Experimental Measurements of the Roughness of Brittle Cracks

Knut Jørgen Måløy,[a] Alex Hansen,[b] and Einar L. Hinrichsen[c]
Fysisk Institutt, Universitetet i Oslo, Postboks 1048, Blindern, N-0316 Oslo, Norway

Stéphane Roux[d]
Centre d'Enseignement et de Recherche en Analyse des Matériaux, Ecole Nationale des Ponts et Chaussées, Central IV, 1 Avenue Montaigne, F-93167 Noisy-le-Grand CEDEX, France
(Received 26 August 1991)

We record the height of the crack surface as a function of position along one-dimensional cuts orthogonal to the crack for six different brittle materials. We find that the width w of this one-dimensional trace as a function of its length L behaves as $w \sim L^{\zeta}$, where $\zeta = 0.87(7)$. This result is in agreement with recent conjectures of a *universal* roughness exponent ζ for these materials.

PACS numbers: 62.20.Mk, 05.40.+j, 64.60.Ht

Even though the fractures appearing in different materials may look very different from each other, much work has been invested over the last five years searching for features in fracture that are quantitatively *universal*. The guiding idea behind these previous studies has been the similarities with critical phenomena. These results have been based mostly on theoretical considerations and computer experiments (see Ref. [1] for a recent review). In the spirit of this guiding idea, it was recently suggested that scaling of the roughness of brittle cracks is universal [2]. We imagine a one-dimensional cut of length L is made through the fracture surface. The height y as a function of position along the cut, x, is then measured. The width of the crack is defined as $w = (\langle y^2 \rangle - \langle y \rangle^2)^{1/2}$, where $\langle y \rangle = \int_0^L y(x)dx/L$, and $\langle y^2 \rangle = \int_0^L y(x)^2 dx/L$. The suggestion of Ref. [2] was then that

$$w \sim L^{\zeta}, \tag{1}$$

where ζ is a *universal* exponent independent of the material. Equation (1) was tested numerically in a two-dimensional fuse model [3], and it was found that $\zeta = 0.7$ to within 10% accuracy for a range of different distributions of fuse strengths, i.e., different disorders. It was furthermore argued in this paper that there might be a connection between brittle fracture and the problem of directed polymers embedded in and interacting with a random medium [4]. Such a connection predicts the roughness exponent to be $\zeta = \frac{2}{3}$ in two dimensions.

The universal scaling of the roughness of cracks has many practical consequences. We mention, for example, that the permeability of oil reservoirs is strongly influenced by the presence of cracks. The roughness of the cracks will result in a length-dependent permeability of the cracks which is different from the simple opening of a straight crack.

Recently, the roughness of *ductile* cracks were studied experimentally by Bouchaud, Lapasset, and Planès [5]. These authors worked with aluminum alloys that received different heat treatments, and found the roughness of the cracks to follow Eq. (1) with a roughness exponent of about $\zeta = 0.8$. Experiments have also recently been performed on two-dimensional piles of collapsible tubes [6]. Here a roughness exponent of 0.7 was found. These experiments come very close to the fuse model studied numerically in Ref. [2].

In this Letter we present an experimental study of the roughness of cracks appearing in different *brittle* materials. We find a roughness exponent of $\zeta = 0.87 \pm 0.07$. This exponent varies little from material to material; see Table I.

We studied six different brittle materials: (1) An Al-Si alloy (AA4253), (2) ARNE steel AISI standard O1 cooled by liquid nitrogen, (3) a graphite sample, (4) porcelain used in high-voltage insulators, (5) bakelite, and (6) plaster of Paris. Samples (1) and (2) were well-defined materials, while samples (3) to (6) were more or less haphazardly chosen. The samples had either a circular or rectangular cross section measuring (1) 10×20 mm, (2) 20 and 25 mm, (3) 51 mm, (4) 25 to 30 mm, (5) 9×80 mm, and (6) 20 to 25 and 85 mm. They were broken by applying a strong shear. No attempts were made to force the cracks to end at certain positions on the surface of the materials, but a small notch was made in order for the crack to start at a well-defined position.

The crack surface was then traced along a straight line

TABLE I. The roughness exponents ζ for the different brittle materials studied. The first column contains the roughness exponents determined from the power spectra, while the second column contains the roughness exponents determined by the return probability histograms. The accuracy of the exponents presented in this table is roughly 10%.

	Power spectrum	Return probability
Al-Si alloy AA4253 (1)	0.82	0.96
ARNE steel (2)	0.88	0.91
Graphite (3)	0.90	0.90
Porcelain (4)	0.75	0.76
Bakelite (5)	0.84	0.89
Plaster of Paris (6)	0.95	0.94

by a slightly weighted needle using essentially the grammophone pickup principle. A mirror was attached to the needle, and a laser beam reflected from it onto a single axis continuous position-sensitive photodetector USD LSC 30-D. The mechanism holding the needle was stationary, while the sample was moved in steps of 0.025 mm by a stepping motor. A height increase of the needle produced a rotation of the mirror which introduced a change in the laser spot position on the detector. As the laser beam moved across the active area of the sensor, output currents were generated which were proportional to the distance of the laser spot to the end contact of the detector. The trace of the crack was then recorded as measurements of the output current of the position-sensitive photodetector for each position of the sample. In order to estimate the resolution of our apparatus, we traced a known structure with steps of height 0.11 and 0.22 mm. From the obtained trace, we estimate our vertical resolution to be 0.005 mm.

We made seven measurements, each containing 700 points for sample (1); 11 measurements, each containing from 400 to 600 points for sample (2); 10 measurements, with 1000 data points for sample (3); 27 measurements, with 450 to 1000 data points for sample (4); 15 measurements, with 700 to 1000 data points for sample (5); and 40 measurements, with 300 to 1200 data points for sample (6).

The trace from a given measurement of a given sample was recorded as $h(x)$ vs x. As pointed out above, we did not attempt to "restrain" the crack to start or end at a particular $h(0)$ or $h(L)$ value. However, this may introduce a drift in the h values which may affect the measurements of the roughness. In order to take this into account, we defined an "average drift" as $d(x)=[h(L)-h(0)]x/L$. This average drift was then subtracted from the data to give $y(x)=h(x)-d(x)$. We based our data analysis on the $y(x)$ vs x data.

We determined the roughness exponents ζ for each sample through two very different methods: One based on the power spectrum of the trace $y(x)$; the other based on the histogram of return probability. In order to estimate the accuracy of these two methods, we tested them on artificially generated fractal landscapes with different known roughness exponents [7].

The power spectrum $P(f)$ is the Fourier transform of the correlation function $\langle y(x+\Delta x)y(x)\rangle$. The power-law equation (1) translates into $P(f)\sim f^{-1-2\zeta}$ [7]. Each trace $y(x)$ from each measurement of each sample was split into (slightly overlapping) pieces containing 256 consecutive data points each. The procedure of transforming $h(x)\rightarrow y(x)$ had to be done on each of these pieces in order to avoid aliasing. We then performed an arithmetic average over each of the obtained power spectra. In Fig. 1 we show these averaged power spectra for all six samples. The corresponding roughness exponents are shown in Table I. The average value of the obtained exponents is 0.86 ± 0.06.

The return probability histogram, $R(\Delta)$, is a measurement of the probability that a height y appearing at a given position x reappears for the first time at a position $x+\Delta$, averaged over all x. In this case, Eq. (1) translates into $R(\Delta)\sim\Delta^{-2+\zeta}$ [8]. Figure 2 shows $R(\Delta)$ averaged over all measurements. The roughness exponents determined from the averaged return probability histograms for all samples are listed in Table I. The average exponent based on these values is $\zeta=0.89\pm0.06$, consistent with that found by the power spectrum method.

Taking into account the 10% accuracy of the roughness exponents measured for the various samples, we find that

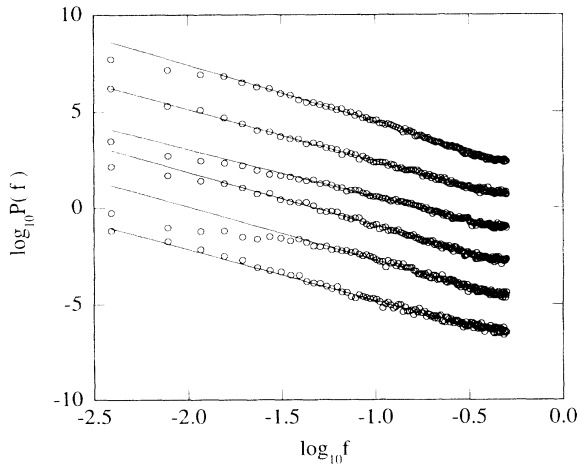

FIG. 1. The power spectrum $P(f)$ as a function of the frequency f for each of the six samples. The data have been moved apart for clarity: Sample (1) corresponds to the lowest data set; the others follow from bottom to top. The straight lines are least-squares fits to the scaling region of each power spectrum. The corresponding roughness exponents are listed in Table I.

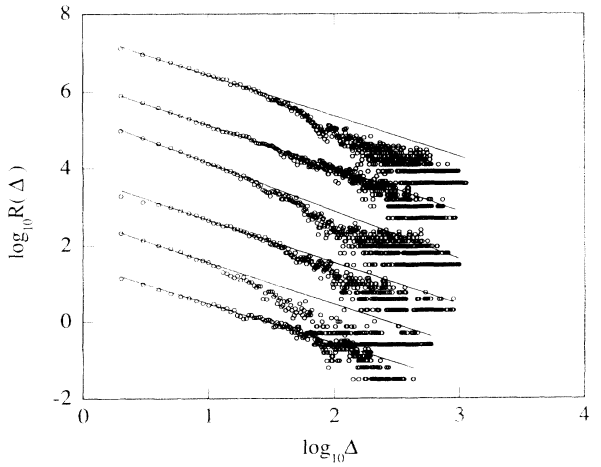

FIG. 2. The return probability histogram $R(\Delta)$ for each of the six samples. The data are presented as in Fig. 1, and the corresponding exponents are listed in Table I. Δ in this figure is measured in units of 0.025 mm.

our data are consistent with a *universal* value for the roughness exponent. Furthermore, comparing our results to those found by Bouchaud, Lapasset, and Planès [5] for ductile fracture, it is not ruled out that the roughness exponents are equal in both cases. This is especially relevant in the case of our Al-Si sample (1) being more brittle than the aluminum samples studied by Bouchaud, Lapasset, and Planès, but still showing an appreciable amount of ductility.

In Ref. [2] an analogy with the directed polymer in a random-medium problem was made. Carrying this analogy over to two-dimensional cracks appearing in three-dimensional materials, we consider the following problem: We associate to each point $\mathbf{r}$ in the material a random variable, "energy" $\epsilon(\mathbf{r})$. Any surface $\mathcal{S}$ cutting the material into an upper and lower piece then has a total energy $E_{\mathcal{S}}=\sum_{\mathbf{r}\in\mathcal{S}}\epsilon(\mathbf{r})$ associated with it. We then search for the surface having the smallest total energy E associated with it,

$$E=\min_{\mathcal{S}} E_{\mathcal{S}}=\min_{\mathcal{S}}\sum_{\mathbf{r}\in\mathcal{S}}\epsilon(\mathbf{r})\,. \tag{2}$$

This random-surface problem is analogous to the random-polymer problem in two dimensions, and the minimum-energy surface will have a roughness of the form of Eq. (1). However, the roughness exponent in this case has been estimated by Kardar and Zhang [9] to be $\zeta=0.50\pm0.08$ by a numerical transfer-matrix technique, while Halpin-Healy estimates $\zeta=\frac{2}{3}$ by an epsilon expansion [10]. Both estimates are appreciably smaller than $\zeta=0.87$ reported in the present Letter for the roughness of brittle cracks, or 0.8 for the roughness of ductile cracks [5].

Ignoring the discrepancy between the estimates for the random-surface roughness exponent of Refs. [9,10] and the experimentally measured crack roughness exponents, we recapitulate the two arguments presented in Ref. [2] why the random-polymer (in the present case, random-surface) problem should be relevant for the crack roughness problem. In the first one, we may think of $\epsilon(\mathbf{r})$ as the local energy necessary to create a microcrack at position $\mathbf{r}$. If we then ignore the inhomogeneities in the stress field induced by the fracture process, Eq. (2) follows as the crack surface will develop in such a way that the total energy necessary to create the crack is as small as possible. This argument has much in common with the celebrated Griffith criterion for crack development—also in that it ignores the inhomogeneities of the stress field [1]. The second argument as to why the random-surface problem is relevant is to note that early in the fracture process microcracks are generated in the material where it is weak, rather than where the stress field is large, since the stress field at this stage is rather uniform. If the weak spots of the material are randomly distributed, so will the microcracks that have been generated. At the late stages of the fracture process when the stress distribution *does* matter, it is plausible to assume that the local stress field will be highest along the surface through the material containing the largest area of voids, or equivalently the smallest area of intact material. If we interpret $\epsilon(\mathbf{r})$ as the local area of intact material, we arrive at Eq. (2) for the surface containing the least area of intact material. Since the stress field is highest in this surface, this is where the final fracture will develop. Both of these arguments contain large approximations, essentially ignoring the strong local inhomogeneities that develop in the stress field through the fracture process. Whether the ignored effects are important enough to change the roughness exponents, or whether the theoretical measurements of the random surface exponent are on the low side whereas our measurements of the crack roughness exponent are on the high side, so that they actually are equal, is at present impossible to determine. Further work to determine the answer to this question is necessary.

We thank T. Arnesen, F. Boger, J. Feder, T. Jøssang, J. Lothe, and J. Schmittbuhl for discussions. K.J.M. was supported by the NAVF, A.H. and E.L.H. by the German-Norwegian Research Cooperation, and S.R. by CRECO Géomatériaux.

[a]Present address: Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA 15260.
[b]Also at Groupe de Physique Cristalline, Université de Rennes I, F-35042 Rennes, France.
[c]Present address: Senter for Industriforskning, Postboks 124, N-0314 Oslo, Norway.
[d]Also at LPMMH, Ecole Supérieure de Physique et de Chimie Industrielles, 10 rue Vauquelin, F-75231 Paris CEDEX 05, France.

[1] *Statistical Models for the Fracture of Disordered Media*, edited by H. J. Herrmann and S. Roux (Elsevier, Amsterdam, 1990).
[2] A. Hansen, E. L. Hinrichsen, and S. Roux, Phys. Rev. Lett. **66**, 2476 (1991).
[3] L. de Arcangelis, S. Redner, and H. J. Herrmann, J. Phys. (Paris), Lett. **46**, L585 (1985).
[4] M. Kardar, G. Parisi, and Y.-C. Zhang, Phys. Rev. Lett. **56**, 889 (1986).
[5] E. Bouchaud, G. Lapasset, and J. Planès, Europhys. Lett. **13**, 73 (1990).
[6] C. Poirier, M. Ammi, D. Bideau, and J. P. Troadec, Phys. Rev. Lett. **68**, 216 (1992).
[7] J. Feder, *Fractals* (Plenum, New York, 1988).
[8] K.-J. Måløy, A. Hansen, E. L. Hinrichsen, and S. Roux (to be published).
[9] M. Kardar and Y.-C. Zhang, Europhys. Lett. **8**, 233 (1989).
[10] T. Halpin-Healy, Phys. Rev. Lett. **62**, 442 (1989).