Comment on "*CP* Asymmetries Induced by Particle Widths: Application to Top-Quark Decays"

In a recent Letter [1], Eilam, Hewett, and Soni discussed the *CP*-violating asymmetries that arise in the decays $t \to q'\bar{q}q$ due to the interference between the tree and penguin amplitudes. The two amplitudes have different weak phases and a relative phase from final-state interactions (FSI), which is dominated by the imaginary part of the tree amplitude, induced by the width of the W boson. However, the calculation in Ref. [1] omits a term in the decay amplitude, which is necessary in order to cancel the contribution to the asymmetry from the rescattering of the final state into itself, a result that is intimately connected to *CPT* invariance [2,3]. Although the standard model predicts asymmetries for the t decays which are too small to be detected, it is possible that higher values may be derived from extensions of the model. Thus, it seems important to give a correct analysis of the problem. I will show that the effect of the missing term is significant for the asymmetry in $t \to u\bar{d}d$, where the rescattering is Cabibbo allowed (the same results will apply to $t \to c\bar{s}s$, but not to $t \to c\bar{d}d$ where the rescattering is suppressed). It will follow from the analysis that equally important asymmetries occur in the semileptonic decays $t \to l^+\nu_l d$, more readily accessible to experiment.

The unitarized decay amplitude $\mathcal{A}_{u\bar{d}d}$ includes the contributions from both the direct decay, $t \to u\bar{d}d$, and the decay via the intermediate states X, $t \to X \to u\bar{d}d$. It is given by $\mathcal{A} = \sqrt{S}\,\mathcal{A}^0$, where $\sqrt{S}$ is the square root of the scattering matrix for the FSI, and the elements of $\mathcal{A}^0$ are the decay amplitudes in the absence of FSI. In the case at hand, the final-state scattering is dominated by the W boson resonance. The important intermediate states are those for which the scattering is Cabibbo allowed: $X = u\bar{d}d, c\bar{s}d, l^+\nu_l d$ (to keep the notation compact, I neglect color and the other lepton families, for now). The eigenvalues of $\sqrt{S}$ are $e^{i\delta}$ for the eigenstate

$$|\alpha\rangle = (|u\bar{d}d\rangle + |c\bar{s}d\rangle + |l^+\nu_l d\rangle)/\sqrt{3}\,, \tag{1}$$

which is the final state of the tree decay and has the W boson resonance (δ is the phase of the Breit-Wigner propagator), and 1 for the other orthogonal states. In the $\{|u\bar{d}d\rangle, |c\bar{s}d\rangle, |l^+\nu_l d\rangle\}$ basis,

$$\sqrt{S} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{1}{3}(-1+e^{i\delta}) \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{2}$$

The $\mathcal{A}^0$ amplitudes are $\mathcal{A}^0_{\rm tree} + \mathcal{A}^0_{\rm peng}$, for $|u\bar{d}d\rangle$, and $\mathcal{A}^0_{\rm tree}$, for $|c\bar{s}d\rangle$ and $|l^+\nu_l d\rangle$. Then,

$$\mathcal{A}_{u\bar{d}d} = (\mathcal{A}^0_{\rm tree} + \mathcal{A}^0_{\rm peng}) + \tfrac{1}{3}(-1+e^{i\delta})2\mathcal{A}^0_{\rm tree} + \tfrac{1}{3}(-1+e^{i\delta})(\mathcal{A}^0_{\rm tree} + \mathcal{A}^0_{\rm peng})\,. \tag{3}$$

The difference in decay rates follows from the interference between the direct decay amplitude [the first term on the right-hand side of Eq. (3)] and the amplitude for the decay via $c\bar{s}d$ and $l^+\nu_l d$ (the second term). There is no contribution from the rescattering of the final state $u\bar{d}d$ (the last term). The result is

$$\Delta_{u\bar{d}d} \equiv |\bar{\mathcal{A}}_{u\bar{d}d}|^2 - |\mathcal{A}_{u\bar{d}d}|^2 = \tfrac{2}{3}\,{\rm Im}(\mathcal{A}^0_{\rm tree}\mathcal{A}^{0*}_{\rm peng})4\sin\delta\,. \tag{4}$$

In the approach of Eilam, Hewett, and Soni [1] the penguin contribution to the rescattering term in Eq. (3) is not included. As a consequence, there is a contribution to the asymmetry from the rescattering of the final state. This would make the result for $t \to u\bar{d}d$ too large by a factor of $\tfrac{3}{2}$, and incompatible with *CPT* invariance [2].

From *CPT* invariance, we know that there must be a quantity analogous to $\Delta_{u\bar{d}d}$ for the decays into the final states $c\bar{s}d$ and $l^+\nu_l d$, so that the asymmetry in the total decay rate vanishes. Accordingly, one proceeds as for the $u\bar{d}d$ case and finds that

$$\Delta_{l^+\nu_l d} = \Delta_{c\bar{s}d} = -\tfrac{1}{2}\Delta_{u\bar{d}d}\,. \tag{5}$$

Once the color factors are included for the nonleptonic decay rates, Eq. (5) still holds provided $\Delta_{l^+\nu_l d}$ now stands for the sum over the semileptonic channels. Thus, there is an asymmetry for each of the semileptonic decays $t \to l^+\nu_l d$ which is one-half the asymmetry for $t \to u\bar{d}d$.

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João M. Soares
Carnegie-Mellon University
Pittsburgh, Pennsylvania 15213

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