Chiral Symmetry Tests in Nonleptonic K Decay

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The existence of empirical tests for the validity of chiral perturbation theory in the analysis of nonleptonic kaon decay is pointed out in the context of $K \to 2\pi, 3\pi$ processes. Comparison with existing data reveals good agreement with the chiral constraints.

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Modern particle physics is based on the validity of QCD as the basis of quark-quark interactions. The study of low-energy processes is thereby severely hindered by the nonlinear and nonperturbative nature of gluonic interactions, limiting the possibilities for rigorous calculations. This situation has been somewhat obviated, however, with the development of methods such as chiral perturbation theory (ChPT) which rely only on basic symmetries of the strong interaction and which therefore are not restricted by the calculational limitations [1]. There is a corresponding price in that ChPT represents an expansion in energy-momentum and is therefore only useful for processes with $E < \Lambda_\chi$, where $\Lambda_\chi \cong 1$ GeV is the chiral scale parameter [2]. However, for such processes ChPT provides a reliable and systematic calculational scheme. The formalism associated with mesonic processes has been carefully developed in a series of papers by Gasser and Leutwyler and successfully applied to a wide range of low-energy processes by these and other authors [3]. Results are generally in excellent agreement and the validity of ChPT can be tested in specific relationships between empirical parameters [4].

A corresponding formalism has been developed for the sector of nonleptonic weak interactions and also applied to a range of different decays [5,6]. In this case, however, the data base is considerably smaller and empirical tests are scarce. The purpose of this Letter is to show that specific tests for the validity of chiral methods exist within the $K \to 2\pi, 3\pi$ sector, and to confront such tests with experimental data. In the following we present a brief review of the chiral formalism and $K \to 2\pi, 3\pi$ phenomenology while constructing these chiral consistency checks. The significance of our results is summarized in a brief concluding section.

First consider the experimental situation. For $K \to 2\pi$ we can perform an isospin decomposition [7,8]

$$A(K^n \to \pi^a \pi^b) = -\sqrt{1/3}\, A_0 \delta^{ab} \bar{s}_{1/2} K^n - (1/\sqrt{6}) A_2 (\bar{s}^a_{3/2} \tau^b K^n + \bar{s}^b_{3/2} \tau^a K^n)\,, \tag{1}$$

where the A_I represent weak decay amplitudes for decay into a $\pi\pi$ final state of total isospin I with phases contained by the Fermi-Watson theorem

$$A_I \equiv i |A_I| \exp i\delta_I\,. \tag{2}$$

A fit to experimental data yields the values given in Table I, with the relative phase between $I=0$ and $2\pi\pi$ final states determined to be $\delta_2 - \delta_0 = -57° \pm 4°$. Here $s_{1/2}, \mathbf{s}_{3/2}$ are "iso-spurions" with $\boldsymbol{\tau} \cdot \mathbf{s}_{3/2} = 0$, which account for the quantum numbers of the nonleptonic weak interaction [8],

$$\bar{s}_{1/2} = (0\ -1)\,, \quad \bar{\mathbf{s}}_{3/2} = \hat{\mathbf{e}}_x(\tfrac{1}{2}\ 0) + \hat{\mathbf{e}}_y(\tfrac{1}{2} i\ 0) + \hat{\mathbf{e}}_z(0\ 1)\,. \tag{3}$$

Similarly for $K \to 3\pi$ we can make an isospin decomposition [7]

$$\begin{aligned} A(K^n \to \pi^a \pi^b \pi^c) = & -\delta^{ab} \bar{s}_{1/2} \tau^c K^n F_{1/2}(s_a, s_b, s_c) + \delta^{ab} \bar{s}^c_{3/2} K^n F_{3/2}(s_a, s_b, s_c) \\ & + i\varepsilon^{abd} (\bar{s}^c_{3/2} \tau^d K^n + \bar{s}^d_{3/2} \tau^c K^n) G_{3/2}(s_a, s_b, s_c) + \text{permutations}\,, \end{aligned} \tag{4}$$

where Bose symmetry requires

$$f(s_a, s_b, s_c) = \lambda_K f(s_b, s_a, s_c) \ \text{with}\ \lambda_K = \begin{cases} +1, & f = F_I\,, \\ -1, & f = G_{3/2}\,, \end{cases} \tag{5}$$

with $s_i = (k - q_i)^2$. We parametrize these functions by keeping terms up to quadratic order in s_i:

$$\begin{aligned} \sqrt{2} F_I(s_a, s_b, s_c) &\equiv \alpha_I - \beta_I Y_c + \sigma_I \zeta_I (Y_c^2 + \tfrac{1}{3} X_c^2) + \sigma_I \xi_I (Y_c^2 - \tfrac{1}{3} X_c^2)\,, \\ \sqrt{2} G_{3/2}(s_a, s_b, s_c) &\equiv (1/\sqrt{3}) \gamma_3 X_c - \tfrac{2}{3} \xi'_3 X_c Y_c\,, \end{aligned} \tag{6}$$

TABLE I. Measured and fitted values for $K \to 3\pi$ expansion parameters, all given in units of 10^{-8} (Ref. [9]).

	Expt.	Lowest order fit [Eq. (9)]	Full fit [Eqs. (9) and (12)]
A_0 (keV)	0.4699 ± 0.0012	0.4698 (fit)	0.4698 (fit)
A_2 (keV)	-0.0211 ± 0.0001	-0.0211 (fit)	-0.0211 (fit)
α_1	91.71 ± 0.32	74.0	91.8
α_3	-7.36 ± 0.47	-4.1	-7.6
β_1	-25.68 ± 0.27	-16.5	-25.6
β_3	-2.43 ± 0.41	-1.0	-2.5
γ_3	2.26 ± 0.23	1.8	2.5
ζ_1	-0.47 ± 0.15		-0.6
ζ_3	-0.21 ± 0.08		-0.02
ξ_1	-1.51 ± 0.30		-1.5
ξ_3	-0.12 ± 0.17		-0.05
ξ_3'	-0.21 ± 0.51		-0.08

where $I = \frac{1}{2}, \frac{3}{2}$, with

$$\sigma_I = \begin{cases} 1, & I = \frac{1}{2}, \\ -2, & I = \frac{3}{2}, \end{cases} \quad Y_c = \frac{s_c - s_0}{m_\pi^2}, \quad X_c = \frac{s_b - s_a}{m_\pi^2}, \quad s_0 = \frac{1}{3} \sum_j s_j . \tag{7}$$

If the phenomenological parameters α_i, β_i, etc., are taken to be approximately real (since $\pi\pi\pi$ phase shifts are presumably small) a phenomenological fit to available $K \to 3\pi$ data reveals the values given in Table I.

To second order in momentum the chiral Lagrangian which describes nonleptonic weak decays must be of the form [5,6]

$$\mathcal{L}^{(2)}_{\Delta S=1} = c_2 \mathrm{Tr}\lambda_6 L_\mu L^\mu + c_3 t^{jl}_{ik} \mathrm{Tr} Q^i_j L_\mu \mathrm{Tr} Q^k_l L^\mu \tag{8}$$

with $L_\mu = iU^\dagger \partial_\mu U$ representing the left-handed weak current. Here

$$U = \exp\left(\frac{i}{F_0} \sum_{j=1}^{8} \lambda_j \phi_j \right)$$

is the chiral matrix, with λ_j representing the SU(3) matrices and F_0 being the pion decay constant to lowest order in chiral symmetry. Also, $(Q^i_j)_{kl} = \delta_{il}\delta_{jk}$ are 3×3 flavor matrices, while $\lambda_6 = Q^2_3 + Q^3_2$ and t^{jl}_{ik} project out the octet and 27-plet components of the weak interaction, respectively. By expanding in powers of the pion field, we can determine the phenomenological parameters A_I, α_i, β_i, γ_i in terms of c_2, c_3 [9]: $\Delta I = \frac{1}{2}$,

$$m_K^2 A_0 = i\sqrt{6} F_\pi \rho (1-\eta)(c_2 - \tfrac{2}{3} c_3) , \tag{9a}$$

$$m_K^2 \alpha_1 = \tfrac{1}{3} \rho (c_2 - \tfrac{2}{3} c_3) , \tag{9b}$$

$$m_K^2 \beta_1 = -\eta\rho (c_2 - \tfrac{2}{3} c_3) ; \tag{9c}$$

$\Delta I = \frac{3}{2}$,

$$m_K^2 A_2 = i (20/\sqrt{3}) F_\pi \rho (1-\eta) c_3 , \tag{9d}$$

$$m_K^2 \alpha_3 = \tfrac{20}{9} \rho c_3 , \tag{9e}$$

$$m_K^2 \beta_3 = \tfrac{5}{3} \eta\rho \frac{5 - 14\eta}{1-\eta} c_3 , \tag{9f}$$

$$m_K^2 \gamma_3 = -\frac{15}{2\sqrt{3}} \eta\rho \frac{3 - 2\eta}{1-\eta} c_3 , \tag{9g}$$

where we have defined $\rho \equiv m_K^4/F_\pi^3 F_K$ and $\eta \equiv m_\pi^2/m_K^2$. At the two-derivative level then we have five requirements of chiral symmetry—i.e., with c_2, c_3 determined from $|A_0|, |A_2|$ we predict the values shown in column two of Table I. Since loop effects and/or terms arising at the four-derivative level produce corrections of $O(m_K^2/\Lambda_\chi^2) \sim 25\%$ we would expect general agreement between the theoretically predicted and experimentally determined values of the five $K \to 3\pi$ parameters but only up to this level of precision, and this is indeed confirmed with the data. Actually these results are not at all new and follow from current-algebra-PCAC (partial conservation of axial-vector current) requirements first written down nearly a quarter-century ago [8]:

$$\lim_{q^c \to 0} A(K^n \to \pi^a \pi^b \pi^c) = -(i/F_\pi) \langle \pi^a \pi^b | [I^c, \mathcal{H}_w] | K^n \rangle + O(m_\pi^2) , \tag{10}$$

where I^c represents the isospin operator. Thus, e.g., Eq. (10) requires, for $K^0 \to \pi^+\pi^-\pi^0$,

$$\lim_{q^0 \to 0} (\alpha_1 - \beta_1 Y_0) = -(i/2F_\pi)\sqrt{2/3} A_0 , \qquad \lim_{q^\pm \to 0} (\alpha_1 - \beta_1 Y_0) = 0 , \tag{11}$$

whose solution is given by Eqs. (9b) and (9c). Similarly one can reproduce Eqs. (9e)–(9g) by working in the

$\Delta I = \frac{3}{2}$ sector.

What *is* new is the ability to identify chiral symmetry structures at the loop and four-derivative level. The most general form for the four-derivative $\Delta S=1$ weak Lagrangian has been given by Kambor, Missimer, and Wyler [6]. The form is lengthy, involving eighty-two separate terms, and will not be required here. What is needed is simply the general result for the contribution from weak counterterms of four-derivative form and $O(m_K^2)$ which has been recently given [9,10]: $\Delta I = \frac{1}{2}$,

$$A_0 = -i\tfrac{2}{3}\sqrt{2/3}F_\pi\rho(G_1+G_2),\quad \alpha_1 = -\tfrac{2}{27}\rho(G_1+G_2+G_3),\quad \beta_1 = -\tfrac{1}{9}\eta\rho(-2G_1-2G_2+G_4),$$
$$\zeta_1 = \tfrac{1}{6}\eta^2\rho G_3,\quad \xi_1 = -\tfrac{1}{6}\eta^2\rho G_4; \tag{12a}$$

$\Delta I = \frac{3}{2}$,

$$A_2 = i(20/3\sqrt{3})F_\pi\rho G_2,\quad \alpha_3 = \tfrac{20}{27}\rho(G_2+2G_5),\quad \beta_3 = \tfrac{5}{18}\eta\rho(10G_2+G_6),\quad \gamma_3 = -(5/4\sqrt{3})\eta\rho(6G_2+G_7),$$
$$\zeta_3 = \tfrac{5}{3}\eta^2\rho G_5,\quad \xi_3 = -\tfrac{5}{24}\eta^2\rho G_6,\quad \xi_3' = \tfrac{15}{8}\eta^2\rho G_7. \tag{12b}$$

Here G_i, $i=1,\ldots,7$, represent various combinations of four-derivative counterterms, whose explicit form can be found in Ref. [9]. The important point is that these order-four counterterms can be determined empirically —a general coefficient in the $K\to 2\pi,3\pi$ isospin decomposition can be written in the form

$$A^i = A^{i(2)}_{\rm tree} + A^{i(2)}_{\rm loop}(\mu) + {}^{\rm wk}A^{i(4)}_{\rm tree}(\mu) + {}^{\rm st}A^{i(4)}_{\rm tree}(\mu), \tag{13}$$

where ${}^{\rm wk}A^{(4)}_{\rm tree}$ (${}^{\rm st}A^{(4)}_{\rm tree}$) is the tree-level contribution from the weak (strong) Lagrangian of order 4 and $A^{(2)}_{\rm loop}(\mu)$ is the one-loop correction arising from $\mathcal{L}^{(2)}_{\Delta S=1}$. The parameter μ represents the scale parameter introduced when renormalizing the dimension-four counterterms and the sum of the last three contributions is μ independent. The loop corrections have been calculated, yielding the values given in Ref. [9]. Using these values, one can then fit to the empirically determined parameters in order to determine the counterterms $G_1,\ldots,G_7$. The result is the fit shown in Table I, column 3, which is certainly impressive and gives confidence in the validity of the chiral approach. One might worry that the quality of the result is simply a consequence of the many free parameters $G_1,\ldots,G_7$. However, we will show that this is not the case and that parameter-free constraints and predictions do exist.

One can go beyond this global fit to generate a set of specific tests which are independent of the chiral counterterms. Specifically, the G_1, G_2 dependence can be absorbed by identifying new parameters

$$c_2' \equiv c_2 - \tfrac{2}{9}m_K^2 G_1,\quad c_3' \equiv c_3 + \tfrac{1}{3}m_K^2 G_2. \tag{14}$$

We can then isolate the dimension-four weak counterterms by defining

$$\tilde{A}^i = A^i_{\rm expt} - A^{i(2)}_{\rm tree} - A^{i(2)}_{\rm loop}(\mu) - {}^{\rm st}A^{i(4)}_{\rm tree}(\mu). \tag{15}$$

Note here that $\tilde{A}^i$ are completely determined since $A^i_{\rm expt}$ are given in terms of the experimental fit, $A^{i(2)}_{\rm tree}$ and $A^{i(2)}_{\rm loop}(\mu)$ are known in terms of the $K\to 2\pi$ parameters c_2' and c_3', respectively, while ${}^{\rm st}A^{i(4)}_{\rm tree}$ are determined in terms of the known strong dimension-four parameters $L_1(\mu)$, $\ldots, L_{10}(\mu)$ given by Gasser and Leutwyler [3]. The chiral relations given in Eqs. (12) are seen then to *require* the following five relations [to $O(m_\pi^2/m_K^2)$]: $\Delta I = \frac{1}{2}$,

$$\tilde{\zeta}_1 = -\tfrac{9}{4}\eta\tilde{\alpha}_1, \tag{16a}$$
$$\tilde{\xi}_1 = \tfrac{3}{2}\eta\tilde{\beta}_1; \tag{16b}$$

$\Delta I = \frac{3}{2}$,

$$\tilde{\zeta}_3 = \tfrac{9}{8}\eta\tilde{\alpha}_3, \tag{16c}$$
$$\tilde{\xi}_3 = -\tfrac{3}{4}\eta\tilde{\beta}_3, \tag{16d}$$
$$\tilde{\xi}_3' = -(3\sqrt{3}/2)\eta\tilde{\gamma}_3. \tag{16e}$$

These conditions result simply from the validity of the ChPT approach at the four-derivative level and the extent to which they are satisfied provides a very nontrivial test of chiral methods. The results of such a test are shown in Table II and are seen to be very successful for the two $\Delta I = \frac{1}{2}$ relations Eqs. (16a) and (16b). The data are not good enough to say anything conclusive about the corresponding $\Delta I = \frac{3}{2}$ relations Eqs. (16c)–(16e) although there could be a possible problem with Eq. (16c). The significance of these results will be assessed in the concluding section. However, before doing so it should be noted that the physics behind these relations is easily discerned from the feature that $\tilde{A}^i$ arises strictly from $A^{i(4)}_{\rm tree}$, i.e., four-derivative terms. Since contributions to Eq. (6)

TABLE II. Predicted and measured values of quadratic $K\to 3\pi$ parameters, all given in units of 10^{-8}. The uncertainties quoted in column two include experimental error bars as well as uncertainties in the strong chiral coefficient and an estimate of neglected $O(m_\pi^2/m_K^2)$ terms.

	Measured experimentally	Predicted from Eq. (16)
ζ_1	-0.47 ± 0.15	-0.47 ± 0.18
ξ_1	-1.51 ± 0.30	-1.58 ± 0.19
ζ_3	-0.21 ± 0.08	-0.011 ± 0.006
ξ_3	-0.12 ± 0.17	0.092 ± 0.030
ξ_3'	-0.21 ± 0.51	-0.033 ± 0.077

must have the form

$$F_I(s_a,s_b,s_c)=k_1^I\,k\cdot q_c\,q_a\cdot q_b+k_2^I(k\cdot q_a\,q_b\cdot q_c+k\cdot q_b\,q_a\cdot q_c)\cong m_K^4\left(\frac{k_1^I}{18}+\frac{k_2^I}{9}\right)[1-\tfrac{9}{4}\eta^2(Y_c^2+\tfrac{1}{3}X_c^2)]$$
$$+\tfrac{1}{12}m_K^4(k_1^I-k_2^I)[\eta Y_c-\tfrac{2}{3}\eta^2(Y_c^2-\tfrac{1}{3}X_c^2)]\,, \tag{17}$$
$$G_{3/2}(s_a,s_b,s_c)=k_3(k\cdot q_a\,q_b\cdot q_c-k\cdot q_b\,q_a\cdot q_c)\cong\tfrac{1}{12}m_K^4k_3(\eta X_c+3\eta^2X_cY_c)\,,$$

we see clearly then that there exists a *required* relation between quadratic and constant, linear contributions arising from $\mathcal{L}^{(4)}_{\Delta S=1}$ [11]. There exist five such relations and these are just those recorded in Eq. (16). Note also that since such four-derivative terms vanish when any soft-pion limit is taken, there exists no constraint on such terms arising from the $K\to 2\pi$ sector.

In conclusion, we have pointed out the existence of *ten* independent tests for the validity of the ChPT approach to nonleptonic kaon decay—five at the two-derivative level and five more at the level of four derivatives. The former are well satisfied up to the $O(m_K^2/\Lambda_\chi^2)$ corrections expected from order-four contributions. The latter are also well satisfied at the level of $\sim 20\%$ for the two cases involving $\Delta I=\frac{1}{2}$ amplitudes. Results are not as good for the three $\Delta I=\frac{3}{2}$ tests. However, this should not be a surprise, as such terms are found from subtracting two much larger numbers (the quadratic amplitudes found in the K^+ and K_L decay amplitudes, respectively) and are particularly sensitive to small errors in either analysis. In addition, electromagnetic effects have been omitted. In the $K\to 2\pi$ case such effects should not be a problem [12]. However, a previous estimate of electromagnetic contributions in the $K\to 3\pi$ system has revealed possibly significant corrections within the $\Delta I=\frac{3}{2}$ sector [13]. Thus any discrepancy within the quadratic $\Delta I=\frac{3}{2}$ terms should not be considered problematic at this time.

We emphasize in closing that a particularly important piece of evidence in this regard should be soon forthcoming in the form of a careful analysis of $2\times 10^6 K_L\to 3\pi^0$ events found in Fermilab E731 [14]. In particular the decay amplitude must have the form

$$A(K_L\to 3\pi^0)=-3(\alpha_1+\alpha_3)-3(\zeta_1-2\zeta_3)(Y^2+\tfrac{1}{3}X^2)\,, \tag{18}$$

and the size of the quadratic term is predicted within the chiral symmetry scheme,

$$\frac{\zeta_1-2\zeta_3}{\alpha_1+\alpha_3}\cong -0.006\pm 0.002\,. \tag{19}$$

Finally, it goes without saying that it would be of great interest to acquire the same sort of high-quality, high-statistics data in the remaining $K^+\to 3\pi$ and $K_L\to\pi^+\pi^-\pi^0$ systems in order to enable a precision test of the ChPT within the $\Delta I=\frac{3}{2}$ sector.

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[1] S. Weinberg, Physica (Amsterdam) **96A**, 327 (1979).

[2] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984); E. Golowich, J. F. Donoghue, and B. R. Holstein, Phys. Rev. D **30**, 587 (1984).

[3] J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) **158**, 142 (1984); Nucl. Phys. **B250**, 465 (1985).

[4] J. F. Donoghue and B. R. Holstein, Phys. Rev. D **40**, 2378 (1989); **40**, 3700 (1989).

[5] G. Ecker, A. Pich, and E. de Rafael, Nucl. Phys. **B291**, 692 (1987).

[6] J. Kambor, J. Missimer, and D. Wyler, Nucl. Phys. **B346**, 17 (1990).

[7] Expressed in more familiar notation we have

$$\sqrt{2}A(K^0\to\pi^0\pi^0)=\sqrt{2/3}A_0+(2/\sqrt{3})A_2\,,$$
$$A(K^+\to\pi^+\pi^0)=(\sqrt{3}/2)A_2\,,$$
$$\sqrt{2}A(K^0\to\pi^+\pi^-\pi^0)=\alpha_1+\alpha_3-(\beta_1+\beta_3)Y_0+(\zeta_1-2\zeta_3)(Y_0^2+\tfrac{1}{3}X_0^2)+(\xi_1-2\xi_3)(Y_0^2-\tfrac{1}{3}X_0^2)\,,$$
$$A(K^+\to\pi^+\pi^0\pi^0)=-\tfrac{1}{2}(2\alpha_1-\alpha_3)+(\beta_1-\tfrac{1}{2}\beta_3-\sqrt{3}\gamma_3)Y_+-(\zeta_1+\zeta_3)(Y_+^2+\tfrac{1}{3}X_+^2)-(\xi_1+\xi_3+\xi_3')(Y_+^2-\tfrac{1}{3}X_+^2)\,.$$

[8] Y. Nambu and Y. Hara, Phys. Rev. Lett. **16**, 875 (1966); C. Bouchiat and Ph. Meyer, Phys. Lett. **25B**, 282 (1967); B. R. Holstein, Phys. Rev. **183**, 1229 (1969).

[9] J. Kambor, J. Missimer and D. Wyler, Phys. Lett. B **261**, 496 (1991).

[10] Several typographical errors in Ref. [9] have been corrected in Eq. (12).

[11] The existence of such relations was first pointed out by E. Golowich *et al.* (Ref. [2]).

[12] F. Abbud, B. W. Lee, and C. N. Yang, Phys. Rev. Lett. **18**, 980 (1967).

[13] A. Neveu and J. Scherk, Phys. Lett. **27B**, 384 (1968).

[14] B. Winstein (private communication).