Metal-Insulator Transition near a Superconducting State

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(Received 20 May 1991)

We show that when the metal-insulator transition occurs near a superconducting state it results in a different critical behavior from that of amorphous metals or uncompensated doped semiconductors. This difference results from the enhancement of the effective electron-electron interaction caused by fluctuations to the superconducting state. This explains the recent experiments of Micklitz and co-workers on amorphous superconducting mixtures Ga-Ar and Bi-Kr.

PACS numbers: 71.30.+h, 74.65.+n

Recently, Micklitz and co-workers [1] measured the critical behavior of conductivity near the metal-insulator transition for amorphous Ga-Ar and found quite surprisingly that it is similar to that of uncompensated doped semiconductors like SiP [2], Si:As [3], Si:Sb [4], and Ge:As [5]. Moreover, this critical behavior is in contrast to all previously measured amorphous metals and semiconductors As:Al [6], Ga:As [7], Ge:Au [8], Ge:Mo [9], Si:Nb [10], and Si:Au [11]. In particular, the critical behavior of the conductivity which is given by $\sigma=\sigma_0[(n-n_c)/n_c]^\mu$ indicates a universal exponent $\mu=1$ for the above amorphous materials whereas amorphous Ga-Ar shows $\mu=\frac{1}{2}$. Micklitz and co-workers [12] also measured the critical behavior of another amorphous mixture, Bi-Kr, and found that it shows a critical exponent $\mu=1$. They have pointed out [1] that the difference between the two materials is that the strength of the spin-orbit coupling is stronger for Bi-Kr. The interesting question which these experiments raise is why are these materials different from the other amorphous metals [6–11] for which the critical behavior is *independent* of the strength of the spin-orbit coupling, being weak for amorphous Si:Al and strong for Si:Au. Moreover, the suggested [13,14] association of the difference in the critical behavior of materials which show $\mu=\frac{1}{2}$ as compared to those which show $\mu=1$ with the strength of spin-orbit coupling is not supported by recent experiments. The uncompensated doped semiconductors show the same critical behavior $\mu\approx\frac{1}{2}$ *independent* of the strength of the spin-orbit coupling which is weak for Si:P but very strong [15] for Si:Sb. Dai, Zhang, and Sarachik [16] have recently tested this point explicitly. They measured the critical exponent of Si:B and showed that it belongs to the same class as uncompensated doped Si, with $\mu=\frac{1}{2}$, in spite of the fact of strong spin-orbit scattering.

In this Letter, we show that the key feature of the Micklitz amorphous mixtures is that they become superconducting, Ga-Ar at $T_c=9$ K and Bi-Kr at $T_c=6$ K. We propose that fluctuations to the superconducting state enhance the effective electron-electron interaction to such an extent that only in this case does the critical behavior depend on the spin-orbit coupling. For nonsuperconducting materials the effective interaction is small and the critical behavior does *not* depend on the spin-orbit coupling as is evidenced by experiment. Our classification in this case of the different classes of critical behavior relies only on the valley degeneracy which occurs only for uncompensated doped semiconductors but is absent in all the amorphous semiconductors.

The most recent study of the electron-electron interaction near the metal-insulator transition is due to Castellani, Kotliar, and Lee [13]. This work is only correct to lowest order in $\epsilon=d-2$. It should also be noted that the results of Castellani, Kotliar, and Lee [13] have been corrected for the case of pure potential scattering by Belitz and Kirkpatrick [14] who illustrated how high-order terms may change results. In their approach [13], the two different classes of critical behavior result from different symmetries of the Hamiltonian. The class of systems which contain potentials which break time-reversal symmetry like spin-orbit scattering should *not* [13] show a localization transition where the diffusion constant $D\to0$, but the metal-insulator transition is driven instead by electron-electron interactions with a critical exponent $\mu=1$. Materials with weak spin-orbit scattering should show [13] a different critical behavior in which the critical exponent $\mu=\frac{1}{2}$ is somewhat an accident. Thus, according to these authors the difference between the two classes is the strength of the spin-orbit scattering, although a definite value of $\mu=\frac{1}{2}$ was not obtained. In Table I, we present all the materials for which the critical behavior has been measured. It is known [15] that the strength of the spin-orbit coupling is proportional to $(\Delta Z)^4$, where ΔZ is the difference between the atomic numbers of the host and the doped atom. We see that for the uncompensated doped semiconductors where the critical exponent is $\mu\approx\frac{1}{2}$, ΔZ ranges from $\Delta Z=1$ for Si:P up to $\Delta Z=27$ for Si:Sb. Thus, the index μ in this class does not seem to depend on the strength of spin-orbit coupling. In *p*-type Si:B, Dai, Zhang, and Sarachik [16] have found an "anomalous" critical exponent $\mu=0.65^{+0.05}_{-0.14}$ in spite of the fact that this material shows clear evidence [16] of strong spin-orbit scattering.

We now turn to the second class in Table I in which a critical exponent $\mu=1$ is shown. Here again ΔZ ranges from $\Delta Z=1$ for Si:Al up to $\Delta Z=55$ for Si:Au. We

TABLE I. Material classification according to the critical conductivity exponent μ and the difference between atomic numbers, ΔZ.

Material	ΔZ	μ
Uncompensated doped semiconductors		
Si:P	1	$\frac{1}{2}$
Ge:As	1	$\frac{1}{2}$
Si:B	18	$\frac{1}{2}$
Si:As	6	$\frac{1}{2}$
Si:Sb	27	$\frac{1}{2}$
Amorphous metals		
Si:Al	1	1
Ga:As	2	1
In:Sb	0	1
Ge:Mo	10	1
Si:Nb	27	1
Si:Au	55	1
Superconducting amorphous mixtures		
Ga:Ar	13	$\frac{1}{2}$
Bi:Kr	47	1

therefore conclude that this class as well as the previous one is independent of the spin-orbit coupling. In view of this, one may well appreciate the surprising fact that the superconducting amorphous mixtures which were developed by Micklitz and co-workers [1,12] seem to depend on the spin-orbit scattering (see Table I).

We first demonstrate that all the nonuniversal critical behavior that has been observed may be classified in a simple manner, by making use of the nonuniversality of the Hartree long-range electron-electron interactions [17]. The electron-electron interaction consists of two parts [18], the exchange which is universal and the Hartree contribution which is material dependent.

The correction to the conductivity due to quantum interference and electron-electron interactions in the weak disorder limit is given at zero temperature by [18-20]

$$\sigma = g^2\sigma_B\left(1-\frac{C_L+C_I}{g^2(K_F l)^2}\right), \tag{1a}$$

where g is the Mott reduction parameter of the density of states near the transition.

For the noninteracting case (where $C_I=0$) we have given [18,19] arguments why we believe that Eq. (1a) is valid down to the transition. For example, at the Ioffe-Regel limit where $K_F l=\pi$, σ vanishes at a value $g\simeq\frac{1}{3}$ at the transition, in agreement with recent numerical calculation [21]. Equation (1a), which was also used by other authors, [22] when extrapolated [18,19] down to the transition yields a critical conductivity exponent $\mu=1$. When interactions are included there are two possible ways to extrapolate Eq. (1a). For $C_I>0$, the previous extrapolation remains valid and yields (as before) $\mu=1$. This is consistent with one-parameter scaling approaches when only the exchange interaction term is considered [8, 17-19]. When, however, $C_I<0$ which occurs when the Hartree terms are also included, the situation is complicated. Castellani, Kotliar, and Lee [13] argued that in this case one needs more than one variable in order to get scaling equations. In this treatment, the exponent μ depends on other thermodynamic critical exponents such as the specific heat and susceptibility.

We use instead [18,19] a self-consistent approach which depends only on the correlation length ξ in the *absence* of interactions. Namely, $\xi\rightarrow\infty$ with an exponent $\nu=1$. This can be understood near the transition as follows. The term $g^2\sigma_B[1-C_L/g^2(K_F l)^2]$ in Eq. (1a) is replaced by $ae^2/\hbar\xi$ (where a is a known constant [18,19]) yielding $\nu=1$. The second term $\sigma_B|C_I|/(K_F l)^2$ is replaced [18,19] by $\beta/D(\xi)\xi$ (β a known constant [18,19]). This leads to

$$\sigma = ae^2/\hbar\xi+\beta/D(\xi)\xi\,. \tag{1b}$$

Near the transition, where $\xi\rightarrow\infty$, using the Einstein relation $\sigma=e^2D(\xi)N(E_F)$ yields $\sigma\propto 1/\xi^{1/2}$. Since the critical exponent of ξ is $\nu=1$ it leads to a conductivity exponent $\mu=\frac{1}{2}$.

Thus, the sign of C_I may determine whether the critical conductivity exponent is $\mu=1$ (for $C_I>0$) or $\mu=\frac{1}{2}$ (for $C_I<0$). We show here that this classification is indeed independent of the spin-orbit coupling as observed experimentally. On the other hand, when a disordered material becomes superconducting the effective electron-electron interaction is increased due to fluctuations from the normal to the superconducting state; the interaction constant C_I then depends crucially on the strength of the spin-orbit coupling.

This explains why superconducting amorphous mixtures may belong simultaneously to two different classes showing a critical exponent $\mu=\frac{1}{2}$ for weak spin-orbit coupling and $\mu=1$ for strong coupling.

The interaction constant C_I may be written [17] as $C_I=C_{\rm exc}+C_{\rm Har}$, where $C_{\rm exc}$ is the contribution from the exchange interaction and leads to a material-independent constant $C_{\rm exc}=1$. The Hartree term is opposite in sign and depends on the density of states. This implies that $C_{\rm Har}$ must depend on the valley degeneracy N_v for uncompensated doped semiconductors ($N_v=6$ for Si and $N_v=4$ for Ge) and on $N_s=3$ possible spin-independent contributions. For strong spin-orbit coupling the spin degeneracy is lifted and $N_s=1$. For strong intervalley scattering the degeneracy is lifted and $N_v=1$. The Hartree contribution to C_I is given by [17,23,24]

$$C_{\rm Har} = -\tfrac{3}{8}N_vN_s\tilde{F}(F)\,, \tag{2}$$

where the strength of the interaction is given by

$$F=\langle v(q)\rangle/v(0)\,. \tag{3}$$

For long-range electron-electron interactions, $F \ll 1$, which leads to $\tilde{F} \ll 1$. Thus, $C_{\rm Har}$ may dominate over $C_{\rm exc} = I$ *only* when we have valley degeneracy. In this case, $|C_{\rm Har}| > C_{\rm exc}$ *even* for $N_s = 1$. Thus, we predict that the inequality $C_I < 0$ is *independent* of spin-orbit coupling for *all* the uncompensated doped semiconductors. Compensation introduces strong intervalley scattering and causes $N_v \to 1$ which reverses the sign of C_I to a positive value, resulting in a critical exponent $\mu = 1$. When $N_v = 1$, $|C_{\rm Har}| < C_{\rm exc}$ even for $N_s = 3$. Therefore, introducing strong spin-orbital coupling which reduced N_s to unity does not affect the sign of C_I, which remains positive.

This implies that materials without valley degeneracy correspond to $C_I > 0$ irrespective of the strength of spin-orbit coupling. Indeed, all the nonsuperconducting amorphous materials belong to this class and show the same critical exponent $\mu = 1$. This results from the fact that for all the amorphous metals and semiconductors $v(q)/v(0)$ decreases rapidly with q, resulting in a small interaction $F \ll 1$ which leads to $|C_{\rm Har}| \ll 1$ and to $C_I > 0$.

We now discuss what happens when the amorphous mixtures become superconducting. There is much evidence [25] that the effective electron-electron interaction is largely increased. Measurements of the Landau-Baber T^2 contribution to the electrical resistivity of superconducting metals above T_c show an enhancement [26] of a few orders of magnitude. This was interpreted as due to phonon mediation which exists even above T_c and increases the effective electron-electron interaction. This increase in $v(q)$ in Eq. (3) arises for $q \approx q_D$, where q_D is the Debye wave vector. Thus, the effective $v(q)$ above T_c is not peaked around $q = 0$ but rather around $q \approx q_D$. This leads to a new situation in which F may even be larger than unity (which is *impossible* for a nonsuperconducting material). In this case, the sign of C_I will depend crucially on the strength of the spin-orbit coupling. For weak spin-orbit coupling, $N_s = 3$ and $C_I < 0$ when $\tilde{F} > \frac{8}{9}$, which implies $F > 0.95$. Such a large F is impossible for nonsuperconducting materials where $F \ll 1$. However, for superconducting mixtures, when the conductivity is measured above T_c, F may obey this inequality. For superconducting materials with strong spin-orbit coupling $N_s = 1$ and $C_I < 0$ is reached only for $\tilde{F} > \frac{24}{9}$ which requires $F \gg 1$ and is difficult to obtain. Thus, superconducting materials may show both classes of critical behavior; a critical exponent $\mu = \frac{1}{2}$ for materials with weak spin-orbit coupling and $\mu = 1$ for strong spin-orbit coupling.

We now propose an experimental test of the above classification of the critical behavior of superconducting amorphous mixtures. For a mixture with weak spin-orbit coupling, as is the case for Ga-Ar, Zint, Rohde, and Micklitz [1] found a critical exponent $\mu = \frac{1}{2}$. We propose to apply a magnetic field which will remove the superconducting state. In this case, F is again reduced (as in nonsuperconducting materials) and we expect the critical exponent to change from $\mu = \frac{1}{2}$ to $\mu = 1$.

In summary, we have shown that the metal-insulator transition of amorphous superconducting mixtures may show two types of critical behavior according to the strength of the spin-orbit coupling. A strong magnetic field will transform the critical behavior of superconducting mixtures with weak spin-orbit coupling ($\mu = \frac{1}{2}$) to behave in the same way as for mixtures with strong spin-orbit coupling ($\mu = 1$).

We acknowledge important discussions with M. Pepper on this subject. M. Kaveh acknowledges a SERC Fellowship at the Cavendish Laboratory.

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