Measurement of Photonic Band Structure in a Two-Dimensional Periodic Dielectric Array

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The photonic band structure in a two-dimensional dielectric array is investigated using the coherent microwave transient spectroscopy (COMITS) technique. The array consists of alumina-ceramic rods arranged in a regular square lattice. The dispersion relation for electromagnetic waves in this photonic crystal is determined directly using the phase sensitivity of COMITS. The experimental results are compared to theoretical predictions obtained using the plane-wave expansion technique. Configurations with the electric field parallel and perpendicular to the axis of the rods are investigated.

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The propagation of electromagnetic radiation in periodic dielectric structures has received much recent experimental and theoretical attention. The existence in such structures of photonic band gaps—frequency intervals in which no photon modes are allowed—leads to a variety of phenomena of both fundamental [1,2] and practical interest [3,4]. Theoretical calculations of the dispersion relations for propagation in both three-dimensional [5–8] and two-dimensional [9] dielectric structures using the plane-wave expansion have become relatively sophisticated. Experimentally, investigations of photonic band-structure phenomena have been confined to measurements at microwave frequencies because of difficulties in fabricating ordered dielectric arrays of optical length scales. Although microwave experiments have been used to determine the frequencies which define photonic band gaps [10,11], and to perform elegant explorations of localized defect modes [11,12], these techniques have not been used to measure the dispersion of radiation at frequencies away from the gaps. Measurement of the photonic dispersion relation over a broad frequency range would provide a direct test of the theoretical formalisms, and would also elucidate the transition in propagation from long wavelengths, where the dielectric array behaves as an effective medium, to shorter wavelengths, where strong scattering leads to dispersion and the opening of photonic band gaps.

In this Letter we investigate the dispersion relation for electromagnetic wave propagation in a periodic dielectric array using the recently developed coherent microwave transient spectroscopy (COMITS) technique [13,14]. COMITS is based on the radiation and detection of picosecond-duration electromagnetic transients with optoelectronically pulsed antennas. It is capable of free-space microwave and millimeter wave measurements over a broad frequency range (15–140 GHz) with good (60:1) polarization sensitivity [15,16]. In particular, the phase sensitivity of the COMITS technique is used to directly measure the dispersion relation in a two-dimensional photonic crystal across the fundamental and higher band gaps. The experimental results are compared to theoretical predictions obtained with the plane-wave expansion method, which has been described elsewhere [7]. In brief, the macroscopic Maxwell's equations are expressed as an eigenvalue equation in reciprocal space and the resulting matrix is then diagonalized to yield the mode frequencies and field patterns. This technique is a simple and powerful method to solve problems in electrodynamics which takes full account of the vector nature of the electromagnetic radiation.

Although there is much interest in three-dimensional photonic crystals, i.e., structures for which there is a photonic band gap in all propagation directions, in these experiments we chose to study a two-dimensional dielectric structure consisting of alumina-ceramic cylinders arranged in a square array. The primary reason for this choice is that the two-dimensional structures are easier to fabricate. However, the COMITS technique is capable of measuring the dispersion relation in three-dimensional systems as well. The results of Ref. [9] were used to determine the optimal choices for cylinder diameter and lattice spacing so that photonic band gaps appear in the experimentally accessible frequency range (15–140 GHz). For the measurements presented here, 0.74 $\pm$0.03-mm-diam 100-mm-long alumina-ceramic rods were arranged with a 1.87-mm lattice constant in a square array. Alumina was chosen because it has a large dielectric constant and low dielectric loss over the microwave and millimeter-wave spectrum [13].

The COMITS experimental setup is shown schematically in Fig. 1. The transmitting and receiving elements are exponentially tapered, coplanar strip antennas fabricated photolithographically on silicon-on-sapphire [14]. The silicon epilayer is subsequently ion implanted to reduce the carrier lifetime to less than 1 ps. Ultrafast optical pulses from a mode-locked, pulse-compressed, and frequency-doubled Nd-doped yttrium-lithium-fluoride laser are arranged in a conventional pump-probe configuration. The 527-nm wavelength pulses are ~1.5 ps wide and have a repetition rate of 240 MHz. A short

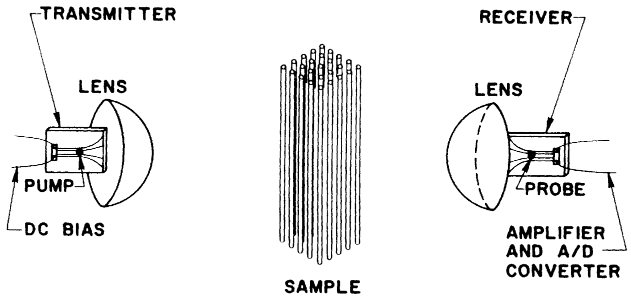

FIG. 1. Experimental setup for coherent microwave transient spectroscopy measurements. The two-dimensional photonic crystal is made of 0.74-mm-diam 100-mm-long alumina-ceramic rods arranged in a square array with a 1.87-mm lattice constant.

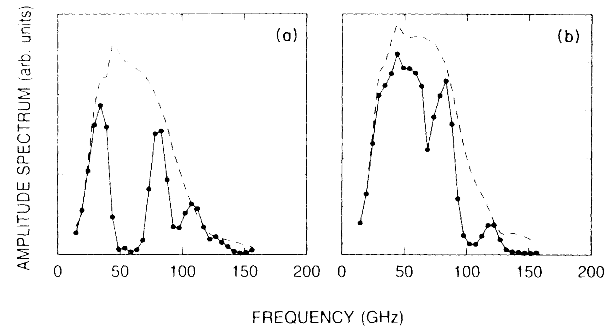

FIG. 2. Amplitude spectra obtained by a numerical Fourier transform of experimentally measured time-domain data. The dashed line represents the reference amplitude spectra obtained with no sample in the beam. Solid circles represent the amplitude spectra transmitted through seven rows of rods with propagation along the ⟨10⟩ direction with E field of the transient radiation polarized (a) parallel and (b) perpendicular to the axis of the rods.

current pulse is generated on the dc-biased transmitter by the pump beam. This pulse propagates down the coplanar strip line, spreading in time to about 7 ps, and is radiated into free space by the exponentially tapered flare. Hemispherical-fused silica lenses are used to collimate the transient radiation from the transmitter and to focus it, after passing through the sample, onto the receiver [14]. The voltage induced on the receiver is measured, by photoconductive sampling, as a function of the time delay between the pump and probe pulses [14]. Signal averaging is performed by adding ~1000 scans with a rapid-scan delay line.

To measure the transmission properties of a dielectric array, the sample is placed in the beam path so that the transient radiation propagates in a plane perpendicular to the axis of the cylinders. The cylinders forming the array were fixed in a machined holder so that the orientation of the sample, with respect to the direction of propagation, could be adjusted. Also, the number of rows in the array can be changed easily. The transverse dimensions of the array were always made larger than the collimated beam size (~3 cm) so that there were no end effects [14]. By suitably rotating the sample we could make measurements with polarization either parallel or perpendicular to the axis of the cylinders [15,16]. By symmetry, waves incident on the rods with polarization either parallel or perpendicular to the rod axis will preserve this polarization.

Time-domain wave forms are recorded first without and then with the sample in the beam path. The amplitude spectrum of the former, obtained by a numerical Fourier transform, is shown as the dashed lines in Fig. 2. Clearly, the pulse contains usable frequency components in the 15–140-GHz frequency range. Because the measured signal is proportional to the time-dependent voltage induced on the receiving antenna, phase information (not shown) is preserved. As we describe below, this phase sensitivity is crucial in determining the dispersion relation for electromagnetic wave propagation in the photonic crystal. The amplitude spectrum of the measured wave form with the E field parallel to the axis of the ceramic rods is shown as the solid line in Fig. 2(a). The corresponding spectrum with E field perpendicular to the axis of the rods is depicted in Fig. 2(b). For both measurements the sample was 7 rows of rods deep in the direction of propagation and 25 columns wide transverse to it. Propagation was along the ⟨10⟩ axis of the lattice. Figure 2(a) clearly indicates the existence of a band gap between 45 and 70 GHz, and the suggestion of other gaps at about 100 and 125 GHz. Very narrow band gaps are not clearly resolved because our frequency resolution is limited to 5 GHz by the 200-ps window of the temporal data [14]. The results presented above are consistent with previous experimental measurements on two-dimensional photonic crystals [11].

Although the amplitude spectrum alone gives an indication of the gaps, it is much more instructive to analyze the phase data to determine the effective refractive index of the photonic crystal and, thus, to determine the full dispersion relation. The complex transmission function of the dielectric array is obtained by dividing the complex Fourier transform of the wave form with the sample by the complex Fourier transform of the reference wave form without a sample. This transmission function contains both amplitude and phase information, and represents the electromagnetic propagation properties of the photonic crystal. Using the known thickness of the dielectric array L and the net phase difference ϕ, the effective microwave refractive index $n(f)$ can be calculated at each frequency [14]:

$$n(f) = (\phi c/2\pi L f) + 1 , \qquad (1)$$

where c is the velocity of light. Using the effective index values, the dispersion relation, f vs k, can be calculated directly with

$$k = (2\pi f/c)\, n(f) . \qquad (2)$$

The measured dispersion relation for propagation along the ⟨10⟩ direction with the E field polarized parallel to the rods is plotted as the solid circles in Fig. 3(a). Overlaid on the measured data are the theoretical predictions (dashed lines) calculated using the plane-wave expansion technique. The agreement between the measured data and the theoretical calculation is generally excellent. In Fig. 3(b) we show the dispersion relation for propagation along the ⟨10⟩ direction but with the polarization rotated such that the E field is perpendicular to the axis of the cylinders. The solid squares in the figure are the experimentally measured values and the dashed lines predictions of theory. Again, the agreement between theory and experiment is excellent. For both polarizations photonic band gaps appear, as expected, at the Brillouin zone boundaries. Although the theory predicts that small gaps open at higher frequencies, as mentioned above, the resolution of the COMITS technique is insufficient to fully resolve them. However, in the amplitude spectra of Fig. 2 there are dips at these points reflecting a strong suppression in the density of states. It should be noted in Fig. 3(b) that there is a large jump in wave vector (i.e., a large phase change) between band 2 and band 3 resulting in an ambiguity in the correct value for the phase, and hence in the correct form of the dispersion relation. This ambiguity can generally be resolved by determining the transmission function due to a single row of rods, accomplished by comparing the transmission through two photonic crystals whose thicknesses differ by one row of rods.

The theoretical calculations also predict the existence of a band which is not seen experimentally, band 3 in Fig. 3(a). Considering the excellent agreement between theory and experiment for the other modes, this result is rather surprising. However, an examination of the field patterns for these bands reveals that a planar wave front (as in our experiment) cannot excite these modes. The reason is as follows. In our experiment we measure the band structure along the ⟨10⟩ direction, which is a special direction of the Brillouin zone. Physically, this means that the fields must be either even or odd upon reflection through the mirror plane shown in Fig. 4. As this figure shows, the fields for band 3 are *odd* under reflection through this mirror plane. However, an incoming plane wave with electric field oriented along the rod axis is *even* with respect to this reflection, and so it cannot excite the mode of band 3. In fact, all of the modes which are seen experimentally are even under this reflection, and all of the modes which are predicted but not found are odd. Symmetry plays an analogous role for radiation of the other polarization. Our calculations show that the lowest three bands in Fig. 3(b) are symmetry allowed and we find experimentally that all three are observed. Theoretically, we predict that band 4 of Fig. 3(b) is symmetry forbidden and we do not observe this band in our experiment. It is possible that we do not observe this band because of the high-frequency limitations of our experiment. Thus, we believe that the missing modes in Fig. 3 do exist in the crystal, but that they do not transmit electromagnetic radiation in this specific experiment.

At long wavelengths, i.e., for frequencies well below the fundamental gap, the dielectric response of the two-dimensional array is well described by effective-medium theory. The orthogonal orientations of the polarization

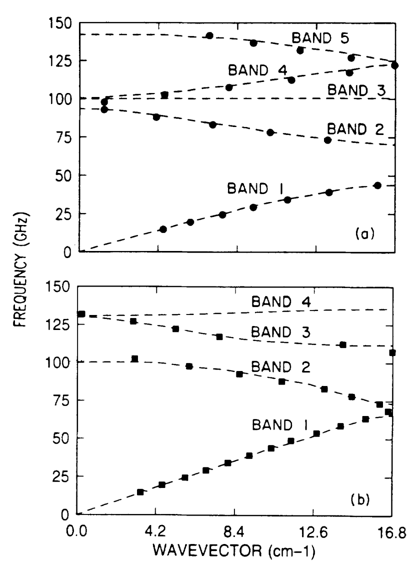

FIG. 3. Dispersion relation for propagation of electromagnetic waves along the ⟨10⟩ direction of a two-dimensional photonic crystal with polarization (a) parallel and (b) perpendicular to the rod axis. The solid symbols are the experimentally determined values and the dashed lines are the predictions of theory.

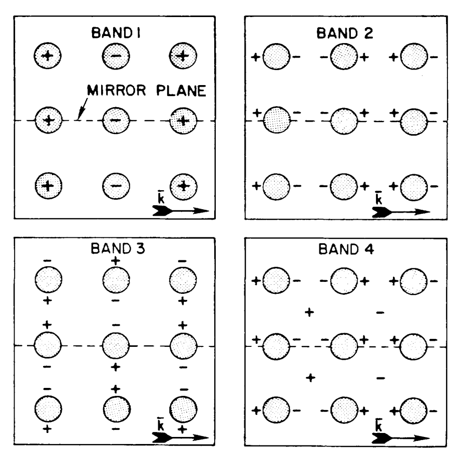

FIG. 4. The symmetry of the electric fields associated with the lowest four photonic bands. For these modes, the electric-field lines run parallel to the rod axis, and so the + (−) signs indicate regions in which the electric field is oriented into (out of) the page. The states depicted lie at the Brillouin zone edge and so the fields alternate sign in neighboring lateral unit cells. The shaded circles (not shown to scale) indicate the square lattice of dielectric rods. The modes along the ⟨10⟩ direction must be even (bands 1, 2, and 4) or odd (band 3) with respect to reflection in the mirror plane, shown as the dashed line.

with respect to the two-dimensional array of cylinders correspond precisely to the configurations considered in effective-medium theory for the limiting cases of maximum (E field parallel to all interfaces) and minimum (E field perpendicular to all interfaces) screening [17]. The dielectric constants for these two situations are given by

$$\epsilon=f_1\epsilon_1+f_2\epsilon_2\,, \tag{3}$$

$$1/\epsilon=f_1/\epsilon_1+f_2/\epsilon_2\,, \tag{4}$$

where f_1 and f_2 are the volume filling fractions occupied by the two media with dielectric constants ϵ_1 and ϵ_2, respectively. For our two-dimensional array of alumina cylinders ($\epsilon_1=8.9$) these expressions predict microwave refractive indices of 1.41 and 1.06 for E field parallel and perpendicular to the cylinders, respectively. Experimentally the lowest-frequency (long-wavelength) refractive index values are 1.47 and 1.11.

In summary, we have investigated the photonic band structure of a two-dimensional dielectric array using the coherent microwave transient spectroscopy technique. The experimentally determined dispersion relations for electromagnetic wave propagation, for E-field polarization both parallel and perpendicular to the rod axis, were in good agreement with theoretical predictions calculated using the plane-wave expansion technique. Certain modes which were predicted theoretically were not observed because coupling between these modes and the incident plane-wave radiation was forbidden by symmetry. Although results for propagation along the ⟨10⟩ direction were presented, similar results were also observed for propagation along the ⟨11⟩ direction. Furthermore, the COMITS technique can also be used to explore the photonic band structure of three-dimensional photonic crystals.

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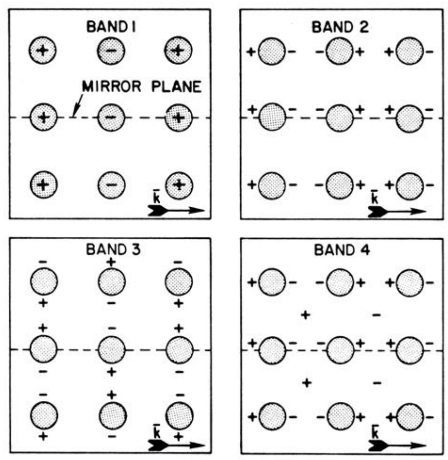

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