1}{2}$. The physical significance of this elementary transformation is illustrated in Fig. 1, where eigenvalues of H are plotted as a function of μ. For $\mu = 0.5$, the pairs with $j = l + \frac{1}{2}$ and $j = (l+2) - \frac{1}{2}$ are degenerate for *all* l values. Furthermore, the splitting of the degenerate pairs follows a $\tilde{l}(\tilde{l}+1)$ rule, where $\tilde{l}$ is the average angular momentum of the pair: $\tilde{l} = \frac{1}{2}[l + (l+2)] = l + 1$. This mapping of the $(ls)j$-coupled single-particle states onto $(\tilde{l}\tilde{s})j$ pairs defines a special (*normal*$\leftrightarrow$*pseudo*) unitary transformation: $U = 2(\eta \cdot \xi - 2l \cdot s + 3)^{-1/2}(\xi \cdot s)$, where the η and ξ respectively create and annihilate oscillator quanta [5,6].

The single-particle Hamiltonian transforms under this mapping as follows:

$$H_0 + v_{ls} l \cdot s + v_{ll} l^2 \underset{\text{pseudo}}{\overset{\text{normal}}{\rightleftharpoons}} \tilde{H}_0 + (4v_{ll} - v_{ls}) \tilde{l} \cdot \tilde{s} + v_{ll} \tilde{l}^2 + (\hbar\omega + 2v_{ll} - v_{ls}) . \tag{2}$$

Since $\hbar\omega + 2v_{ll} - v_{ls}$ is a constant, the pseudo form, $\tilde{H}$

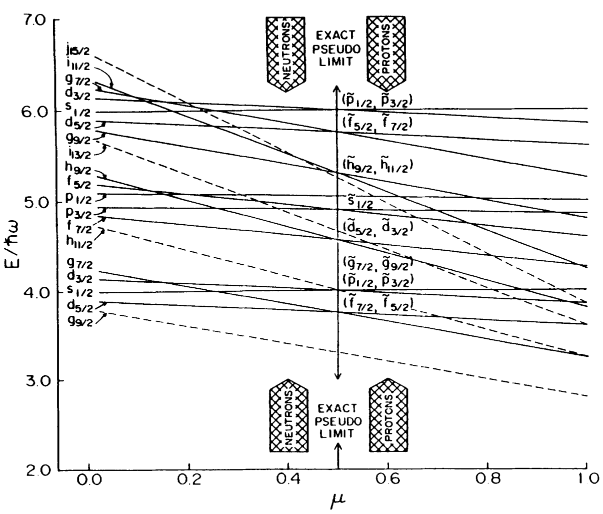

FIG. 1. Eigenvalues of the single-particle Hamiltonian $H/\hbar\omega = n - \kappa(2l \cdot s + \mu l^2)$, where $\mu = 2v_{ll}/v_{ls}$ and $\kappa = -v_{ls}/2\hbar\omega \approx \frac{1}{4}A^{-1/3}$, for the specific value $\kappa = 0.05$ and $0.0 \le \mu \le 1.0$. The $j = (l+2) - \frac{1}{2}$ and $j = l + \frac{1}{2}$ levels are degenerate for $\mu = 0.5$, which can be duplicated by the simpler pseudo-oscillator Hamiltonian $\tilde{H}/\hbar\tilde{\omega} = \tilde{n} - \kappa\mu\tilde{l}^2$ when $\hbar\tilde{\omega} = \hbar\omega$ and $\tilde{n} = n - 1$ with $\tilde{l} = l - 1 = \tilde{n}, \tilde{n} - 2, \ldots, 1$ or 0 and $\tilde{s} = \frac{1}{2}$. Each pseudoshell is accompanied by a unique parity intruder level (shown as dashed) with $j = (n+1) + \frac{1}{2} = n + \frac{3}{2}$ from the shell above. As indicated, empirical results place medium and heavy mass nuclei close ($\mu_\pi \sim 0.6$ and $\mu_\nu \sim 0.4$) to the $\mu = 0.5$ value require for exact pseudospin symmetry.

$=\tilde{H}_0+\tilde{v}_{ls}\tilde{l}\cdot\tilde{s}+\tilde{v}_{ll}\tilde{l}^2$, has the *same* excitation spectrum as the normal one $(H=H_0+v_{ls}l\cdot s+v_{ll}l^2)$ when $\hbar\tilde{\omega}=\hbar\omega$, $\tilde{v}_{ls}=4v_{ll}-v_{ls}$, and $\tilde{v}_{ll}=v_{ll}$. This transformation is important, because $v_{ls}\approx 4v_{ll}$, so $\tilde{v}_{ls}\approx 0$. As specifically indicated in Fig. 1, $\mu_\nu\approx 0.4$ and $\mu_\pi\approx 0.6$ (ν for neutrons and π for protons); this places medium and heavy nuclei close to the exact pseudospin limit ($\mu=0.5$) of the theory (cf., e.g., Ref. [7]). Indeed, the average μ value is almost exactly 0.5. For these nuclei, the familiar single-particle shell-model Hamiltonian can therefore be replaced by a less familiar, but equivalent, pseudo form which is inherently simpler due to its much smaller spin-orbit term.

The pseudospin scheme maps the normal-parity $(j=\frac{1}{2},\frac{3}{2},\ldots,n-\frac{1}{2})$ levels of the nth oscillator shell onto levels of a pseudo-oscillator shell with $\tilde{n}=n-1$. For example, the $(3s_{1/2},2d_{3/2},2d_{5/2},1g_{7/2})$ levels of $n=4$ are mapped onto the $(2\tilde{p}_{1/2},2\tilde{p}_{3/2},1\tilde{f}_{5/2},1\tilde{f}_{7/2})$ orbitals of $\tilde{n}=3$. The $j=n+\frac{1}{2}$ orbital ($1g_{9/2}$ for $n=4$) defects from the valence space and joins the shell below, while the $j=(n+1)+\frac{1}{2}=n+\frac{3}{2}$ level ($1h_{11/2}$ from the $n=5$ shell for the $n=4$ case) intrudes into the valence space from the shell above. Unique-parity intruder configurations couple to normal-parity states only through excitations involving pairs of particles and are therefore usually handled as weak-coupled, direct-product structures.

(3) Relativistic mean-field results.—The pseudospin concept may be better understood by comparing an intuitive result for v_{ll} with relativistic nuclear mean-field predictions for v_{ls}. The origin of the l^2 term in H is in the flatness of the mean field in the interior region, as compared with the quadratic oscillator form $[V(r)=\frac{1}{2}M\omega^2r^2]$. In the large mass limit $(A\to\infty)$ the potential approaches that of a spherical well of finite depth. If this spherical well is replaced by one with an infinite depth, the single-particle energies are given by

$$E_{nl}=(\hbar^2/2MR^2)x_{nl}^2\,, \tag{3}$$

where M is the nucleon mass, R is the radius of the well, and the x_{nl} are zeros of spherical Bessel functions. These zeros are approximately given by the result $x_{nl}^2\approx[(\frac{1}{2}n+1)\pi]^2-l(l+1)$. Table I illustrates the dependence of x_{nl} on l for the $n=4$ case. The results show that the splitting follows an $l(l+1)$ rule. Therefore,

$$v_{ll}=-\hbar^2/2MR^2\,. \tag{4}$$

A determination of v_{ll} using the Klein-Gordon equation leads to the same conclusion when the kinetic energy is a small fraction of the nucleon mass.

Next, consider the strength of the spin-orbit coupling. Starting with the usual Dirac equation (with only the time component of the scalar and vector potentials taken into account) and using a nonrelativistic reduction of the relativistic mean-field theory, the spin-orbit interaction is given by

$$V_{ls}=\frac{\hbar^2}{2M}\frac{2}{r}\frac{d}{dr}\left[\frac{1}{1-B\rho/\rho_0}\right]l\cdot s\,. \tag{5}$$

In this expression, ρ and ρ_0 are respectively the nucleon density at radius r and the nuclear matter density. The dimensionless quantity B in (5) is related to the strength of the scalar and vector coupling constants. The spin-orbit strength v_{ls} can be obtained from the average of V_{ls} over the region inside radius R,

$$v_{ls}=\frac{-\hbar^2}{2MR^2}\frac{6B}{1-B}\,. \tag{6}$$

In determining this result, the fact that $d\rho/dr$ vanishes everywhere, except near the surface of the nucleus, has been used.

It follows from Eqs. (4) and (6) that the ratio

$$\mu=\frac{2v_{ll}}{v_{ls}}=\frac{1-B}{3B} \tag{7}$$

is independent of mass number. Furthermore, to obtain $\mu=0.5$ requires $B=0.4$. In the simplest version of the theory, $B=\frac{1}{2}(B_s+B_v)$, with its scalar ($i=s$) and vector ($i=v$) components given by $B_i=g_i^2\rho_0/\mu_i^2Mc^2$, where μ_i and g_i respectively denote meson masses and coupling constants. Using this expression for B, the Nambu-Jona-Lasinio (NJL) model [8,9]—which in its modern form starts with massless quarks and generates hadron masses out of the vacuum by spontaneous symmetry breaking, and which has also been used to predict the coupling constants and masses appearing in a relativistic nuclear field theory—gives the result $\mu=0.686$ shown in Table II. As also shown in the table, results for the original Walecka model [10] and a derivative coupling model due to Zimanyi and Moszkowski [11]—which gives a more realistic equation of state for nuclear matter, which

TABLE I. Zeros (x_{nl}) of spherical Bessel functions and differences of their squares ($x_{n0}^2-x_{nl}^2$) compared with the simple $l(l+1)$ approximation for the $n=4$ case.

n	l	x_{nl}/π	x_{nl}^2	$x_{n0}^2-x_{nl}^2$	$l(l+1)$
4	0	3.000	88.83	0.00	0
4	2	2.895	82.72	6.11	6
4	4	2.605	66.98	21.85	20

TABLE II. Comparison of $\mu=2v_{ll}/v_{ls}$ values for various relativistic mean field theories. Exact pseudospin symmetry requires $\mu=0.5$. Results given are for $\rho_0=0.16$ nucleon/fm^3 and a nuclear binding energy of -16 MeV.

	B_s	B_v	B	μ
NJL [a]	0.339	0.316	0.327	0.686
Walecka [a]	0.487	0.368	0.427	0.447
Zimanyi [b]	0.252	0.088	0.344	0.635

[a] $B=\frac{1}{2}(B_s+B_v)$.
[b] $B=\frac{2}{3}B_s+2B_v$ (including recoil and exchange effects).

includes the effect of nucleon recoil, and when extended to include exchange correlations—also yield reasonable results for μ.

(4) Pseudospin dynamical symmetry.—A consequence of good pseudospin symmetry is that a $\tilde{L}\tilde{S}$ coupling scheme (with distinct $\tilde{S}$ multiplets that are decoupled and ordered) is expected to be a good starting point for describing heavy-nuclei, many-particle phenomena. The $\tilde{m}$-particle valence spaces ($\tilde{m}=\tilde{m}_\pi$ for protons and $\tilde{m}=\tilde{m}_\nu$ for neutrons, which occupy different major shells) divide into subspaces: $\tilde{S}=0,1,2,3,\ldots,\tilde{S}_{\max}$ for $\tilde{m}$ even or $\tilde{S}=\frac{1}{2},\frac{3}{2},\frac{5}{2},\ldots,\tilde{S}_{\max}$ for $\tilde{m}$ odd, with $\tilde{S}_{\max}=\min(\frac{1}{2}\tilde{m},\tilde{N}-\frac{1}{2}\tilde{m})$, where $\tilde{N}=\frac{1}{2}(\tilde{n}+1)(\tilde{n}+2)$ is the pseudoshell degeneracy. The proton-neutron $\tilde{L}\tilde{S}$-coupled states ($\tilde{\alpha}$ labeling $\tilde{L}$ multiplicity), $|\Psi^J\rangle=|[(\tilde{\alpha}_\pi\tilde{L}_\pi,\tilde{\alpha}_\nu\tilde{L}_\nu)^{\tilde{L}}\times(\tilde{S}_\pi,\tilde{S}_\nu)^{\tilde{S}}]^J\rangle$, with $\tilde{S}_\pi=\tilde{S}_{\pi_{\min}}$, $\tilde{S}_\nu=\tilde{S}_{\nu_{\min}}$, and $\tilde{S}=\tilde{S}_{\min}=|\tilde{S}_{\pi_{\min}}-\tilde{S}_{\nu_{\min}}|$, are expected to dominate, because realistic interactions favor pseudospace symmetric (pseudospin antisymmetric) configurations.

This truncation to the lowest $\tilde{S}_\pi$, $\tilde{S}_\nu$, and $\tilde{S}$ multiplets is usually insufficient to reduce the model space to a reasonable and workable size. Fortunately, another symmetry for strongly deformed nuclei can be invoked to effect a further truncation. Just as for light nuclei, $\tilde{S}\tilde{U}(3)$ of the pseudo oscillator which lies between $\tilde{U}(\tilde{N})$ and $\tilde{S}\tilde{O}_{\tilde{L}}(3)$ can be used to organize the states within each $\tilde{S}$ multiplet according to their deformation [12]. In this case the deformation is realized in terms of the pseudo (not normal) space symmetry. Nonetheless, this gives rise to strongly enhanced $B(E2)$ transition strengths, because the electric quadrupole operators $\tilde{Q}_E$ and Q_E are known to differ very

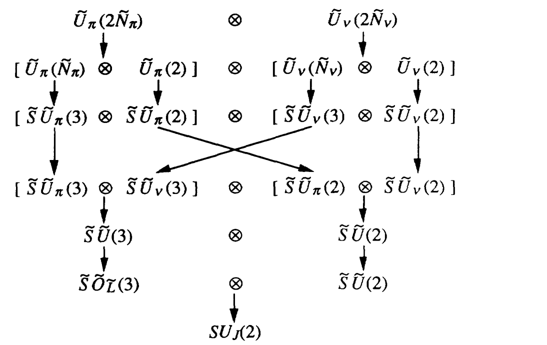

FIG. 2. Group structure of the pseudospin dynamical symmetry model. The proton and neutrons fill different major shells, and within each of these subshells the structures $\tilde{U}(2\tilde{N})\supset\tilde{U}(\tilde{N})\otimes\tilde{S}\tilde{U}_{\tilde{S}}(2)$, with $\tilde{U}(\tilde{N})\supset\tilde{S}\tilde{U}(3)\supset\tilde{S}\tilde{O}_{\tilde{L}}(3)$, organize the allowed normal-parity configurations according to their pseudospace deformation. The many-particle dynamics insure that the most deformed of these lie lowest and the least deformed highest. The $\tilde{S}\tilde{U}(3)$ strong coupling limit, which is motivated by a deformation reenforcement principle, likewise organizes the combined proton-neutron space according to its deformation. Particles distributed in the unique parity intruder orbitals tend to reenforce this picture.

little from one another [13].

Of the various coupling schemes that can be built with these group structures, the $\tilde{S}\tilde{U}(3)$ strong-coupling limit shown in Fig. 2 is the most natural (cf. Ref. [14]). In the dynamical symmetry limit, when the interaction is expressed solely in terms of group invariants, the corresponding eigenvalue spectrum is given by

$$\begin{aligned} E\{\tilde{m}_\pi\tilde{S}_\pi(\tilde{\lambda}_\pi,\tilde{\mu}_\pi);\tilde{m}_\nu\tilde{S}_\nu(\tilde{\lambda}_\nu,\tilde{\mu}_\nu);\tilde{\rho}(\tilde{\lambda},\tilde{\mu})\tilde{K}(\tilde{L}\tilde{S})J\} = {}& C_{\tilde{m}_\pi}\tilde{m}_\pi+C_{\tilde{m}_\nu}\tilde{m}_\nu+C_{\tilde{S}_\pi}\tilde{S}_\pi(\tilde{S}_\pi+1)+C_{\tilde{S}_\nu}\tilde{S}_\nu(\tilde{S}_\nu+1)+C_{\tilde{S}}\tilde{S}(\tilde{S}+1) \\ & -\tfrac{1}{2}\chi\pi C_2(\tilde{\lambda}_\pi,\tilde{\mu}_\pi)-\tfrac{1}{2}\chi\nu C_2(\tilde{\lambda}_\nu,\tilde{\mu}_\nu)-\tfrac{1}{2}\chi C_2(\tilde{\lambda},\tilde{\mu}) \\ & +C_{\tilde{L}}\tilde{L}(\tilde{L}+1)+C_{\tilde{K}}\tilde{K}^2+C_J J(J+1)\,. \end{aligned} \quad (8)$$

This choice is consistent with a deformation reenforcement principle which recognizes the favored configuration to be the one with maximum overlap of maximally deformed proton and neutron spatial configurations. In (8), $C_2(\lambda,\mu)$ is the second-order SU(3) invariant with eigenvalue $\lambda^2+\mu^2+\lambda\mu+3(\lambda+\mu)$. An explicit form for an operator that has K^2 as its eigenvalue is known in the limit $L\ll[C_2(\lambda,\mu)]^{1/2}$ [15]. The constants in (8) are related to the effective interaction. For example, the χ's are given by the strengths of the quadrupole-quadrupole interactions; the $C_{\tilde{S}}$'s are related to centroid separations of the pseudospin multiplets; $C_{\tilde{L}}$ is the inertia parameter; $C_{\tilde{K}}$ determines the band splitting; etc. The J^2 term can be replaced by $\tilde{L}\cdot\tilde{S}=\frac{1}{2}[J^2-\tilde{L}^2-\tilde{S}^2]$ and used to fine tune the placement of the $\tilde{L}(\tilde{L}+1)$ bands with respect to $\tilde{S}$.

When the pseudospin dynamical symmetry picture applies, there are $\tilde{L}(\tilde{L}+1)$ bands—one for each pseudospin orientation—that differ in *total* angular momenta ($\mathbf{J}=\tilde{\mathbf{L}}+\tilde{\mathbf{S}}$) by integer (even A compared with even A) or half-integer (odd A with even A) amounts. This seems to affirmatively answer the question of "whether low-lying collective states having alignment 1 would occur in a nucleus with rather good pseudospin symmetry" [16]. In considering this matter, it is important to emphasize that the alignment can be either proton or neutron in origin, or a combination. In particular, a consequence of good pseudospin symmetry is the prediction of $2\tilde{S}+1$ *identical* $\tilde{L}(\tilde{L}+1)$ bands with J values given by $J=\tilde{L}-\tilde{S}$ in the first, $\tilde{L}-\tilde{S}+1$ in the second, ..., $\tilde{L}+\tilde{S}$ in the last. The model further predicts (since on the average $\mu_\pi>0.5$ and $\mu_\nu<0.5$) that in odd-A proton nuclei the $J=\tilde{L}+\frac{1}{2}$ series should fall below the $J=\tilde{L}-\frac{1}{2}$ sequence and vice versa for odd-A neutron systems. Indeed, for the $^{151}_{65}Tb_{86}$ case ($\tilde{S}=\frac{1}{2}$) an excited superdeformed band of the $J=\tilde{L}+\frac{1}{2}$ has been reported and taken as evidence for the goodness of the pseudo SU(3) picture [17].

(5) Conclusions.—The origin and consequences of the

$\mu = 2v_{ll}/v_{ls} \approx 0.5$ result was examined. Actual estimates for μ are (0.60 and 0.65) for protons with ($50 < Z < 82$ and $Z > 82$), and (0.42 and 0.33) for neutrons with ($82 < N < 126$ and $N > 126$), respectively. These values are sufficiently close to $\mu = 0.5$ that the many-particle extension of the single-particle picture is expected to have good total pseudospin symmetry, provided the residual interaction is a pseudospin scalar operator. Examples include pairing, the surface delta interaction, and $\tilde{Q}\cdot\tilde{Q}$, which generates $\tilde{L}(\tilde{L}+1)$ rotational sequences in the decoupled pseudo spaces. At a more fundamental level, good pseudospin symmetry was shown to be consistent with relativistic mean field results for v_{ls} and v_{ll}.

Further consequences of good pseudospin symmetry were noted; particularly, the appearance of identical bands. Strong deformation in the pseudo-space part of the many-particle basis gives rise to $\tilde{L}(\tilde{L}+1)$ rotational sequences for each of the $2\tilde{S}+1$ orientations of the pseudospin. That these bands yield strongly enhanced $B(E2)$ strengths follows because $\tilde{Q}_E \approx Q_E$. A prediction of the theory is that many additional, strongly deformed bands should be found when the detectors with high efficiency and multiple-coincidence capability that are currently under construction come on line.

We have assumed that the particles in the intruder levels do not affect the dynamics in a significant way. This assumption is justified so long as intruder pair alignment (common in normally deformed systems) which results in backbending, accompanied by weakened $E2$ transition strengths, is not observed. Superdeformed rotational sequences appear to conform to this assumption, namely, intruder level particles apparently keep an internal structure that changes (at most) slowly as a function of increasing angular momentum of the system.

This work (J.P.D.) was supported in part by the U.S. National Science Foundation.

[1] A. Arima, M. Harvey, and K. Shimizu, Phys. Lett. **30B**, 517 (1969).
[2] K. T. Hecht and A. Adler, Nucl. Phys. **A137**, 129 (1969).
[3] A. Bohr, I. Hamamoto, and B. R. Mottelson, Phys. Scr. **26**, 267 (1982).
[4] R. D. Ratna Raju, J. P. Draayer, and K. T. Hecht, Nucl. Phys. **A202**, 433 (1973).
[5] J. P. Draayer, in Proceedings of Symmetries in Physics, Cocoyoc, Morelos, México, 3–7 June 1991, edited by A. Frank and K. B. Wolf (Springer-Verlag, Berlin, to be published).
[6] O. Castaños, M. Moshinsky, and C. Quesne, Phys. Lett. (to be published).
[7] A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, Reading, 1975), Vol. II.
[8] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); **124**, 246 (1961).
[9] H. Reinhardt, Phys. Lett. B **188**, 263 (1987).
[10] B. D. Serot and J. D. Walecka, in *Advances in Nuclear Physics*, edited by J. W. Negele and E. Vogt (Plenum, New York, 1986), Vol. 16.
[11] J. Zimanyi and S. A. Moszkowski, Phys. Rev. C **42**, 1416 (1990).
[12] J. P. Elliott, Proc. R. Soc. London A **245**, 128 (1958); **245**, 562 (1958).
[13] O. Castaños, J. P. Draayer, and Y. Leschber, Ann. Phys. (N.Y.) **180**, 290 (1987).
[14] K. T. Hecht, *Annual Review of Nuclear Science* (Academic, New York, 1973), Vol. 23, p. 123.
[15] H. Naqvi and J. P. Draayer, Nucl. Phys. (to be published).
[16] F. S. Stephens *et al.*, Phys. Rev. Lett. **64**, 2623 (1990); **65**, 301 (1990).
[17] W. Nazarewicz, P. J. Twin, P. Fallon, and J. D. Garrett, Phys. Rev. Lett. **64**, 1654 (1990); see also T. Byrski *et al.*, Phys. Rev. Lett. **64**, 1650 (1990).