Statistics of Cosmological Gamma-Ray Bursts

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A phenomenological model of gamma-ray burst spectra is used to calculate the statistics of gamma-ray bursts originating at cosmological distances. A model of bursters with no source evolution in a $q_0 = \frac{1}{2}$ Friedmann cosmology is in accord with recent observations of the differential $V/V_{\rm max}$ distribution. The data are best fit with an average peak burst luminosity of $4(\pm 2)\times 10^{51}$ ergs s^{-1} and a present-day source emissivity of 940 ± 440 bursts/(10^{10} yr)Mpc^3. A spectral test of the cosmological hypothesis is proposed.

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Recent observations [1-3] with the Burst and Transient Source Experiment (BATSE) on board the Gamma Ray Observatory (GRO) show that gamma-ray bursts (GRBs) are isotropically distributed on the celestial sphere, yet the burst size distribution deviates significantly from a $-\frac{3}{2}$ slope expected for an isotropic, homogeneous population of sources. The BATSE data showed statistical agreement with isotropy for both the strongest and weakest bursts. These findings confirm previous studies [4,5], but now at a much lower detector threshold ($\sim 10^{-7}$ erg cm^{-2}s^{-1}). The deviation of a source population from uniformity is characterized by the average of a sample of $V/V_{\rm max}$ values [6]. For the BATSE sample of 126 GRBs, $\langle V/V_{\rm max}\rangle = 0.35 \pm 0.03$, significantly lower than the value of 0.5 expected for a uniform source population.

Paczyński [7] has recently summarized the arguments in favor of a cosmological origin for the bursters, as originally proposed in Ref. [8]. Plausible scenarios for the release of $\gtrsim 10^{50}$ ergs of energy on a time scale $\lesssim 1$ s include collisions or mergers of strange stars, neutron stars, or neutron stars and black holes (see references cited in [7]). The cosmological hypothesis has failed, however, to provide a spectral model that agrees with the characteristic spectral form of GRBs deduced from the wealth of available data, nor does it account for the origin of spectral features observed in a small fraction ($\approx$ 10%-20%) of GRBs [9]. In the absence of a convincing model for the spectra of GRBs at cosmological distances, we investigate whether the observed burst statistics are in accord with the cosmological scenario by employing a phenomenological description of burst spectra.

We generalize the cosmological test of source number counts [10] to a bursting population with a spectral form characterized by the parameter α. We assume an isotropic, homogeneous universe with burst sources whose density and bursting rate depend only on epoch t_1. Let $\dot n(L,\alpha;t_1)dL\,d\alpha$ represent the differential number of bursts per unit proper time per unit proper volume occurring at time t_1, which have luminosity between L and $L+dL$ and spectral parameter α in the range α to $\alpha+d\alpha$. For a Robertson-Walker metric, the burst rate per unit (observer) proper time t_0 per differential solid angle $d\Omega$ is given by

$$\frac{d\dot N}{d\Omega} = c\left|\frac{dt_1}{dt_0}\right| \dot n(L,\alpha;t_1)R^2(t_1)r_1^2\,dt_1\,dL\,d\alpha\,, \qquad (1)$$

where $r_{1(0)}$ and $R(t_{1(0)})$ are the radial coordinate and expansion scale factor at time $t_{1(0)}$.

Let the source spectral power be denoted by $P(\epsilon;\alpha)$, where $\epsilon = h\nu/m_e c^2$ is the dimensionless photon energy. The total luminosity of a source with spectral parameter α is therefore given by

$$L = \int_0^\infty d\epsilon \oint d\Omega_s\, P(\epsilon;\alpha)\,, \qquad (2)$$

where Ω_s is the solid angle element measured at the source. The flux density is given by

$$S(\epsilon;\alpha) = \frac{P[\epsilon(1+z);\alpha](1+z)}{d_L^2}\,, \qquad (3)$$

where the redshift z is given by the well-known relation $dt_1/dt_0 = R(t_1)/R(t_0) = (1+z)^{-1}$. The luminosity distance d_L is defined so that the energy flux $l = L/4\pi d_L^2$, and is given for a matter-dominated Friedmann cosmology by

$$d_L = \frac{R^2(t_0)r_1}{R(t_1)} = \frac{c}{H_0 q_0^2}\{zq_0 + (q_0-1)[(2q_0 z+1)^{1/2}-1]\}\,. \qquad (4)$$

Here $H_0 = 100h$ km s^{-1}Mpc^{-1} is Hubble's constant and q_0 is the deceleration parameter. Noting that $|dt_1/dz| = H_0^{-1}(1+z)^{-2}(1+2q_0z)^{-1/2}$, we obtain the observed differential burst rate per steradian for bursts with flux density S_ϵ at photon energy ϵ given by

$$\frac{d\dot N}{d\Omega\,dS_\epsilon} = \frac{c}{H_0}\int_0^\infty dL \int_{-\infty}^\infty d\alpha \int_0^\infty dz\,(1+z)^{-7}(1+2q_0z)^{-1/2} d_L^2 \dot n(L,\alpha;z)\,\delta\left[S_\epsilon - \frac{P[\epsilon(1+z);\alpha](1+z)}{d_L^2}\right]. \qquad (5)$$

This relation generalizes the burst emissivity [11] to a matter-dominated Friedmann cosmology, valid for $z \lesssim 10^2$.

Let the detector threshold at photon energy ϵ be denoted by S_d. The $V/V_{\max}$ value of a burst is given by $v \equiv V/V_{\max} = (S_\epsilon/S_d)^{-3/2}$, and the differential $V/V_{\max}$ distribution for bursts is given from Eq. (5) by the expression

$$\frac{d\dot{N}}{dv} = \frac{8\pi S_d c}{3v^{5/3}H_0}\int_0^\infty dL \int_{-\infty}^\infty d\alpha \int_0^\infty dz\,(1+z)^{-7}(1+2q_0z)^{-1/2}d_L^2\dot{n}(L,\alpha;z)\,\delta\left[S_d v^{-2/3} - \frac{P[\epsilon(1+z);\alpha](1+z)}{d_L^2}\right]. \quad (6)$$

The function

$$P(\epsilon;\alpha) = \begin{cases} P_0, & \epsilon \le \bar{\epsilon}, \\ P_0\bar{\epsilon}/\epsilon, & \bar{\epsilon} \le \epsilon \le \epsilon_{\max} \end{cases} \quad (7)$$

provides a good characterization of the spectra of bursts in the Konus [12] and Signe [13] catalogs, with the parameter $\alpha \to \bar{\epsilon} \approx 1$. Assuming that the burst emission is unbeamed, Eq. (2) implies that $P_0 = L/4\pi\bar{\epsilon}\ln(e\epsilon_{\max}/\bar{\epsilon})$. Solar Maximum Mission observations [14] show that the maximum photon energy $\epsilon_{\max} \gtrsim 12$. The model results are only weakly dependent on $\epsilon_{\max}$, so we let $\epsilon_{\max} = 200$, in agreement with observations of 100-MeV photons in the spectra of bright bursts [15]. Errors of factors of 2–5 in the source luminosity can result, however, if the burst spectra are assumed to cut off at a few MeV.

For simplicity, we assume a standard-candle luminosity function for the bursters and model the z dependence of the burst evolution through the relation $\dot{n}(L,\bar{\epsilon},z) = \dot{n}_0(1+z)^p f(\bar{\epsilon})\delta(L-L_0)$, where $f(\bar{\epsilon})$ is normalized to unity. If there is no source evolution, $p=3$. Inverting the δ function in Eq. (6) to solve the z integral gives

$$\frac{d\dot{N}}{dv} = \frac{8\pi S_d \dot{n}_0}{3v^{5/3}}\frac{c}{H_0} \times \int_0^\infty d\bar{\epsilon}\,\frac{f(\bar{\epsilon})(1+z_v)^{-7+p}d_L^4(z_v)}{(1+2q_0z_v)^{1/2}P(\bar{\epsilon};L_0)|A(z_v)|}, \quad (8)$$

where $A(z_v)$ is the derivative of the argument of the δ function evaluated at the value of z_v which solves the δ function. Values of z_v must, in general, be calculated numerically. Analytic results are possible, however, for representation (7) when $z \le \bar{z} = \bar{\epsilon}/\epsilon - 1$ with $q_0 = \frac{1}{2}$, and when $z > \bar{z}$, for general q_0. In the former case,

$$A(z) = 1 - \frac{4(1+z)c}{H_0 d_L}\left[1 - \frac{1}{2\sqrt{z+1}}\right] \quad (9a)$$

and

$$z_v(\bar{\epsilon}) = k(k+2), \quad (9b)$$

where

$$k = H_0 P_0^{1/2} v^{1/3}/2cS_d^{1/2}. \quad (9c)$$

When $z > \bar{z}$,

$$A(z) = \frac{2c\bar{\epsilon}}{q_0 H_0 \epsilon d_L}\left[1 + \frac{q_0 - 1}{(2q_0z+1)^{1/2}}\right] \quad (10a)$$

and

$$z_v = K - 1 + q_0 + [(1-q_0)^2 + 2K(q_0^{1/2} - q_0^{-1/2})^2]^{1/2}, \quad (10b)$$

where

$$K = 2q_0 k(\bar{\epsilon}/\epsilon)^{1/2}. \quad (10c)$$

For a $q_0 = \frac{1}{2}$ universe, Eq. (10b) reduces to $z_v = K - 1/2 + \sqrt{K + 1/4}$.

The BATSE detector triggers on photons in the 50–300-keV band, with a threshold flux of ≈ 1.0 photon/cm^{2}s [2]. The average photon energy in this band is $\cong 140$ and 110 keV for intrinsic burst *photon* spectra with spectral indices of 1 and 2, respectively. We therefore assign the detector threshold $S_d \cong (0.5m_ec^2\ \mathrm{cm}^{-2}\mathrm{s}^{-1})\epsilon^{-1}$ at a mean photon energy ≈ 125 keV (i.e., $\epsilon \approx 0.25$). We choose two extreme cases [16] for $f(\bar{\epsilon})$ that should bracket the true distribution. In the first case, we describe $f(\bar{\epsilon})$ by a Gaussian peaking at $E_{\rm peak} = 200$ keV with a FWHM of 200 keV. This corresponds to the observed distribution of Konus bursts. In the second case, we let $f(\bar{\epsilon})$ be represented by a Gaussian with $E_{\rm peak} = 1000$ keV and a FWHM of 1000 keV. This case corresponds to an intrinsic distribution with detector triggering biases that arise from a distribution of incident spectra. We note that if the cosmological hypothesis is correct, the source spectral distribution should be narrower, because the Konus sample represents a convolution of sources at different redshifts. Because the results are not very sensitive to the assumed value of $E_{\rm peak}$, a self-consistent calculation, which takes into account different values of the spectral turnover at different epochs, is unnecessary.

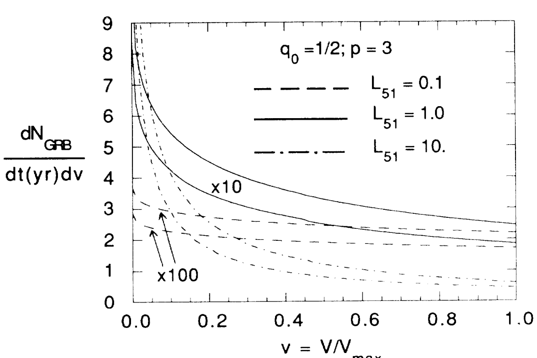

FIG. 1. Calculated differential $V/V_{\max}$ distribution for standard-candle GRBs with no source evolution in a $q_0 = \frac{1}{2}$ universe. Curves are labeled by peak luminosity in units of 10^{51} ergs s^{-1}. The lower and upper curves of each pair represent extreme cases of intrinsic GRB spectral variations, as discussed in the text. Curves are multiplied by the factors shown for ease of presentation, with the present-day source emissivity normalized to $\dot{n}_0 = 1$ burst/(10^{10} yr)Mpc^3.

Figure 1 shows the differential V/V_{max} distribution for standard-candle bursts with peak luminosities $L_{51}=0.1$, 1, and 10, where $L_{51}=L_0/10^{51}$ ergs s^{-1}. Here we treat the case of no source evolution, and let $q_0=\frac{1}{2}$ and $h=1$ throughout. The lower and upper curves of each pair in Fig. 1 are results for Gaussian spectral parameter distributions peaking at 200 and 500 keV, respectively, as described above. The present-day source emissivity $\dot{n}_0$ is normalized to 1 burst/(10^{10} yr)Mpc^3. As can be seen, the effects of an expanding universe on the size distribution of 10^{50} ergs s^{-1} GRBs are small, since the characteristic sampling distance to such bursts is $\ll c/H_0$. Thus $\langle V/V_{max}\rangle \approx 0.5$. For brighter bursts, the expansion of the universe reduces the presently observed luminosity and burst rate of GRB sources at large redshifts, that is, for those with $V/V_{max}\rightarrow 1$. The characteristic redshift z_{char} of the faintest bursts can be obtained by calculating the z value of a burst with $V/V_{max}=1$ whose spectrum is defined by the peak of the Gaussian distribution. Values of z_{char} and $\langle V/V_{max}\rangle$ are listed in Table I for the displayed curves.

Figure 2 shows the effect of source evolution on bursts with luminosity $L_{51}=1$. Here we let $p=1$, 3, and 5, representing source decay, no source evolution, and source increase at early times, respectively. The consequence of an increase in the number of sources at early times is an approximate counterbalance of the effects of universal expansion for a $p=5$ law, as indicated in Table I.

The differential V/V_{max} distribution for 126 GRBs observed with BATSE [3] is shown in Fig. 3. The error bars are statistical. The full sky burst rate is normalized to the observed rate by multiplying by a factor of 2.75 [1]. Also shown in Fig. 3 is the no-evolution ($p=3$), standard-candle source model with $L_{51}=4$, which gives the best fit obtained by minimizing χ^2 with respect to the centroid of the error bars. The source luminosity obtained by requiring $\langle V/V_{max}\rangle=0.35$, in agreement with the BATSE result, is $L_{51}\cong 3$. The source luminosities and present-day source emissivities obtained by varying the parameters to provide acceptable fits to the data are $L_{51}=4\pm 2$ and $\dot{n}_0=940\pm 440$ bursts/(10^{10} yr)Mpc^3, respectively.

The simplicity of the proposed model and the reasonable agreement of model results with data suggest that further study of the cosmological hypothesis is warranted. A second component of GRBs originating from galactic neutron stars is not excluded, however, provided that the sampling distance of the detector does not greatly exceed the scale height of the local neutron-star population. Such a component could be the source of line signatures observed in some GRB spectra. Discovery of a spatial anisotropy in bursts characterized by peak flux, morphology, or spectral signature would provide strong evidence in favor of a second, and possibly local, component [17]. But the lack of clear-cut distinctions [18] in sources whose absolute luminosities vary by factors ranging from 10^4 to 10^{14} argues in favor of a one-component model.

A straightforward test of the cosmological hypothesis is to examine the dependence of the mean energy of the soft gamma-ray spectral break in GRBs as a function of the V/V_{max} values of the bursts. For the $p=3$ model used to fit the data, the ratio of the mean break energies for the brightest and weakest bursts in the BATSE sample should equal ~ 2 (see Table I). Spectral modeling of

TABLE I. Dependence of z_{char} and $\langle V/V_{max}\rangle$ on burst luminosity and model of source-density evolution.

	L_0	$E_{peak}=200$ keV		$E_{peak}=500$ keV	
	(10^{51} ergs s^{-1})	z_{char}	$\langle V/V_{max}\rangle$	z_{char}	$\langle V/V_{max}\rangle$
$p=3$	0.1	0.19	0.47	0.23	0.47
	1.0	0.63	0.41	0.69	0.41
	4.0	1.2	0.32	1.6	0.34
	10.0	1.8	0.26	2.8	0.27
$p=1$	1.0	0.63	0.35	0.69	0.35
$p=5$	1.0	0.63	0.46	0.69	0.47

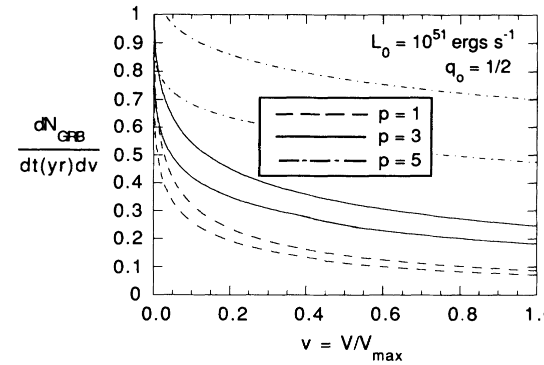

FIG. 2. Same as Fig. 1, but now showing the effects of the GRB source density evolving according to the law $(1+z)^p$, where $p=1$, 3, and 5.

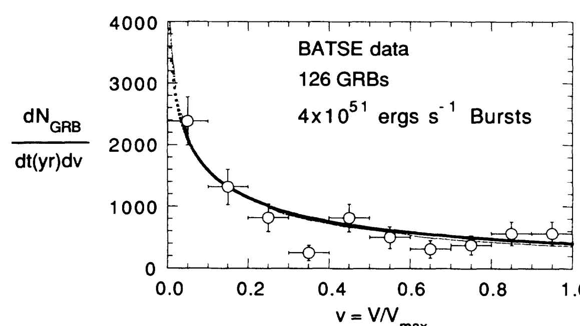

FIG. 3. Data points show the differential V/V_{max} distribution observed with BATSE on GRO [3], and the curves show the best fits to the data for a model with no source evolution. The heavy curve shows the model results with $E_{peak}=1000$ keV, and the nearly coincident light curve gives the model results with $E_{peak}=200$ keV.

GRBs to test this correlation is strongly encouraged. Unfortunately, the lack of a distinct correlation cannot falsify the cosmological hypothesis because the intrinsic dispersion of source spectra combined with a general model of source evolution may conceal the redshift effect. The discovery of such a correlation would, however, be extremely important, because a given value of $V/V_{\max}$ is related in a unique way to z for a given cosmological model. Should the standard-candle assumption hold, spectral observations of a large sample of GRBs could in principle constrain values of q_0 and h.

Except in supernova explosions, sources emitting 10^{51-52} erg s^{-1} for a few-tenths to a few tens of seconds are unmatched in nature. This is equivalent to the conversion of $\sim 0.1\%-1\%$ of the energy of one solar mass of material into x rays and gamma rays. The generation of this amount of energy in a compact region will produce an expanding fireball, but calculations of fireball spectra [19,20] have not succeeded in reproducing the characteristic GRB spectral form. In view of the evidence from GRO in favor of the cosmological origin of gamma-ray bursts, an examination of these and other models, including electrodynamic and general relativistic effects, neutrino interactions, and pair production and annihilation processes, is now necessary.

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[1] C. A. Meegen *et al.*, Nature (London) **355**, 143 (1991).
[2] G. J. Fishman (private communication).
[3] BATSE News Service (BNEWS), data of 29 October 1991.
[4] E. P. Mazets, in *Proceedings of the Nineteenth International Cosmic Ray Conference, La Jolla, 1985* (U.S. GPO, Washington, DC, 1985), Vol. 9, p. 424.
[5] J.-L. Atteia *et al.*, Astrophys. J. Suppl. **64**, 305 (1987).
[6] M. Schmidt, Astrophys. J. **151**, 393 (1968).
[7] B. Paczyński (to be published).
[8] V. V. Usov and G. V. Chibisov, Astron. Zh. **52**, 192 (1975) [Sov. Astron. **19**, 115 (1975)].
[9] For reviews, see *Gamma Ray Bursts,* edited by E. P. Liang and V. Petrosian (Wiley, New York, 1986); J. C. Higdon and R. E. Lingenfelter, Annu. Rev. Astron. Astrophys. **28**, 401 (1990); A. K. Harding, Phys. Rep. **206**, 327 (1991).
[10] S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972).
[11] C. D. Dermer and K. C. Hurley (to be published).
[12] E. P. Mazets *et al.*, Astrophys. Space Sci. **80**, 3–143 (1981).
[13] K. C. Hurley (private communication).
[14] S. M. Matz, D. J. Forrest, W. T. Vestrand, E. L. Chupp, G. H. Share, and E. Rieger, Astrophys. J. Lett. **288**, L37 (1985).
[15] G. H. Share, S. M. Matz, D. C. Messina, P. L. Nolan, E. L. Chupp, D. J. Forrest, and J. F. Cooper, Adv. Space Res. **6**, 15 (1986).
[16] J. C. Higdon and R. E. Lingenfelter, Astrophys. J. **307**, 197 (1986).
[17] R. E. Lingenfelter and J. C. Higdon (to be published).
[18] R. E. Lingenfelter and J. C. Higdon (to be published).
[19] B. Paczyński, Astrophys. J. Lett. **308**, L43 (1986).
[20] J. Goodman, Astrophys. J. Lett. **308**, L47 (1986).