Predictive Framework for Fermion Masses in Supersymmetric Theories

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The fermion masses and mixings are derived from a small number of input parameters. The resulting six predictions are consistent with data and have interesting consequences for future experiments. The top quark is heavy, near 188 GeV; its precise mass is sensitive to V_{cb}.

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The standard model is unlikely to be a fundamental theory; it contains 18 arbitrary parameters, 13 of which are the fermion masses and mixing angles. In a fundamental theory, these should be calculable from a few inputs just as the hydrogen spectral lines follow from quantum mechanics. We are very far from such a theory for fermion masses. We would be fortunate to have an analog of Balmer's formula since it might lead us to the fundamental theory. The framework described here is, at best, an attempt to obtain such a formula. In contrast to Balmer, who fitted many spectral lines in terms of one input parameter, we will need seven inputs to explain the 13 fermion masses and mixing angles. Thus, we make six predictions. Although this is an improvement on previous attempts that made only two predictions [1], a lot more needs to be accomplished. Each of the seven inputs can be traced to a fundamental problem that we fail to solve: (1) Three of the input parameters are needed to explain the hierarchy of the three down quark masses d, s, and b; (2) another three parametrize the hierarchy of the three up quarks u, c, and t; and (3) one parameter describes CP violation.

Had we succeeded in relating the up to the down quark masses, we would get away with just four inputs [2]. Our attempts to find such a relation are frustrated by the largeness of $m_t/m_b \sim 40$ at the weak scale. Such a large number [even larger at the grand unification theory (GUT) scale] cannot be obtained as a group-theoretic Clebsch-Gordan coefficient. Thus, we are stuck with seven input parameters and any further reduction necessitates the solution of one of the aforementioned fundamental problems. Conversely, even though we do not solve any new fundamental problems, we still manage to find a framework that makes six predictions. Also, at present there appears to be no fundamental reason for using more than seven parameters to fit the fermion masses.

In supersymmetric models there is an additional parameter, namely, the ratio of the vacuum expectation values (VEVs) of the two Higgs fields that necessarily occur in these theories; thus we have one extra input and output but the same number of predictions.

It is worth stressing that our objective is *not* to present a single theory of fermion masses. We would have to be incredibly lucky to run into *the* correct theory, especially since we still have seven parameters to explain. Instead we focus on a framework that can be the consequence of large classes of theories and is flexible enough to allow for future reductions in the number of input parameters. The framework is defined solely by gross features that are relevant for determining the light fermion mass spectrum.

Previous attempts.— (a) Fritzsch matrices: Fritzsch [1] suggested that light quarks get their masses by sequentially mixing with their nearest heavier neighbor. He predicts

$$V_{cb} = \sqrt{s/b} - \sqrt{c/t} \,,$$

implying a top mass no larger than 90 GeV, which is probably excluded.

(b) Grand unified predictions: In the Georgi-Glashow SU(5) model with a 5 multiplet of Higgs fields, the down quarks start out degenerate with their corresponding leptons. Even though this works well for the third generation [3], it predicts the troublesome relation

$$d/s = e/\mu \,,$$

which appears to be wrong by 1 order of magnitude. This led Georgi and Jarlskog to introduce the 45-dimensional Higgs multiplet [4]. This multiplet, when coupled to a given family, say, the second one, gives $\mu = -3s$ at the GUT scale. Georgi and Jarlskog make use of this multiplet and obtain the following Yukawa matrices for the down quarks and electrons:

$$D = \begin{pmatrix} 0 & F & 0 \\ F & E & 0 \\ 0 & 0 & D \end{pmatrix}, \quad E = \begin{pmatrix} 0 & F & 0 \\ F & -3E & 0 \\ 0 & 0 & D \end{pmatrix}. \qquad (1)$$

The elements D and F arise from the VEV of 5's of Higgs fields and the entries E and $-3E$ from a **45** of Higgs fields. The zeros are forced by discrete symmetries. The up matrix has the Fritzsch form

$$U=\begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix}. \tag{2}$$

We will refer to the matrices D, E, and U given by Eqs. (1) and (2) as having the Georgi-Jarlskog texture.

One could imagine choosing this texture at the weak scale for U and D alone (ignoring the leptons and any reference to GUTs) as suggested recently [5]. Since the down quark mass matrix is diagonal in the two heaviest generations, one has

$$V_{cb} \simeq \sqrt{c/t}\,. \tag{3}$$

This implies a very heavy top quark in the 220–800 GeV region [5], which, when compared to electroweak data, is seen to be unacceptably large.

Running textures.—The unacceptably large top quark mass is a consequence of Eq. (3), which in turn follows from the Georgi-Jarlskog matrices of Eqs. (1) and (2). In deriving the value of the top quark mass from Eq. (3), the low-energy values of m_c and V_{cb} were used [5]; thus implicitly assuming that Eqs. (1) and (2) were valid at the electroweak scale. In a grand unified theory, this assumption is not justified. Thus, the fermion masses have the Georgi-Jarlskog texture of Eqs. (1) and (2) only at M_{GUT} where the theory is defined. At energies below M_{GUT} the form of the mass matrices can change. In particular, zero entries can become nonzero and this can significantly change the connection between masses and mixing angles. For example, a nonvanishing 22 entry in the up quark mass matrix U will change Eq. (3) and can therefore fix the heavy top quark mass problem.

The statement that zero entries in the mass matrices can evolve becomes evident when we recall that such zeros originate at the GUT scale as a result of a (typically discrete) symmetry Z. If, as is often the case, Z is spontaneously broken at M_{GUT} then at low energies there is no symmetry to protect the zeros; as a result they become nonvanishing but *calculable* quantities at the weak scale.

Under what conditions is Z spontaneously broken at M_{GUT}? The implementation of Z requires the existence of several Higgs doublets (belonging to various 5's and a 45 in the Georgi-Jarlskog model); if most of these doublets become superheavy and are not available below M_{GUT} then Z cannot be implemented and is necessarily broken [6]. Such is the case in minimal low-energy models where only one Higgs field couples to quarks of a given charge.

Supersymmetric grand unified theories [7,8] are also examples of such a theory. There, the successful prediction of $\sin^2\theta_W$ necessitates the minimal Higgs particle content [7–9]. Thus, the zeros necessarily evolve in this theory and the disasterous Eq. (3) may be fixed. In the following we present a framework in which this is indeed the case.

Our framework.—Our objective is not to focus on a single grand unified theory but to propose a general framework which can result from a very large class of theories. Only the features of the framework relevant to predicting the fermion mass spectrum are of interest to us. These are as follows.

(1) Grand unification: We work in the context of GUTs, so that we can relate quark and lepton masses. This leads to an economy of parameters; we save ourselves from having to introduce three additional new parameters to describe the hierarchy of the three lepton masses e, μ, and τ.

(2) Low-energy supersymmetry: The successful prediction of $\sin^2\theta_W$ makes it preferable to work in supersymmetric (SUSY) GUTS [7–10]. In such a theory, we have two Higgs doublets; thus the fermion mass matrices include a new parameter $\tan\beta$, the ratio of Higgs VEVs.

(3) Georgi-Jarlskog texture: The mass matrices will have the Georgi-Jarlskog form [Eqs. (1) and (2)] at M_{GUT}.

(4) SO(10): The gauge group will be SO(10) or E(6) instead of SU(5). There are many reasons for this. In SO(10) the mass matrices can be automatically symmetric. This is important since otherwise we are forced to introduce an extra eighth parameter for no fundamental reason and reduce the predictive power of our model. Also in SO(10) we can relate neutrino to quark masses and make predictions about the light Majorana neutrino masses. In addition, the Georgi-Jarlskog factor of -3 relating quark and lepton masses can be easily achieved in several ways as a consequence of the Pati-Salam subgroup contained in SO(10).

(5) Complex parameters: To allow for CP violation we shall start with all the parameters A, B, C, D, E, and F that appear in the mass matrices being complex.

It is immediately seen that in our framework the top quark is necessarily heavy: Recall that we have to avoid the relation $V_{cb}=\sqrt{c/t}$ at low energies. This equation is valid at M_{GUT} since it is a direct consequence of the GUT scale mass matrices given by Eqs. (1) and (2). Thus, to avoid it we must ensure that V_{cb} runs between the grand and weak scales; this can only happen if the top quark Yukawa coupling is large $\lambda_t \sim 1$ [11].

Inputs versus outputs.—The parameter counting for the quarks is as follows. The U and D Yukawa matrices have nine nonvanishing entries. We have nine fields at our disposal—three doublets and six singlets—thus eight relative phases that can be used to get rid of all but one of the complex phases. For convenience we use this phase freedom to make A, B, C, D, and E real and keep F complex, and the mass matrices Hermitian. Thus we have seven real parameters A, B, C, D, E, the magnitude of F (call it F from now on), and its phase ϕ. A, B, and C describe the hierarchy of up quark masses; D, E, and F that of down quarks or electrons.

The lepton mass matrix E can easily be made real by using the phase freedom of the six fields—three doublets and three charged singlets. In this paper we will not consider neutrino masses. Thus the seven parameters A, B, C, D, E, F, and ϕ in the fermion Yukawa matrices, as well as $\tan\beta$, determine the 13 masses and mixing angles and $\tan\beta$, itself, leading to six predictions.

We will take as inputs and outputs the following quantities:

$$\text{Inputs: } m_e,\ m_\mu,\ m_\tau,\ m_b,\ |V_{cb}|,\ m_c,\ \frac{m_u}{m_d},\ |V_{cd}|\,,$$

$$\text{Outputs: } m_d,\ m_s,\ m_t,\ \sin\beta,\ \left|\frac{V_{ub}}{V_{cb}}\right|,\ \phi\,.$$

Predictions [12].—(α) Top quark mass and $\sin\beta$: The top quark mass is given by

$$m_t=\frac{m_c m_b}{m_\tau}\frac{1}{V_{cb}^2}f(g_3,g_2,g_1)\,,$$

where f is a known function of just gauge couplings. By substitution we obtain

$$m_t=(188\text{ GeV})\left(\frac{m_b}{4.25\text{ GeV}}\right)\left(\frac{m_c}{1.27\text{ GeV}}\right)\left(\frac{0.053}{V_{cb}}\right)^2. \tag{4}$$

Also, a general expression for m_t is

$$m_t=\frac{v}{\sqrt{2}}\sin\beta\lambda_t=(174\text{ GeV})\sin\beta\lambda_t\,, \tag{5}$$

where λ_t is the top quark Yukawa coupling. Equations (4) and (5) imply that both $\sin\beta$ and λ_t must be near 1. Since the fixed point of λ_t is near 1 ($\lambda_t^{\text{fp}}=1.09$) and is attractive, it follows that λ_t will be at its fixed point for all practical purposes. This was anticipated earlier. Furthermore, combining the above equations we find

$$\sin\beta=0.94\left(\frac{4.25}{m_b\text{ (GeV)}}\right)^5\left(\frac{m_c}{1.22\text{ GeV}}\right)\left(\frac{0.053}{V_{cb}}\right)^2 \times\left[\left(\frac{4.25}{m_b\text{ (GeV)}}\right)^{12}-0.08\right]^{-1/2}. \tag{6}$$

Thus $\tan\beta$ is large; this has important consequences for the SUSY phenomenology that we are presently studying. Since perturbativity in SUSY requires $m_t<190$ GeV, Eq. (4) implies an upper limit on m_b,m_c. Equation (6) implies a lower limit on V_{cb} of order 0.052. The experimental value is $V_{cb}\simeq0.044\pm0.009$.

(β) d, s, and ϕ: We find $d\simeq6.2$ MeV, $s\simeq156$ MeV. These should be compared with the Gasser-Leutwyler values of $d=8.9\pm2.6$ MeV and $s=175\pm58$ MeV [13]. Our s/d is a bit on the large side, but not uncomfortably so [14,15]. These predictions are, of course, just a SUSY version of the predictions of Georgi and Jarslkog. The CP violating phase ϕ is given by

$$\sin\phi=0.91^{+0.05}_{-0.13}\,.$$

(γ) Cabibbo-Kobayashi-Maskawa (CKM) matrix, V_{ub}/V_{cb}: The CKM matrix is given by

$$V=\begin{pmatrix} c_1-s_1s_2e^{-i\phi} & s_1+c_1s_2e^{-i\phi} & s_2s_3 \\ -c_1s_2-s_1e^{-i\phi} & c_1e^{-i\phi}-s_1s_2 & s_3 \\ s_1s_3 & -c_1s_3 & e^{i\phi}\end{pmatrix},$$

where

$$s_1=\sin\theta_1=0.196\,,$$

$$s_2=0.05\left(\frac{m_u/m_d}{0.6}\frac{1.25\text{ GeV}}{m_c}\right)^{1/2}=\left|\frac{V_{ub}}{V_{cb}}\right|.$$

This value of V_{ub}/V_{cb} is in the low end of the acceptable range; however, the uncertainties in extracting V_{ub}/V_{cb} from data are so large [16] that this does not yet pose a problem for our framework. However, a more accurate evaluation of this ratio will provide an important test of the model.

A complete analysis of our framework will soon appear in an upcoming paper [12]. In addition, our model makes several interesting predictions on K and B physics that are the subject of another paper [12]. We find that CP asymmetries in neutral B meson decays will also provide a crucial precision test of this framework.

In conclusion, we have presented the most predictive framework for fermion masses known to us. It involves seven plus one inputs, each of which corresponds to a yet-unsolved problem and makes six predictions. The central values for our predictions are $m_t\simeq188$ GeV, $s/d\simeq25$, $\sin\beta\simeq0.9$–1.0, $\sin\phi\simeq0.91$, $d\simeq6$ MeV, and $V_{ub}/V_{cb}\simeq0.05$. In addition, V_{cb} is constrained to be bigger than $\simeq0.052$.

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