Perturbations from Cosmic Strings in Cold Dark Matter

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(Received 10 July 1991; revised manuscript received 27 September 1991)

We present a systematic linear analysis of the perturbations induced by cosmic strings in cold dark matter. We calculate the power spectrum and find that the strings produce a great deal of power on small scales. We show that the perturbations on interesting scales are the result of many uncorrelated string motions, which indicates a more "Gaussian" distribution than was previously supposed.

PACS numbers: 98.80.Cq, 98.60.Eg

Cosmic strings are widely considered to be possible seeds for galaxies and larger-scale structure (see, for example, [1-9] and other papers which have followed). The idea is that a network of strings formed as defects in a symmetry-breaking phase transition in the early Universe. Such a network is expected to rapidly approach a "scaling solution" in which most of the properties of the network are independent of the initial details. As the network evolves, the strings attract other matter gravitationally. In this way perturbations are seeded which can grow, via gravitational collapse, into galaxies and larger-scale structure.

Veeraraghavan and Stebbins [10] have developed a formalism with which to study linear perturbations induced in the surrounding matter by sources such as strings. We have applied this formalism to a numerical realization of a cosmic string network for the case of a flat universe with cold dark matter (CDM) and radiation. We describe the results in terms of a simple model which can be easily extrapolated beyond the dynamical limitations of the numerical work, and adapted to different pictures of the string network. A complete discussion of this work will appear in [11]. This Letter reports the main results, which differ in major ways from earlier results based on more heuristic approaches.

The degree to which string networks have been shown to exhibit scaling behavior on cosmic time scales is still controversial [12]. For this work, however, we will assume that a realistic long-string network is well represented by a self-similar "scaling solution" [13] in which the statistical properties of the network at a given time can be described in terms of a single comoving scale $\xi(t)$ defined by

$$\rho_L = \mu/\xi^2 , \tag{1}$$

where ρ_L is the energy density in a long string and μ is the mass per unit length of the string. (Except where stated otherwise, we use units where $c=G=1$.) The long strings describe random walks of step size ξ, and also have a mean separation of $O(\xi)$. The strings move around at relativistic speeds, but the general properties of the network at two different times are related by simply rescaling ξ appropriately. The degree to which the long string is straight on scales smaller than ξ is a subject of current debate. One possibility is that small-scale structure contributes to a renormalization of μ and a possible decrease of the string's bulk velocity, but does not affect the gross scaling properties.

The time evolution of the string network is a process of dilution by the cosmological expansion and equilibration, whereby the long strings are continually chopped up into the statistically favored loops. As a result ρ_L decreases, and ξ grows, scaling roughly with the Hubble length, $R_H \equiv a/\dot{a}$ (where a is the cosmological scale factor). This evolution is basically determined by the rate of loop production.

In this paper we will calculate, using linear theory, the overdensity, $\delta(x) \equiv [\rho(x) - \bar{\rho}]/\bar{\rho}$, in matter today ($\bar{\rho}$ is the average density). We may write δ as the convolution of the string "source" density with a suitable Green function, integrated over all time. The source density is a component of the string stress-energy density: $\Theta_+(x) \equiv \Theta_{00}(x) + \Theta_{ii}(x)$. In Fourier space,

$$\delta_{\mathbf{k}} = \delta_{\mathbf{k}}^{i} + 4\pi(1+z_{\rm eq}) \int_{\eta_i}^{\infty} T(\mathbf{k};\eta') \tilde{\Theta}_+(\mathbf{k},\eta') d\eta' , \tag{2}$$

where T is $\tilde{T}_2^{\varsigma}$ from [10] and $\eta \equiv \int^t dt/a$. Here $\delta_{\mathbf{k}}^{i}$ is the initial perturbation, which "compensates" the string density, evolved to today. This compensation term is important because on large scales it cancels the string density and therefore suppresses the amplitude of inhomogeneities. The issue of compensation is a crucial one in these calculations and it is discussed in [10,11,14]. In [11] we conclude that the effect of $\delta_{\mathbf{k}}^{i}$ will be well approximated if we make the substitution $T \rightarrow T/[1 + (k_c/k)^2]$ and drop $\delta_{\mathbf{k}}^{i}$ from Eq. (2). This is essentially a long-wavelength cutoff which sets in at wave number k_c.

Thus modified, Eq. (2) can be squared and averaged over directions in order to study $P(k)$, the power spectrum:

$$(2\pi)^3 P(k) \delta^{(3)}(\mathbf{k}-\mathbf{k}') = 16\pi^2(1+z_{\rm eq})^2 \int_{\eta_i}^{\infty} \int_{\eta_i}^{\infty} T(k;\eta_1) T(k';\eta_2) \langle \tilde{\Theta}_+(k,\eta_1) \tilde{\Theta}_+(k',\eta_2) \rangle d\eta_1 d\eta_2 . \tag{3}$$

The picture can be simplified by noting that although there are two time integrals, the only significant contributions come from times when η_1 and η_2 are reasonably close. That is because the strings' configurations are uncorrelated when separated by a sufficiently long time. For large enough values of $\eta_1 - \eta_2$, $\langle \tilde{\Theta}_+(k,\eta_1)\tilde{\Theta}_+(k,\eta_2)\rangle$ will be negligibly small. As long as the correlation time of the strings is small compared with the time over which $T(k;\eta)$ varies, we can model $P(k)$ by

$$P(k) = 16\pi^2(1+z_{\text{eq}})^2\mu^2\int_{\eta_i}^{\infty} |T(k;\eta')|^2 F(k;\eta')d\eta' , \quad (4)$$

where

$$(2\pi)^3 F(k;\eta')\delta(k-k') \approx \int_{-\infty}^{+\infty} \langle \tilde{\Theta}_+(\mathbf{k},\eta')\tilde{\Theta}_+(\mathbf{k}',\eta'')\rangle d\eta'' . \quad (5)$$

The "structure function" $F(k;\eta)$ represents the power spectrum of the coherent string motions. In order to represent a scaling string network, we write $F(k;\eta) = \mathcal{F}(k\xi/a)$. [For some of our calculations T does vary by factors of the order of 1 over the string correlation time, but we do not expect this to introduce major errors, other than small changes in the normalization.]

We have chosen a form for $\mathcal{F}(k\xi/a)$ which is motivated by cosmic string physics [15]:

$$\mathcal{F}(k\xi/a) = \frac{2}{\pi^2}\overline{\beta^2\Sigma}\frac{\chi^2}{\xi^2}\left[\frac{1}{1+2(k\chi/a)^2}\right] , \quad (6)$$

where the overbar indicates an energy-weighted average over the string, β gives the (microscopic) velocity of the string, Σ is proportional to the surface density of the wakes produced, and χ ($\propto \xi$) gives the curvature scale of wakes. The form factor goes as k^{-2} on scales smaller than χ/a which is characteristic of the 2D surfaces swept out by the long strings, while for larger scales $\mathcal{F}$ has a white-noise (k^0) form characteristic of uncorrelated pointlike objects. We show in [11] that with the appropriate choice of parameters this model fits the Albrecht and Turok (AT) string network very well. Equation (6) does not include the effects of string loops which we have found to be not extremely important and whose contribution would differ widely between the different simulations.

This form factor is appropriate for both smooth strings and strings with very-small-scale structure which, if the small-scale structure is not resolved, may be thought of as one dimensional but with a renormalized string tension μ_r and a macroscopic bulk velocity β_b [16]. Using the results of [17],

$$\Sigma = \frac{\mu_r}{\mu}\frac{\beta_b}{(1-\beta_b^2)^{1/2}} + \frac{(1-\beta_b^2)^{1/2}}{2\beta_b}\left[\frac{\mu_r^2-\mu^2}{\mu\mu_r}\right] . \quad (7)$$

For smooth strings $\overline{\beta^2\Sigma} \sim 0.5$. For wiggly strings we use the results of [18] to estimate $\overline{\beta^2\Sigma} \sim 1.2$ in the radiation era and less in the matter era. For this estimate we have taken $\beta_b = 0.3$ in Eq. (7) which we think is a reasonable guess of the typical velocity averaged over the small-scale structure and not over the large-scale curvature.

For our calculations, the nature of the cosmic strings is completely specified once one fixes k_c, ξ, $\overline{\beta^2\Sigma}$, and χ. We consider three different models for these quantities: (1) the AT model [$k_c = 4\pi/\eta$ and $\xi(\eta)$ given by Eq. (3.7) in [13], $\overline{\beta^2\Sigma} = 0.5$, and $\chi = 0.58\xi$] which has been fitted to the simulation in [13] which exhibits no small-scale structure, (2) the "X" (extreme) model ($k_c = 2\pi/\eta$, $\xi = \chi = a\eta$, and $\overline{\beta^2\Sigma} = 0.5$) which is close to the old picture of string networks where the curvature scale of the strings is close to the horizon, and (3) the "I" (intermediate) model [$k_c = 2\pi/\eta$ and ξ given by Eq. (3.7) in [13] but with chopping efficiency $c_r = 0.16$, twice that for the AT model, $\overline{\beta^2\Sigma} = 1.2$, and $\chi = 2\xi$] which is motivated by the results of Bennett and Bouchet [19] and Allen and Shellard [20]. Their simulations tend to show both larger ξ, due to a greater chopping efficiency, and a larger curvature scale χ, at least partly due to the small-scale structure on their strings. For the AT, I, and X models $\xi/a\eta$ varies from 0.066, 0.140, and 1 in the radiation era to 0.121, 0.185, and 1 in the matter era. The X model gives both ξ and χ their largest plausible value which is much larger than any of the recent simulations suggest. Since the number of wakes in the X model is minimized, the prominence of individual wakes will be maximized.

We plot $4\pi(\lambda/2\pi)^{-3}P(\lambda)$ for the three models in Fig. 1 along with the same curve for Harrison-Zel'dovich (HZ) perturbations (as produced in inflationary cosmologies) with CDM [21] (dot-dashed curve). All the curves have the standard normalization of unit variance of $\delta m/m$ in an $R = 8h^{-1}$ Mpc sphere, chosen so the clustering of CDM reproduces the observed galaxy clustering [$h \equiv H_0/(100$ km/sec Mpc)]. Thus μ is different for each curve (see Table I). The slope of all the curves is λ^{-4} for large λ, because the scaling properties of the strings induce the same behavior in $P(\lambda)$ as does the scaling properties of the HZ perturbations. The transition between

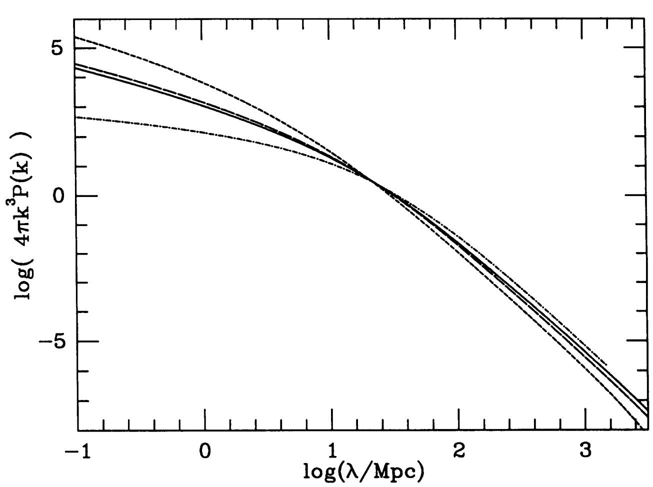

FIG. 1. The logarithm of $4\pi(\lambda/2\pi)^{-3}P(\lambda)$ vs $\log(\lambda)$ for inflation (dot-dashed), AT strings (short-dashed), X strings (solid), and I strings (long-dashed). We use $h = \Omega_0 = 1$, normalized at $8h^{-1}$ Mpc.

TABLE I. Fit parameters (α's) for Eq. (8) when k is in units of h^2/Mpc. The values of μ are fixed by the standard $8h^{-1}$ Mpc normalization.

	α_1	α_2	α_3	α_4	α_5	$10^6\mu$ $h=1$	$10^6\mu$ $h=0.5$
AT	1.77	2.30	1.80	0.520	0.00127	4.5	6.2
X	0.98	12.0	6.2	2.11	0.000345	16	27
I	6.80	4.70	4.40	1.55	0.000455	1.8	2.8

radiation and matter domination exposes the differences between strings and HZ perturbations. Smaller wavelengths, which enter the horizon in the radiation era, have much more power in the string spectrum compared with the HZ spectrum.

The integration over conformal time in Eq. (4) covers the entire history of a given scale. At early enough times, any given scale is far outside the Hubble radius and is unperturbed. Around the time when R_H [which always grows faster than $a(t)$] catches up with the comoving scale λ, the string perturbations become uncompensated and $P(\lambda)$ starts to receive significant contributions. The power continues to receive contributions at all later times, but perturbations produced later have less time to grow, and this can diminish their relative contribution to $P(\lambda)$.

One can fit the curves in Fig. 1 to about 10% accuracy with the function of the form

$$4\pi k^3 P(k)=\frac{4\pi k^4 h^4 \alpha_1^2 \mu^2}{1+(\alpha_2 k)+(\alpha_3 k)^2+(\alpha_4 k)^3}\left(\frac{1}{1+(\alpha_5/k)^2}\right)^2 \tag{8}$$

if one takes the α's given in Table I. The last factor in Eq. (8) represents a deviation from λ^{-4} behavior at very large wavelengths, which occurs because the very largest scales have just entered the horizon and have yet to receive their full complement of perturbations.

As originally pointed out by Vachaspati [22], it is tempting to think that individual string wakes could be responsible for the sheetlike structures in the distribution of galaxies observed on scales of around $50h^{-1}$ Mpc [23]. One way this could occur is if the wakes produced when the strings had a comoving coherence scale of $\sim 50h^{-1}$ Mpc contributed significantly to the overdensity surrounding that wake when averaged over the galaxy scale ($\sim h^{-1}$ Mpc). Indeed for a wake to produce a distinctive sheetlike overdensity one needs a single coherent string motion to produce the dominant perturbation on a *range* of scales, so that the single wake dominates inside many different size volumes which contain the wake. This allows the "sheet" to look both thin in one direction and wide in the other two. The variance of $\delta m/m$ in a sphere of radius R averaged over space is given by

$$\left.\frac{\delta m^2}{m^2}\right|_R=\int |w(kR)|^2 P(k) 4\pi k^2 dk\,,$$

where $w(x)=3[\sin(x)-x\cos(x)]/x^3$. The same sphere if centered on a planar wake will receive a contribution $\delta m/m=\frac{3}{2}\Delta R^{-1}$ from that wake. Here $\Delta(\eta)$ is the distance out to which matter has been accreted, which depends on the time η when the wake was produced, and is proportional to Σ. (We take $\Sigma=1$ for AT and X strings, and $\Sigma=2.5$ for I strings.) For a wake to be sheetlike $\delta m/m$ must be comparable to or exceed the rms $\delta m/m$ on a range of scales. As we shall see this is probably not the case.

In Fig. 2 the curved lines show the rms $\delta m/m$ vs R. The straight lines show the contribution of an individual wake to the $\delta m/m$ in a sphere centered on that wake. For each of our three models we calculate the single-string $\delta m/m$ for wakes produced at two different times. One time corresponds to when $\chi=50h^{-2}$ Mpc and the other is chosen so that Δ takes on its maximum value, $1.5\times10^6\Sigma\mu$ Mpc, which occurs at about $z=1.7z_{\text{eq}}$ [24]. We terminate the single-wake lines at $R=\chi/a$, the curvature scale of the wakes. For the AT strings the lines are always well below the rms curve. For I strings the lines are close to the rms curve for large R, but fall away with decreasing R on scales where the rms curve is steeper than R^{-1}. The single-wake lines always lie above the rms for the X strings.

We can use Fig. 2 to make two points. First, the $\chi=50h^{-2}$ Mpc wakes do not give a dominant contribution to $\delta m/m$ on scales relevant to galaxy formation ($R\sim1$ Mpc). The maximal wakes, for example, have a larger $\delta m/m$, and there are many more of them, since their mean separation is much smaller.

Second, in neither the AT nor the I model do *any* of the wakes meet the aforementioned criteria to produce in-

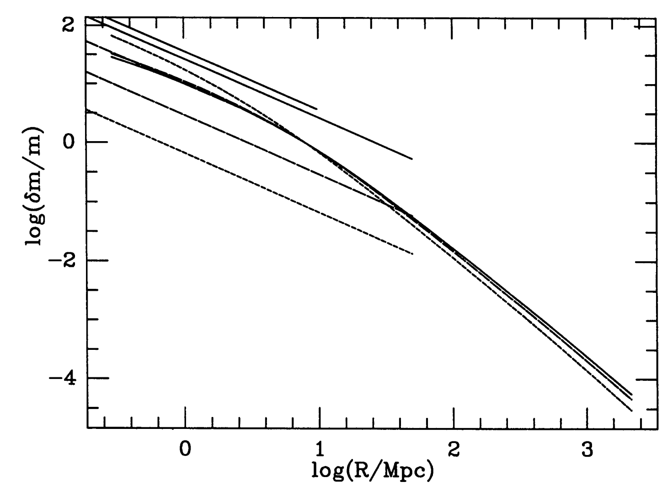

FIG. 2. The value of $\delta m/m$ in a ball of radius R, vs R for AT strings (short-dashed), X strings (solid), and I strings (long-dashed). The curves give the rms $\delta m/m$, and the straight lines give $\delta m/m$ in a ball centered on a single wake. We use $h=\Omega_0=1$, normalized at $8h^{-1}$ Mpc. The line for the AT maximal wake, with $\chi=0.4h^{-2}$ Mpc, lies on top of the line for the I maximal wake and has been removed for clarity.

itially sheetlike perturbations. Thus sheetlike density patterns are not imprinted by the strings. This does not mean that some sort of strong biasing mechanism could not lead to a sheetlike distribution of galaxies, nor that when a given scale begins to go nonlinear that it will not produce sheetlike Zel'dovich pancakes. These pancakes are a property of gravitational clustering and not evidence for "stringiness." Also, since on all scales many wakes are required to produce the rms, we expect the *linear* density field to be fairly Gaussian. If something very different from any of the simulations, such as the X model, were really how string networks behaved, then a less-Gaussian, more-sheetlike initial density field would be expected.

In conclusion, our picture of perturbations from cosmic strings in CDM has been significantly changed by a more systematic analysis. The perturbations on scales less than $10h^{-1}$ Mpc are not dominated by a few wakes with coherence length greater than $10h^{-1}$ Mpc, but rather by much smaller wakes. Galaxy-sized inhomogeneities will not be confined to sheets with a coherence scale very much greater than the galactic scale. The resulting spectrum has much more power on small scales, as compared with the Harrison-Zel'dovich spectrum, and also as compared with earlier pictures of perturbations from strings. The contributions of many string motions with many different coherence scales make it unlikely that distinctive "stringy" features will show up strongly in the matter distribution today.

Even given the uncertainties in the relation between the linear perturbation spectrum and the distribution of luminous matter, the string spectrum does not look good. When one normalizes at $8h^{-1}$ Mpc the amplitude on large scales is smaller than is predicted by inflation. Current thinking (e.g., Ref. [25]) suggests that the inflationary CDM spectrum is in trouble for lack of power on large scales. The string-induced CDM spectrum should have even greater problems.

We thank Robert Brandenberger for useful conversations. This work was supported in part by the DOE and NASA (Grant No. NAGW-2381) at Fermilab.

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