Statistical Field Theory of Multiparticle Density Fluctuations

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We present a three-dimensional statistical field theory of density fluctuations motivated by the Ginzburg-Landau theory of superconductivity. The free field theory yields Yukawa-type two-particle cumulant correlations and no higher-order ones, in agreement with high-energy heavy-ion data. We mention the extension of our model to describe hadronic reactions and indicate how it may be related to QCD. We predict the multiplicity distribution of particles produced in heavy-ion collisions.

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There is much interest in studying the underlying dynamics of multiparticle production in ultrarelativistic heavy-ion collisions. In particular, the possibility of detecting the onset of a phase transition from hadronic matter to quark-gluon plasma has created excitement in this field [1]. So far there are no conclusive predictions of signatures for quark matter. However, the study of unusually large density fluctuations observed in high-energy hadronic and heavy-ion reactions seems promising, especially to study the poorly understood dynamics of particle production [2].

Several experiments have recently measured these fluctuations via the bin-averaged factorial moments, defined as [3]

$$F_p(\delta y)=\frac{1}{M}\sum_{m=1}^{M}\frac{\langle n_m(n_m-1)\cdots(n_m-p+1)\rangle}{\langle n_m\rangle^p}=\frac{1}{M(\delta y)^p}\sum_{m=1}^{M}\int_{\Omega_m}\prod_i dy_i\,\frac{\rho_p(y_1\cdots y_p)}{\langle\rho_m\rangle^p}, \tag{1}$$

where n_m is the number of particles in bin m, M is the total number of bins, δy is the rapidity bin size ($\delta y=Y/M$), and ρ_p is the p-particle density correlation function. In all experiments the factorial moments are found to increase with decreasing bin size [4], for which there is presently no generally accepted theoretical explanation. Existing event generators fail to reproduce the data [5].

The connection between factorial moments and correlations was previously emphasized, in particular that higher-order moments have large combinatoric contributions from two-particle correlations [6]. Therefore, in order to examine the true higher-order correlations, we express F_p in terms of bin-averaged cumulant moments (for simplicity we here consider only rapidity "space"),

$$\begin{aligned}&F_2=1+K_2,\quad F_3=1+3K_2+K_3,\\&F_4=1+6K_2+3(K_2)^2+4K_3+K_4,\end{aligned} \tag{2}$$

where

$$K_p(\delta y)=\frac{1}{M(\delta y)^p}\sum_m\int_{\Omega_m}\prod_i dy_i\,k_p(y_1\cdots y_p), \tag{3}$$

and

$$\begin{aligned}
k_2(1,2)&=\frac{\rho_2(y_1,y_2)}{\langle\rho(y_1)\rangle\langle\rho(y_2)\rangle}-1,\\
k_3(1,2,3)&=\frac{\rho_3(y_1,y_2,y_3)}{\langle\rho(y_1)\rangle\langle\rho(y_2)\rangle\langle\rho(y_3)\rangle}-\sum_{\rm perm}^{(3)}\frac{\rho_2(y_1,y_2)}{\langle\rho(y_1)\rangle\langle\rho(y_2)\rangle}+2,\\
k_4(1,2,3,4)&=\frac{\rho_4(y_1,y_2,y_3,y_4)}{\langle\rho(y_1)\rangle\langle\rho(y_2)\rangle\langle\rho(y_3)\rangle\langle\rho(y_4)\rangle}-\sum_{\rm perm}^{(4)}\frac{\rho_3(y_1,y_2,y_3)}{\langle\rho(y_1)\rangle\langle\rho(y_2)\rangle\langle\rho(y_3)\rangle}\\
&\quad-\sum_{\rm perm}^{(3)}\frac{\rho_2(y_1,y_2)\rho_2(y_3,y_4)}{\langle\rho(y_1)\rangle\langle\rho(y_2)\rangle\langle\rho(y_3)\rangle\langle\rho(y_4)\rangle}+\sum_{\rm perm}^{(12)}\frac{\rho_2(y_1,y_2)}{\langle\rho(y_1)\rangle\langle\rho(y_2)\rangle}-6,
\end{aligned} \tag{4}$$

etc. If there are no true dynamical correlations, the cumulant moments K_p vanish.

The KLM and EMU01 Collaborations have measured factorial moments in proton-nucleus and nucleus-nucleus collisions. In Fig. 1 we present cumulants K_2 for NA22 [7] (π-p collision at $\sqrt{s}=22$ GeV), KLM [8] [p-emulsion (em) and O-em at 200 GeV/nucleon], and EMU01 data [9] (S-Au at 200 GeV/nucleon), all referring approximately to the same c.m. energy per particle. K_2 decreases in going from lighter to heavier projectiles, especially in the case of sulfur.

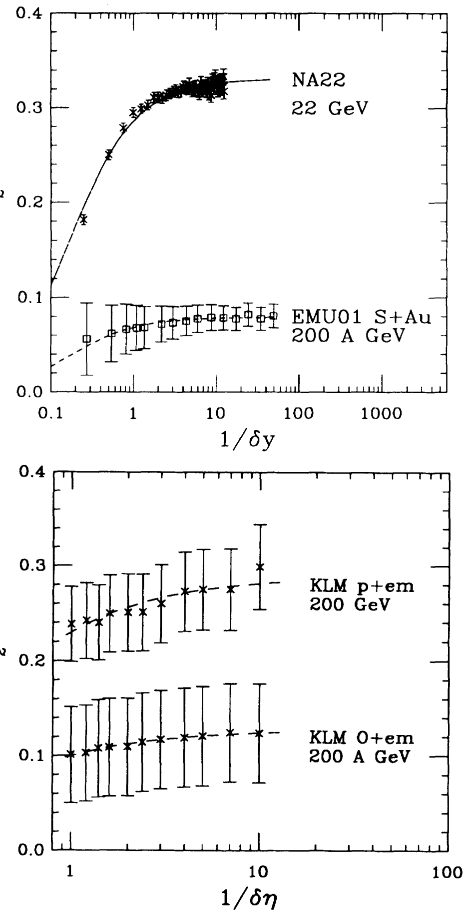

FIG. 1. Theoretical results for K_2 (dashed line) compared with NA22 [7], KLM [8], and EMU01 data [9].

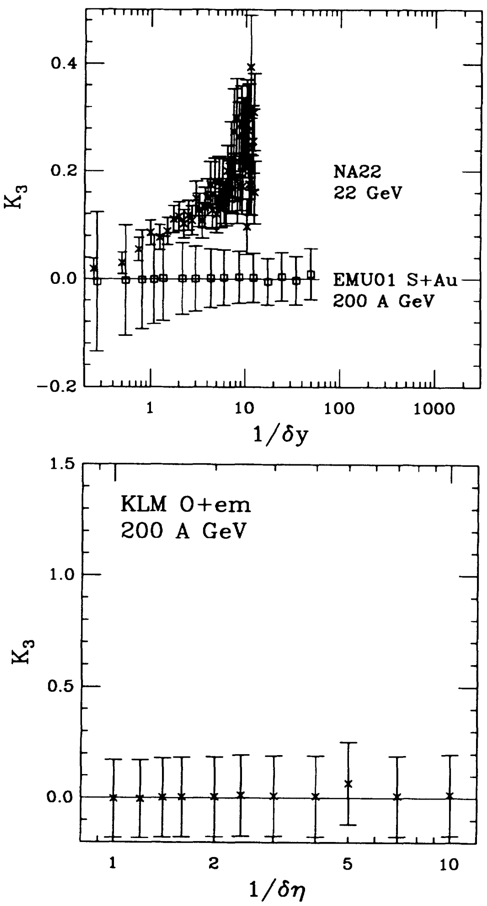

FIG. 2. The cumulant K_3 for NA22 [7], KLM [8], and EMU01 data [9].

Higher-order cumulants K_3, K_4, and K_5 for hadronic and nuclear collisions are given in Figs. 2–4 in Ref. [10]. Since they all exhibit similar dependence on δy, in Fig. 2 we only present K_3. While in hadronic collisions K_3 and K_4 are non-negligible (e.g., K_3 gives a 20% contribution to F_3 at small δy), in nucleus-nucleus collisions at the same energy these cumulants are compatible with zero. Thus, in high-energy heavy-ion collisions, the observed increase of higher-order factorial moments F_p [8,9] is entirely due to dynamical two-particle correlations. This conclusion holds even in a higher-dimensional analysis. The KLM Collaboration has done a two-dimensional analysis (rapidity y, azimuthal angle φ) of the factorial moments [8]. Their measured two-dimensional cumulant K_2 is increasing with decreasing bin size ($\delta\varphi\delta y$) faster than in the one-dimensional case, but the higher-order cumulants are still consistent with zero [11]. Preliminary NA35 data for O-Au at 200 GeV/nucleon indicate that there are no true dynamical higher-order correlations present in *any* dimension [12].

Up to now there has been no theory to describe these phenomena. We present a statistical field theory of density fluctuations which shows the observed features and which is motivated by the Ginzburg-Landau (GL) theory [13]. However, we take the conservative view that no (first- or) second-order phase transition should be implied by any model as long as there is no compelling evidence in the data, in contrast to other approaches in the literature. For example, there has been some controversy about attempts to interpret intermittency in high-energy collisions as a signal of a second-order phase transition by assuming a two-component model (one component corresponds to the critical behavior) [14] or by fitting one-dimensional factorial moments with straight lines (i.e., assuming that 1D moments do not saturate at small δy) and then analyzing whether these slopes follow monofractal (signal of quark-gluon plasma) or multifractal behavior (signal of cascading). Detailed discussion of these models can be found in Ref. [15].

The large number of particles produced in ultrarelativistic heavy-ion collisions justifies a statistical description of particle production. The formal analogy with statistical mechanics of a one-dimensional "gas" in rapidity space was first pointed out by Feynman and Wilson [16] and later developed by Scalapino and Sugar [17] and many others [18]. In the case of superconductivity the microscopic BCS theory provided the derivation of GL theory and its phenomenological parameters. We hope that our macroscopic theory of multiparticle production and density fluctuations can eventually be derived from a fundamental theory such as QCD. Two approaches presently can be envisaged: (i) Describe the hadronization process in an effectively confining theory involving quarks and gluons, which is modeled in analogy to type-II superconductors; and (ii) describe the emerging hadronized phase by an effective theory of the QCD (scalar) condensate, such as the sigma model. One would attempt to integrate out time-dependent modes of the chosen model to obtain a statistical description of (quasi)particle density fluctuations. Then, the observed intermittent behavior is conjectured to arise as a relatively low-energy phenomenon, in contrast with, e.g., self-similar cascading of high-energy partons, conjectured to yield intermittency via parton-hadron duality.

In the particle-production problem the field (order parameter) describes density fluctuations. The random field ϕ is assumed to depend on the rapidity of the particle ($y=\frac{1}{2}\ln[(E+p_\parallel)/(E-p_\parallel)]$) and its transverse momentum. Even though particles produced in high-energy collisions need not be in thermal equilibrium, we can still introduce a functional of the field ϕ, $F[\phi]$, which plays a role analogous to the free energy in equilibrium statistical mechanics. In principle, it should be derived from the underlying dynamics. We chose

$$F[\phi]=\int_0^Y dy\int_{p_t\le p_{t,\max}}\frac{d^2p_t}{P^2}[a^2(\partial_y\phi)^2+a^2(\nabla_{(p_t/P)}\phi)^2+M^2\phi^2+V(\phi)]\,, \tag{5}$$

where Y is the rapidity gap between projectile and target, and a and M are phenomenological parameters depending on control parameters of the considered reaction, such as total energy, mass number(s), impact parameter, etc. We introduced a scale factor P, such that all constants and ϕ remain dimensionless. Presently we drop the interaction term $V(\phi)$, since a free field theory of density fluctuations accurately describes multiparticle heavy-ion data. That is, if, in analogy with the order parameter of a critical liquid-gas system, we suitably identify ϕ ($\mathbf{z}\equiv\mathbf{p}_t/P$),

$$\phi(y,\mathbf{z})=\rho(y,\mathbf{z})/\langle\rho(y,\mathbf{z})\rangle-1\,, \tag{6}$$

so that $\langle\phi\rangle\equiv0$. Here $\langle\rho(y,\mathbf{z})\rangle$ is the average (over all events) particle density. Note that we differ from the original Scalapino-Sugar model [17], in which ϕ^2 is identified with particle density leading to expressions for the two- and three-particle cumulants which do not agree with the experimental data we consider. Introducing a "partition function,"

$$Z=\int\mathcal{D}\phi e^{-F[\phi]}\,, \tag{7}$$

physical quantities are given by ensemble averages appropriately weighted by $F[\phi]$. We remark here that hadronic reactions require an interacting theory to be studied separately. Field correlations are given by $[\mathbf{x}\equiv(y,\mathbf{z})]$

$$\langle\phi(\mathbf{x}_1)\phi(\mathbf{x}_2)\cdots\phi(\mathbf{x}_p)\rangle=\frac{1}{Z}\int\mathcal{D}\phi e^{-F[\phi]}\phi(\mathbf{x}_1)\phi(\mathbf{x}_2)\cdots\phi(\mathbf{x}_p)\,. \tag{8}$$

Via Eq. (6) field correlations yield cumulant particle correlations:

$$\langle\phi(y_1)\phi(y_2)\rangle=k_2(1,2),\quad \langle\phi(y_1)\phi(y_2)\phi(y_3)\rangle=k_3(1,2,3)\,,$$
$$\langle\phi(y_1)\phi(y_2)\phi(y_3)\phi(y_4)\rangle=k_4(1,2,3,4)+\sum_{\text{perm}}^{(3)}k_2(1,2)k_2(3,4)\,, \tag{9}$$

etc.; see Eqs. (4) for $k_{2,3,4}$. These relations hold in any dimension.

A standard calculation gives the two-particle correlations [assuming periodic boundary conditions on ϕ, $p_{t,\max}/P=Y$ so that we have a simple cubic integration domain in Eq. (5), and for sufficiently energetic reactions $Y\gg0$ so that surface effects can be neglected],

$$\langle\phi(\mathbf{x}_1)\phi(\mathbf{x}_2)\rangle=(\gamma/2\pi\xi)e^{-|\mathbf{x}_1-\mathbf{x}_2|/\xi}/|\mathbf{x}_1-\mathbf{x}_2|,\quad \langle\phi(y_1)\phi(y_2)\rangle=\gamma e^{-|y_1-y_2|/\xi}\,, \tag{10}$$

where $\gamma=1/4aM$ and $\xi=a/M$ and the second equation applies for the one-dimensional case considered below. Note that the three-dimensional correlation function has a singular, Yukawa-type form. All higher-order odd-power field correlations vanish and even-power ones can be expressed in terms of two-field ones:

$$\langle\phi(y_1)\phi(y_2)\phi(y_3)\rangle=0,\quad \langle\phi(y_1)\phi(y_2)\phi(y_3)\phi(y_4)\rangle=\sum_{\text{perm}}^{(3)}\langle\phi(y_1)\phi(y_2)\rangle\langle\phi(y_3)\phi(y_4)\rangle\,,$$
$$\langle\phi(y_1)\phi(y_2)\phi(y_3)\phi(y_4)\phi(y_5)\rangle=0\,, \tag{11}$$

etc., which result again is valid in any dimension. Equations (9)–(11) imply exponential or Yukawa-type two-particle and vanishing higher-order cumulant correlations ($k_{n\ge3}=0$). In the following we restrict ourselves to a one-dimensional analysis. A detailed two- and three-dimensional study of our theory will be presented elsewhere.

We get the cumulants K_p by integrating the cumulant correlations k_p over the appropriate rapidity region δy,

$$K_2=2\gamma\xi^2(\delta y/\xi-1+e^{-\delta y/\xi})/\delta y^2\,, \tag{12}$$

and $K_{n\geq 3}=0$. Here the phenomenological parameters a and M [cf. Eq. (5)] are assumed to be constant with respect to rapidity [19].

Our result that there are no higher-order correlations is consistent with all experimental data (see Figs. 2–4 in Ref. [10]). In Fig. 1 we compare our predictions for K_2 (dashed lines) with the data. For NA22 we need $\gamma=0.331$ and $\xi=2.18$, for proton-emulsion $\gamma=0.29$ and $\xi=1.39$, and for KLM and EMU01 heavy-ion data $\gamma=0.13$ and $\xi=1.28$ (for O-em) and $\gamma=0.08$ and $\xi=2.14$ (for sulfur-gold). The value of the correlation length $\xi=a/M$ usually determines how far a system is from a critical point; when $\xi\to\infty$ the system goes through a second-order phase transition. Our fitted values for ξ do not indicate any critical behavior of the system, assuming it would survive the projection of 3D theory on one dimension. However, to study this and related questions, we will apply our theory to the two- and three-dimensional cases [20]. Approaching a critical point or, more generally, a phase transition will presumably change the behavior of two-particle correlations. Therefore, the appealing possibility of studying the phase transition from hadronic matter to quark-gluon plasma by measuring three-dimensional density fluctuations certainly deserves further investigation providing further constraints on our model.

Another interesting prediction here is the probability distribution of producing n particles in a high-energy heavy-ion collision. In principle, to obtain this distribution theoretically one needs to know the correlations to all orders. Since in heavy-ion collisions there are no correlations beyond two-particle ones, we easily find the generating function $G(z)$ [$\ln G(z)=\sum_p (z-1)^p K_p \bar{n}^p/p!$ or $G(z)=\sum_n z^n P_n$]. Thus [21],

$$G(z)=\exp[\bar{n}(z-1)+K_2\bar{n}^2(z-1)^2/2]\,. \tag{13}$$

From the generating function, we obtain the following probability distribution:

$$P_n=\frac{e^{(K_2\bar{n}^2/2)-\bar{n}}}{n!}\left(\frac{K_2\bar{n}^2}{2}\right)^{n/2} \times(-i)^n H_n\left\{-i\left[\frac{(K_2\bar{n}^2-\bar{n})^2}{2K_2\bar{n}^2}\right]^{1/2}\right\}. \tag{14}$$

It has two parameters, $\bar{n}$ and K_2, and is narrower than the corresponding hadronic distribution, which can be very well described by the negative binomial distribution. Therefore, one can test our statistical theory by a direct comparison of the distribution given by Eq. (14), not only with the first few moments of the distribution function [Eqs. (1) and (2)], but with the experimental measurement of the full distribution function for producing n (charged) particles in high-energy heavy-ion collisions.

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