Electroweak Bubbles: Nucleation and Growth

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In the standard electroweak theory, if the Higgs boson mass is comparable with the weak scale, the electroweak transition is weakly first order, and proceeds via bubble nucleation. It is shown that the Universe supercools beyond the point where phase equilibrium is possible. When true-vacuum bubbles nucleate, they expand at velocities of the order of the speed of light until they fill the Universe. These considerations are important for the recently proposed electroweak baryogenesis mechanisms.

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One of the most striking consequences of the Higgs mechanism in the standard electroweak theory is the symmetry-breaking phase transition which would have occurred in the very early Universe. Recently, there has been considerable interest in the idea that the matter-antimatter asymmetry apparent in today's Universe could have been generated at this electroweak transition. The proposed mechanisms make use of the remarkable anomalous baryon-number-violating processes in the standard model, which occur rapidly at high temperatures [1].

In simple extensions of the standard model, involving more than one fundamental Higgs doublet, these baryon-number-violating processes may be "biased" at the electroweak phase transition, during the very brief period when the Higgs fields move from the "false" unbroken-symmetry vacuum to the "true" symmetry-breaking vacuum, naturally producing a baryon asymmetry [2,3] (see [4,5] for earlier speculations on these lines, and [6,7] for alternate scenarios). The biasing occurs through a chain of connected events. *CP*-violating terms in the Higgs potential drive an axionlike *C*-odd scalar field as the Higgs fields roll from the false to the true vacuum. The *C*-odd field in turn couples to the gauge fields in the effective action for the theory [2,3], producing a "force" driving the Chern-Simons number positive. Finally, this produces a baryon excess through the axial anomaly. This scheme was elaborated in [8], where it was argued that it naturally produces an asymmetry of the observed order of magnitude. However, the estimate presented there was certainly crude, and there is still some debate over it [9].

The aim of the present Letter is to improve on previous treatments of bubble nucleation, such as that presented in [8], and make a first step towards understanding the process of bubble growth. A more complete discussion will be presented elsewhere [10]. My main conclusion is that the transition proceeds via bubble nucleation with the bubbles expanding to fill space at close to the speed of light. This result conflicts with the widespread belief that the bubble walls move very slowly in weakly first-order phase transitions [7,11]. The main point is not to overcount the effect of particle collisions with the bubble wall, which are already taken into account in the finite-temperature "effective potential." As I show below, if the fluid in which the wall is immersed is in local thermal equilibrium, and the temperature is constant across the wall, the force on the wall does not increase with its velocity. It is only departures from equilibrium which slow the wall.

Electroweak baryogenesis is only possible if the Higgs boson mass is low, at least in the calculable, weak-coupling regime [4,12,13]. The heavier the Higgs boson, the weaker the phase transition, and the less suppressed baryon violation is *after* it, so that the baryon asymmetry produced during the transition may be subsequently erased. In the two-Higgs-doublet theory, this confines one to the mass range $m_H < 120$ GeV for the lightest observable Higgs boson [13], not far above the current experimental lower bound of 50 GeV.

I will focus on the one-Higgs-doublet theory for simplicity, and where numerical values are involved will assume (unless stated otherwise) that $m_H=50$ GeV and the top mass $m_t=100$ GeV, both for definiteness. In the two-Higgs-doublet theory we are really interested in for baryogenesis, the phase transition is qualitatively and quantitatively very similar to the $m_H=50$ GeV minimal model I discuss here [13].

For Higgs boson masses of this order, the electroweak transition is weakly first order [14]. A reasonable approximation to the free energy near the transition is (see, e.g., [13])

$$F(\phi,T)=-\mathcal{N}_{\rm eff}\frac{\pi^2T^4}{90}+\frac{\gamma}{2}(T^2-T_c^2)\phi^2-\delta T\phi^3+\frac{\lambda}{4}\phi^4\,, \tag{1}$$

where $\mathcal{N}_{\rm eff}=106.75$ is the number of relativistic $[m(T)/T\ll 1]$ degrees of freedom at this temperature, $\phi=\sqrt{2}(\Phi^\dagger\Phi)^{1/2}$ with Φ the standard Higgs doublet, $\gamma=\frac{1}{4}(2m_W^2+m_Z^2+2m_t^2)/\phi_0^2$, $\delta=(2m_W^3+m_Z^3)/4\pi\phi_0^2$, with $\phi_0=250$ GeV the Higgs vacuum expectation value at zero temperature ($\phi_0=\sqrt{\mu/\lambda}$ in the usual notation), and $T_c=\sqrt{\mu/\lambda}$ is the temperature at which the $\phi=0$ configuration becomes unstable, equal to 100 GeV for $m_H=50$ GeV. This form for $F(\phi,T)$ arises in the high-temperature expansion, i.e., for all masses $m/T\ll 1$, for weak but not too weak quartic coupling, formally $g^4\ll\lambda\ll g^2$, by straightforward application of the standard results [15]. The negative, nonanalytic cubic term which makes the transition first order has an analog in the theory of the superconducting transition, where in the type-I regime (analogous to $m_H<m_W$), the transition is

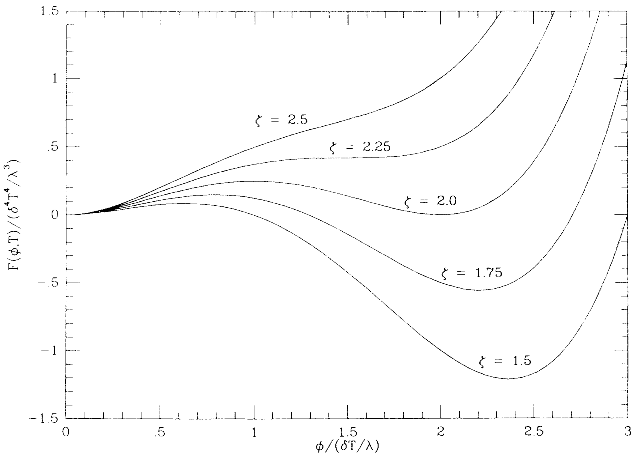

FIG. 1. The finite-temperature free energy $F(\phi,T)$ at several values of the dimensionless temperature $\zeta=\chi(1-T_c^2/T^2)$ around the electroweak phase transition.

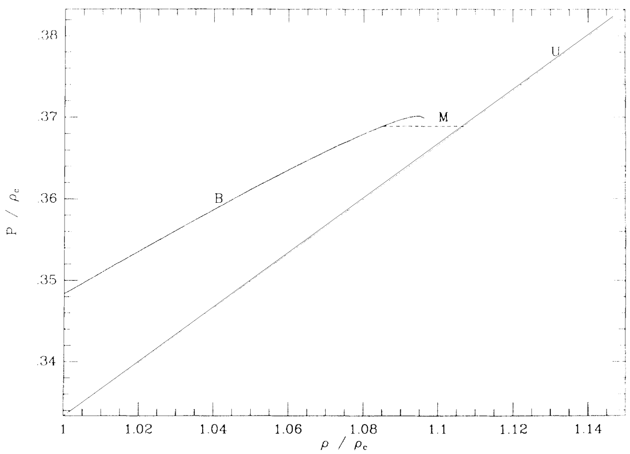

FIG. 2. The pressure-energy diagram for the electroweak theory. U denotes the unbroken phase ($\phi=0$), B the broken phase ($\phi\neq0$), and M the mixed phase. Densities and pressures are given in units of the energy density of the U phase, ρ_c, at $T=T_c$ ($\zeta=0$). Below this temperature the U phase is unstable. The latent heat of the transition is given by the length of the M line, $L\approx4\epsilon\chi\rho_c$ for $\chi\gg1$. For clarity, the situation for $\epsilon=10^{-4}$, $\chi=50$ is shown. The realistic case has a smaller value of ϵ and the B-phase line is correspondingly closer to the U line.

also thought to be weakly first order [16].

It is convenient to define a dimensionless "temperature" $\zeta=\chi(1-T_c^2/T^2)$, with $\chi=\lambda\gamma/\delta^2\gg1$ ($\chi=50$ for $m_H=50$ GeV, $m_t=100$ GeV). The form of the free energy at various values of ζ is shown in Fig. 1. Starting at high temperature, $\zeta\gg1$, there is a unique minimum of $F(\phi,T)$ at $\phi=0$, corresponding to the unbroken-symmetry phase (U). At $\zeta=2.25$ a second higher F minimum appears at nonzero ϕ. At $\zeta=2$ the minima are degenerate, and as ζ falls below 2, the $\phi\neq0$ minimum corresponding to the broken-symmetry phase (B) becomes favored. Finally, on cooling below $\zeta=0$, the $\phi=0$ minimum disappears. Had nucleation of the B phase not yet occurred, the phase transition would proceed by "spinodal decomposition," i.e., through ϕ rolling down to the B-phase minimum.

Minimizing $F(\phi,T)$ with respect to ϕ and using the standard thermodynamic relations, one finds that the pressure $P=-F$ and energy density $\rho=F-T\,dF/dT$ of the two phases are given by

$$\rho_U=\mathcal{N}_{\rm eff}\pi^2T^4/30,\quad \rho_B=\rho_U[1+\epsilon(A-B)],$$
$$P_U=\tfrac{1}{3}\rho_U,\quad P_B=\tfrac{1}{3}\rho_U(1+\epsilon A), \tag{2}$$

where $A=\frac{3}{4}\zeta(\zeta-3)-3(\zeta-\frac{9}{4})(\frac{3}{2}+\sqrt{9/4-\zeta})$ and $B=2(\chi-\zeta)(\frac{9}{4}+\frac{3}{2}\sqrt{9/4-\zeta}-\frac{1}{2}\zeta)$. The parameter $\epsilon=\delta^4/\lambda^3(\pi^2N_{\rm eff}/30)^{-1}$ is very small, $\epsilon\approx1.5\times10^{-5}$ for $m_H=50$ GeV, showing that the Higgs field makes a tiny contribution to the energy and pressure of the Universe in the regime of interest. These equations yield the pressure-energy diagram shown in Fig. 2.

At high temperatures, the Universe begins in the unbroken-symmetry phase U. As it cools, ρ and P decrease until the temperature $T_{\rm eq}$ (corresponding to $\zeta=2$) is reached, where both phases have the same pressure. In true thermal equilibrium the system would then enter a mixed phase, with the fraction of the Universe f remaining in the U phase determined from the total energy density through the relation $\rho=\rho_{U,T_{\rm eq}}f+\rho_{B,T_{\rm eq}}(1-f)$. When the density reaches $\rho_{B,T_{\rm eq}}$, the Universe is in the pure B phase.

There is, however, a large free-energy barrier suppressing the nucleation of B-phase bubbles in the U phase, so the Universe supercools. If nucleation occurs when the energy density is greater than $\rho_{B,T_{\rm eq}}$, then the B-phase bubbles will grow only until the Universe reaches the appropriate mixed phase fraction. But if the density drops below $\rho_{B,T_{\rm eq}}$ before nucleation, bubbles will grow until they fill space. For small ϵ and large χ this corresponds to ζ falling below $2(1-\epsilon\chi^2)\approx1.9$ in the case of interest. As we shall see, bubble nucleation is suppressed until ζ falls below ≈1.6.

The bubble nucleation rate may be calculated using the standard formalism described, for example, in Refs. [11,17]. Bubbles nucleate at a rate given by

$$\Gamma=M^4(S_3/\pi T)^{3/2}e^{-S_3/T}, \tag{3}$$

where $M^2=\gamma(T^2-T_c^2)$ and S_3 is the O(3)-invariant bubble action, calculated by minimizing the functional

$$S_3=4\pi\int r^2dr\left[\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2+F(\phi,T)-F(0,T)\right]$$
$$=4\pi T\int R^2dR\left[\frac{1}{2}\left(\frac{df}{dR}\right)^2+\frac{\lambda^3}{\delta^2}[\tfrac{1}{4}(f^2+\zeta-3)^2+f(\zeta-2)-\tfrac{1}{4}(\zeta-2)^2+\zeta-2]\right], \tag{4}$$

where $f=\phi(\lambda/\delta T)-1$ and $R=(\delta/\lambda)^2 Tr$ are dimensionless. The critical bubble action is a stationary point of (4), with boundary conditions $d\phi/dr(0)=0$, $\phi(\infty)=0$. It is simple to solve the differential equation for $f(r)$ using a "shooting" method, adjusting the magnitude of f at the origin until f asymptotes to -1 at large radius. The result depends only on ζ and the dimensionless quantity $a=\lambda^3/\delta^2$. I find that over the most important interval, $1.3<\zeta<1.8$, the results are very well approximated by $S_3/T=C(2-\zeta)^{-1.6}$, where for Higgs boson mass $m_H=50$ GeV, $a=0.13$ and $C=30$. For comparison, with $m_H=60$ GeV, $a=0.4$, the results fit the same form with $C=16.5$. Neither the "thin-wall" nor the "thick-wall" approximation presented in [11] is good in the regime of interest—the bubble wall is thick but all three terms in (1) are important. As a check, I used a variational ansatz $\phi=\phi(0)\exp(-R^2/L^2)$ and stationarized to find $\phi(0)$ and L analytically. This gives results in good agreement for $\zeta<1.6$, but breaks down at $\zeta=1.68$. At larger ζ the solution approaches thin-wall behavior, looking more like a step function than a Gaussian.

If the B-phase bubbles grow at a speed v, then the fraction of the Universe remaining in the U phase at a time t is given by

$$f_U=\exp\left[-\int_0^t dt'\frac{4\pi}{3}v^3(t-t')^3\Gamma(t')\right], \qquad (5)$$

with t the cosmic time. To a good approximation the time dependence of the prefactor in Γ may be ignored,

$$\Gamma\approx t_c^{-4}(m_{\rm Pl}/T_c)^4\exp(-S_3/T)\approx t_c^{-4}\exp(160-S_3/T)\,.$$

Now change variables to $\zeta=\chi(1-T_c^2/T^2)=\chi(1-t/t_c)$ and expand S_3/T about the point where it equals 160, $S_3/T(\zeta')\approx 160+D(\zeta'-\zeta_0)$, with $\zeta_0\approx 1.65$ and $D=160a/(2-\zeta_0)\approx 730$. The phase transition occurs close to this point. The ζ' integral is dominated by a saddle point, and the exponent in (5) grows to unity at a value of $\zeta\approx\zeta_0+D^{-1}\ln[v^3/(D\chi)^4]\approx 1.6$. At the saddle point, $\zeta'-\zeta\approx 3/D$, corresponding to the ratio of the bubble radius to the horizon $v(t-t')/2t\approx 3v/2D\chi\approx 4\times 10^{-5}v$. So when the bubbles collide and fill space, they are very much smaller than the horizon, but very much larger than the correlation length $m_H(T)^{-1}\approx 10T^{-1}\approx 10^{-15}t$. During the growth of these bubbles, the temperature of the Universe decreases by $\approx 4\times 10^{-5}$, a negligible effect.

We are now ready to discuss how fast the bubbles grow after they nucleate. The standard formalism for shocks [18] employs energy and momentum conservation across the front, treating the matter as a fluid in equilibrium on either side. However, this gives only two equations, for three unknowns—the wall velocity, and the temperature and velocity of the fluid behind it. The extra equation which is needed is the equation of motion of the Higgs field ϕ. The terms in this equation involving fields getting their mass from ϕ may be averaged in the appropriate thermal state.

Particle collisions are precisely the origin of the temperature-dependent terms in (1), and the corresponding terms in the equation of motion for ϕ. Consider one of the particle species that receives its mass $m(\phi)$ from ϕ. If the wall is at rest, the particle mass is a function of space, $m=m(x)$, varying smoothly from 0 to m across the wall. During a collision with the wall, energy and momentum parallel to the wall are conserved, but momentum perpendicular to the wall, k_x, is not. A particle incident from the U phase is reflected if $k_x<m$, and transmitted with a lower momentum if $k_x>m$. One incident from the B phase suffers an acceleration towards the U phase. In each case the wall receives an impulse back towards the B phase. We may calculate the force exerted by the particle on the wall from energy conservation. In the rest frame of the wall, $k_x^2(x)+m(x)^2=$const. It follows that $F_x=-dk_x/dt=[dm^2(x)/dx]v_x/2k_x=[dm^2(x)/dx]/2\omega_k$, with $\omega_k=(\mathbf{k}^2+m^2)^{1/2}$. Adding up the effect of all the particles on the wall, the total pressure is given by

$$\begin{aligned}P_{\rm collisions}&=\int_{-\infty}^{\infty}dx\frac{dm^2(x)}{dx}\int\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}\frac{1}{e^{\beta\omega_k}-1}\\&=T\int_0^{m^2}dm^2\frac{d}{dm^2}\int\frac{d^3k}{(2\pi)^3}\ln(1-e^{-\beta\omega_k})\\&=F(m,T)-F(0,T)=\frac{m^2T^2}{24}-\frac{m^3T}{12\pi}+O(m^4)\end{aligned} \qquad (6)$$

for bosons, and similarly for fermions. These are precisely the temperature- and ϕ-dependent terms in (1).

This collisional pressure on the wall changes if the wall is moving through the fluid. If the mean free time τ between particle collisions is considerably shorter than the time taken for the wall to pass a given point, given by $l/\gamma_v v$ with $l\approx m_H^{-1}(T)\approx 10T^{-1}$, then the fluid will to a good approximation be in local thermal equilibrium. This condition is certainly fulfilled if the wall velocity relative to the fluid v is substantially less than the speed of light, and even for mildly relativistic velocities—the Universe is at this epoch filled with a very dense quark-gluon plasma. If there is local thermal equilibrium in the cosmic rest frame, then in the rest frame of the bubble wall at each point x the phase-space density is given by the Lorentz boosted form of that in (6), $n_k'=(\exp\{\gamma\beta(x)[\omega_k-v(x)k_x]\}-1)^{-1}$, with $m(x)$, $\beta(x)$, and $v(x)$ the local particle mass, inverse temperature, and velocity of the fluid (note that the phase-space density is a scalar under Lorentz transformations). Changing variables to $k_x'=\gamma_v[k_x-v(x)\omega_k]$ one finds that the velocity dependence completely disappears. While the apparent number density of particles in the rest frame of the wall rises (due to Lorentz contraction), they have higher energy, and the wall is more transparent to them. These opposing effects exactly cancel.

The result for the collisional pressure is then given by

$$P_{\text{collisions}}=\frac{m^2\overline{T^2}}{24}-\frac{m^3\overline{T}}{12\pi}+O(m^4)\,, \tag{7}$$

with $\overline{T^2}=[\int dm^2 T^2(m)]/m^2$, $\overline{T}=[\int dm^3 T(m)]/m^3$ being "mass weighted" averages of the powers of temperature across the wall. The only way to slow the wall while remaining in local thermal equilibrium is to raise the "average" temperature across it to the phase equilibrium value. However, as noted above, the latent heat of the transition is tiny, so the temperature variation across the wall is small. In this situation, the wall is accelerated up to a speed close to the speed of light.

The assumption of local thermal equilibrium breaks down if the wall moves fast enough that $l/\gamma_v v \ll \tau$. One can make the opposite assumption, that there is *no* thermalization in the vicinity of the wall, and use the thermal distribution functions appropriate to the medium on either side. In this case, one finds a collisional pressure which rises linearly with the velocity of the wall [10]. Equating this with the excess pressure of the true-vacuum phase, one determines a mildly relativistic terminal velocity $v\approx 0.1$. Departure from equilibrium does produce a mechanism for slowing the wall, but the actual velocity is likely to be considerably larger than 0.1 since in the realistic case τ is certainly much less than $10l\approx 100T^{-1}$. It is intriguing that the final wall velocity depends so strongly on the microscopic details of the thermalization process [10].

The analysis given here is certainly incomplete, but will I hope serve to stimulate more detailed study. My main conclusion, that bubble walls in weakly first-order transitions propagate at velocities of the order of the speed of light, is new, but an accurate determination of the velocity clearly requires more work. In [10], a detailed derivation of the velocity dependence of the collisional pressure acting on the wall is given, starting from the Higgs field equation of motion. This requires an analysis of the Boltzmann equation in order to determine the departure from local equilibrium in the particle phase-space density across the bubble wall.

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[1] V. Kuzmin, V. Rubakov, and M. Shaposhnikov, Phys. Lett. **155B**, 36 (1985); F. Klinkhamer and N. Manton, Phys. Rev. D **30**, 2212 (1984); P. Arnold and L. Mclerran, Phys. Rev. D **36**, 581 (1987). For a recent numerical study see, e.g., J. Ambjorn, T. Askgaard, H. Porter, and M. Shaposhnikov, Phys. Lett. B **244**, 497 (1990).

[2] N. Turok and J. Zadrozny, Phys. Rev. Lett. **65**, 2331 (1990).

[3] N. Turok and J. Zadrozny, Nucl. Phys. **B358**, 471 (1991).

[4] M. E. Shaposhnikov, Pis'ma Zh. Eksp. Teor. Fiz. **44**, 364 (1986) [JETP Lett. **44**, 465 (1986)]; Nucl. Phys. **B287**, 757 (1987); **B299**, 797 (1988).

[5] L. McLerran, Phys. Rev. Lett. **62**, 1075 (1989).

[6] A. G. Cohen, D. B. Kaplan, and A. E. Nelson, Phys. Lett. B **245**, 561 (1990); Nucl. Phys. **B349**, 727 (1991); Phys. Lett. B **263**, 86 (1991).

[7] M. Dine, P. Huet, M. Singleton, and L. Susskind, Phys. Lett. B **257**, 351 (1991).

[8] L. McLerran, M. Shaposhnikov, N. Turok, and M. Voloshin, Phys. Lett. B **256**, 451 (1991).

[9] M. Dine, P. Huet, and M. Singleton, SCIPP Report No. 91/08, 1991 (to be published), and A. Cohen, D. Kaplan, and A. Nelson, ITP report, 1991 (to be published), argue that there are extra suppression factors but recent numerical simulations in a 1+1 analog theory indicate the supposed suppression does not actually occur, supporting the original estimate of [8]; see D. Grigoriev, M. Shaposhnikov, and N. Turok, Phys. Lett. B (to be published).

[10] L. Mclerran and N. Turok (to be published).

[11] A. D. Linde, Nucl. Phys. **B216**, 421 (1983).

[12] A. I. Bochkarev, S. Kuzmin, and M. E. Shaposhnikov, Phys. Lett. B **244**, 275 (1990).

[13] N. Turok and J. Zadrozny, Nucl. Phys. B (to be published).

[14] D. A. Kirzhnits and A. D. Linde, Ann. Phys. (N.Y.) **101**, 195 (1976).

[15] L. Dolan and R. Jackiw, Phys. Rev. D **9**, 3320 (1974).

[16] B. Halperin, T. Lubensky, and S. K. Ma, Phys. Rev. Lett. **32**, 292 (1974).

[17] I. Affleck, Phys. Rev. Lett. **46**, 388 (1981).

[18] M. Gyulassy, K. Kajantie, H. Kurki-Suonio, and L. McLerran, Nucl. Phys. **B237**, 477 (1984).