Spatial Structure of Screening Propagators in Hot QCD

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We use numerical simulations to study the spatial structure of a quark and an antiquark in the imaginary-time excitations which mediate the Debye screening of color singlet sources in the high-temperature phase of QCD. We find that these correlation functions are very similar to the zero-temperature wave functions of the corresponding particles. This result contrasts with results on the ρ and nucleon screening lengths for these sources, which are well described by a gas of free or weakly interacting quarks.

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There is strong evidence that at high temperatures QCD enters a regime where the spontaneous breaking of chiral symmetry seen at low temperatures disappears, and chiral symmetry is only slightly broken by the small quark masses. Symmetry restoration could happen either because QCD has become a theory of free or nearly free quarks or because chiral symmetry is realized through the parity doubling of color singlet hadronic modes. Much of the evidence for chiral symmetry restoration comes from the study of the spatial screening of color singlet hadronic sources. Operationally these studies are identical to the standard computation of the hadron spectrum on the lattice, except that the simulation is done at high temperature, so the size of the lattice in the Euclidean time direction is small, and the propagators are measured in the spatial direction [1]. Parity doubling is clearly seen in these propagators—the pion and σ screening lengths become equal, as do the ρ and a_1 and the nucleon and its parity partner. Of course, this behavior would be seen in a theory of free quarks as well as in a theory of color singlet excitations.

It has recently been emphasized that the screening masses of the ρ and a_1 propagators are remarkably close to twice the lowest Matsubara frequency, or twice the energy of a low mass free quark. Moreover, the agreement is improved when finite lattice corrections are incorporated into the calculation of the Matsubara frequency [2]. Also, the nucleon screening length is reasonably close to 3 times the Matsubara frequency. However, the pion-sigma screening length is lower than twice the Matsubara frequency. These results have suggested models of high-temperature QCD in which the pion and σ are considered to be composite particles but the excitations with the quantum numbers of the ρ and nucleon consist of two or three weakly interacting quarks, respectively [3]. In this Letter we report on a study of the spatial structure of the screening of color singlet sources, with results that appear to be in conflict with models of screening by two or three free or weakly interacting quarks.

Wave functions of hadrons, or at least the valence quark components of hadrons, can be calculated in lattice simulations by evaluating the quark and antiquark propagators from some source at an initial Euclidean time to spatially separated points at a later Euclidean time [4]. To get a nonzero result this calculation must either be done in a fixed gauge such as the lattice Coulomb gauge or the propagators must be parallel transported to a common point. In this work we use the Coulomb gauge. Thus we evaluate (for a meson)

$$\psi(\mathbf{x}) \propto \sum_{\mathbf{y}} \langle \mathcal{O}(t=0)\bar{q}(\mathbf{y},\tau)\Gamma q(\mathbf{y}+\mathbf{x},\tau)\rangle . \qquad (1)$$

Here $\mathcal{O}$ is an operator that creates a quark and an antiquark at $t=0$ and Γ is the appropriate Dirac matrix for the desired meson. The Euclidean time separation τ should be made large enough so the result is independent of τ, apart from a normalization factor that decays at a rate $\exp(-m_H\tau)$ determined by the hadron mass m_H. Because we are interested in chiral symmetry we use Kogut-Susskind quarks, for which a U(1) subgroup of the continuum chiral symmetry remains unbroken by the nonzero lattice spacing. With Kogut-Susskind quarks the mesons must be constructed from the appropriate combinations of propagators to different lattice sites. The easiest way to handle this construction is to consider the quark fields to be defined on a lattice of spacing $2a$. Thus we compute wave functions for spatial separations that are multiples of $2a$. For example, the γ_3 component of the ρ-b_1 propagator at spatial separation $2\hat{\mathbf{e}}_2$ is given by

$$\psi_{\mathrm{VT},z}(2\hat{\mathbf{e}}_2) = \sum_{\mathbf{y}} \langle \phi^\dagger(\mathbf{y},\tau)(-1)^{y_3}\phi(\mathbf{y}+2\hat{\mathbf{e}}_2,\tau)\rangle , \qquad (2)$$

where $\phi(\mathbf{y},t) = M^{-1}(U)S(t=0)$. Here M is the fermion hopping matrix, S is the source, e_2 is the unit vector in the y direction, and $\langle\ \rangle$ averages over gauge configurations

U. These are just the conventional operators used in staggered fermion mass spectrum calculations, except that the quark is displaced by $2\mathbf{x}$, with $\mathbf{x}$ some lattice vector, from the antiquark [5].

In practice, the sum over spatial lattice points, a convolution, is accomplished by Fourier transforming the propagator, squaring the magnitude of the result, Fourier transforming back, and combining the results in each 2^3 site of the doubled lattice with the appropriate signs. In these Fourier transforms each 2^3 block is treated as a single variable, so for a lattice dimension of 16 we use an 8-point transform.

We began by computing wave functions at low temperature, using $16^3\times 24$ lattices stored in a previous hadron spectrum calculation. These lattices had $6/g^2=5.445$ and $m_q=0.025$, with two flavors of dynamical quarks. This value of g is the crossover value to the high-temperature regime for a lattice with six time slices and the same quarks [6], so the present simulation at the same g and m_q on a $16^2\times 24\times 4$ lattice is a simulation at a temperature $T\approx 1.5T_c$, where T_c is the crossover temperature to the high-temperature regime. We computed propagators on 48 cold lattices, using a wall source at $t=0$. Specifically, we set the source to one on the (0,0,0) site of each 2^3 cube at $t=0$. With this source, the meson wave functions for free quarks are constant—exactly independent of the quark-antiquark spatial separation at any Euclidean time. The actual results for the pion and ρ wave functions are pictured in Figs. 1 and 2. It can be seen that the wave functions are reasonably well contained in the box. As has been found in calculations with Wilson quarks, the ρ is significantly larger than the pion. The ρ wave function shows the γ_z ρ $(\bar\psi\gamma_z\psi)$, so the wave function at separation (0,0,2) is not expected to equal the wave function at separation (0,2,0). However, the wave functions at (2,0,0) and (0,2,0) should be equal, and have been averaged together here. In general, we have averaged over all the lattice symmetries, including reflections

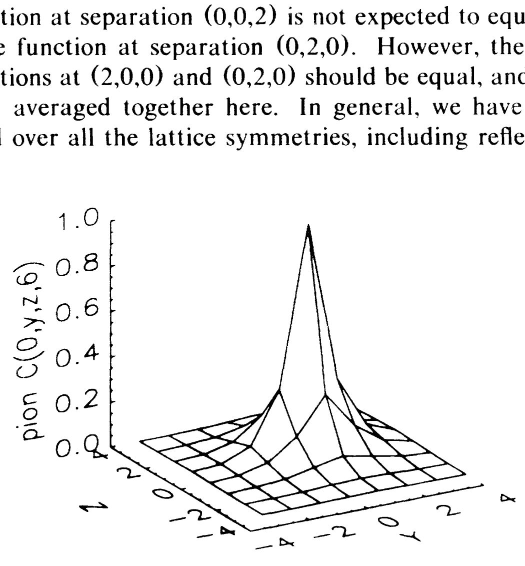

FIG. 1. The pion wave function at $T=0$. We show the y-z slice with $x=0$, at Euclidean time $\tau=6$. The wave function is defined on the doubled lattice, so the 16^3 spatial lattice yielded an 8×8 wave function.

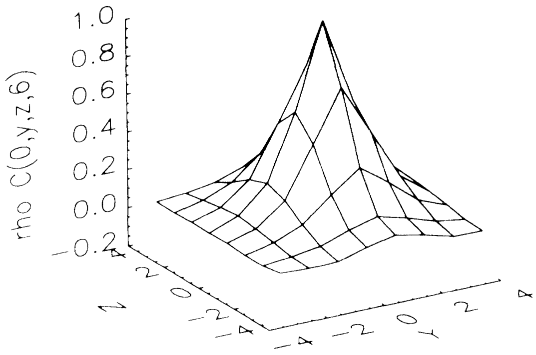

FIG. 2. The ρ wave function at $T=0$. We show the y-z slice with $x=0$. This wave function is for the γ_z component of the ρ, so it is not necessarily symmetric under $y\leftrightarrow z$.

of the spatial coordinates, interchanges of all the spatial coordinates for the pion wave function, interchanges of the x and y coordinates for the γ_z ρ, etc., and $t\rightarrow N_t-t$.

The ρ wave function shown in Fig. 2 is for the VT, or ρ-b_1, propagator [5]. In this channel the source couples more strongly to the ρ than to the b_1, and the b_1 is heavier so its signal dies away compared to the ρ signal. For the PV, or ρ-a_1, channel and the SC, or π_2-σ, channel, the coupling to the positive-parity particle is relatively larger, and it is necessary to separate the two parities. We will present results for this in a later paper.

We then evaluated similar "wave functions" for screening propagators in a simulation at the same values of $6/g^2$ and m_q, on a $16^2\times 24\times 4$ lattice. This involves interchanging z and t in with respect to the zero-temperature wave-function calculation. We measured propagators on 156 lattices in this run. In this case we measured the expectation values of quark and antiquark propagators separated in the (x,y,t) plane for fixed z. Now "Coulomb" gauge fixing acts on the x,y,t gauge links. For the VT propagator we used only the γ_x and γ_y components, with sign factors $(-1)^x$ and $(-1)^y$, respectively, and for the PV propagator only the γ_z component, with sign factor $(-1)^{x+y}$. The results are shown in Figs. 3 and 4, together with the low-temperature results. In these figures we show the wave function versus the Euclidean magnitude of the displacement $\mathbf{x}$ in $\psi(\mathbf{x})$. [For the high-temperature results we show correlators for the quark and antiquark at the same Euclidean time t, but separated in the x and y directions, using a source in the (x,y,t) plane at $z=0$.] The striking fact is that the spatial structure of the screening correlators closely resembles the corresponding low-temperature wave functions. We emphasize, and we have checked, that because the source is uniform in x and y, for free quarks the result is independent of the spatial separation. In other words, for free quarks the wave function is a constant extending over the entire lattice.

One can imagine several possibilities for the structure

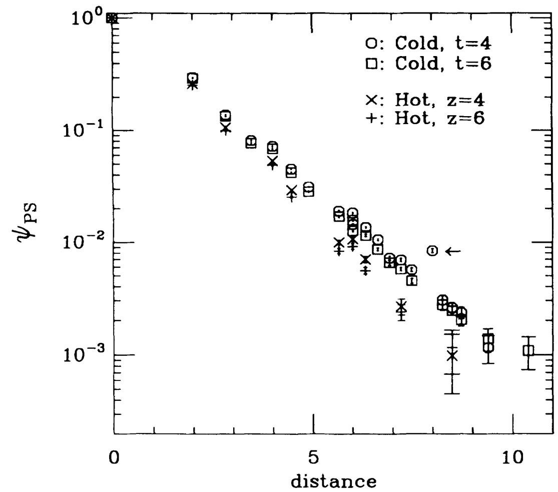

FIG. 3. The pion wave function at $T=0$ and the screening pion correlation function at $T\approx 1.5T_c$. We show all distances on the lattice, with the functions normalized to 1 at the origin. For the cold case we show results at two Euclidean times, to show that the wave function is roughly independent of time, and for the hot case we show two values of z. For the hot case we show correlators for the quark and antiquark separated in the x and y directions, but at the same value of t. The anomalously high point indicated by the arrow at $d=8$ is the wave function at (8,0,0), which is halfway around the periodic lattice with $L=16$. Other points at similar distances, such as (6,4,4), are lower. We have omitted all other points where any component of the displacement was 8.

of these screening correlators: (1) Free quarks, perhaps with a temperature-dependent mass. In this case $\psi(\mathbf{x})$ would be constant (the quarks would remain uncorrelated in the coordinate $\mathbf{x}$, as they were created). This case seems to be ruled out. (2) Unbound quarks in an attractive screened potential. In this case we would expect that

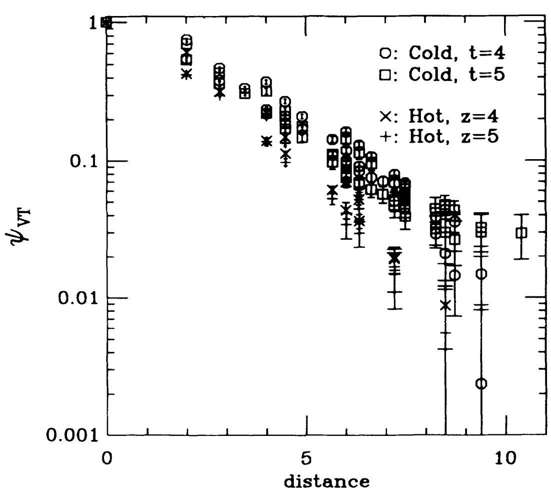

FIG. 4. The ρ (VT) wave function at $T=0$ and the screening ρ correlation function. Here we plot only displacements with all components less than $L/2=8$.

$\psi(\mathbf{x})$ would go to a constant at large $\mathbf{x}$. The data limit such behavior. (3) Bound quarks with an attractive screened interaction. (4) Bound quarks with a confining interaction.

From one point of view our result is not unexpected [7]. Briefly, at high temperatures our lattice is short in one direction, effectively becoming three dimensional for long distance observables. Thus we are measuring wave functions of the zero-temperature states of QCD on the compactified spatial manifold $S^1\times R^2$. In the pure Yang-Mills version of such a theory, large spatial Wilson loops are expected to show an area law, and separated point sources, a linearly rising potential. The very same evidence is offered for confinement at zero temperature, suggesting that the compactified theory is also confining. From this point of view it is not surprising that the screening correlators we have measured should be similar to the zero-temperature (four-dimensional) wave functions. On the other hand, a number of previous results are consistent with a picture of the high-temperature regime as a plasma of weakly correlated quarks. Among these are the dramatic rise in the Polyakov loop, corresponding to a drop in the free energy of a color triplet test charge, the similarity of the energy density to a gas of free quarks and gluons, the increase in baryon number susceptibility at the crossover [8], and the closeness of the ρ and nucleon screening masses to 2 or 3 times the Matsubara frequency. (Reference [9] discusses this seeming paradox and a possible resolution.)

Finally, we are well aware that the screening propagators are not directly observable in experiment. To relate these states to real-time finite-temperature plasma excitations requires an analytic continuation about which lattice simulations so far provide little information. Nonetheless, because they are analytically related to measurable quantities, they must be accommodated in phenomenological models. A picture of screening by hadronlike objects leaves unexplained the fact that the screening mass of the ρ is close to twice the Matsubara frequency. A coherent picture of high-temperature QCD which accounts in detail for all of the simulation results does not appear to be at hand.

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