Low-Temperature Properties of Two-Dimensional Frustrated Quantum Antiferromagnets

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Though much of the recent work on 2D quantum canted antiferromagnets has been devoted to the study of disordered states, there is evidence that some of these systems have Néel ground states, even for spin $\frac{1}{2}$. Following Chakravarty, Halperin, and Nelson, we study the quantum nonlinear sigma model suited to these systems and obtain for the correlation length $\xi = C_\xi [A/(k_B T)^{1/2}] e^{2\pi B/k_B T}$, where A and B depend on the spin stiffnesses and spin-wave velocities renormalized by quantum fluctuations. We discuss experiments on ^{3}He adsorbed on Grafoil.

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Since at the classical level the effect of frustration is to reduce the order, frustrated quantum spin systems are expected to exhibit nonconventional disordered ground states [1]. Such an effect would be of importance for some theories of high-T_c superconductivity [2,3]. For this reason, quantum frustrated spin models have recently been the object of intensive numerical as well as analytical work [4-8]. Though most of the theoretical interest has been devoted to the study of states not having long-range order, it is likely that some frustrated models may exhibit long-range order in their ground states [6-8], even for spin $\frac{1}{2}$. To our knowledge, there have been no studies of the low-temperature properties of frustrated quantum antiferromagnets assuming a noncollinear ordered ground state (which will be referred to as a Néel state in the following). In particular, the expression for the correlation length for these systems is still lacking. Such an expression would be of interest in discussing experiments on doped La_2CuO_4 [9,10] and on ^{3}He [11] adsorbed on a graphite substrate. In this work, we extend the analysis of Chakravarty, Halperin, and Nelson (CHN) [12,13] to quantum canted Heisenberg models with zero net magnetization. We study the quantum nonlinear sigma (QNLσ) model suited to these systems and give the general expression of their correlation length.

Years ago, Halperin and Saslow [14] showed that the long-distance physics of frustrated quantum Heisenberg models may be described by a hydrodynamical theory, provided the total magnetization is zero. They predicted the existence of three spin-wave excitations resulting from the complete breaking of the O(3) spin-rotation group. The linear spectrum and the interactions of these spin-wave excitations are described by a QNLσ model whose action is entirely determined by symmetries. This action was first derived from the lattice by Dombre and Read [15] in the special case of the $D=2$ triangular Heisenberg antiferromagnet (AFT). They wrote it in terms of a rotation matrix of SO(3) which is the relevant order parameter for such a model:

$$S = -\frac{1}{2}\int_0^\beta d\tau \int d^2x \{\mathrm{Tr}[P_0(g^{-1}\partial_0 g)^2 + P_\perp (g^{-1}\partial_i g)^2]\}, \qquad (1)$$

where $g(x,\tau)\in SO(3)$, $\partial_\mu = (\partial_0, \partial_i) = (\partial/\partial\tau, \partial/\partial x_i)$, $i=1,2$, β is the inverse temperature, and $P_\mu = \mathrm{diag}(p_{1\mu}, p_{2\mu}, p_{3\mu})$, $\mu = 0, \perp$, are diagonal matrices which are *a priori* independent. It is important to notice that action (1) is not particular to the AFT model. In fact, if not for the anisotropy between space and time directions, action (1) would be that of a standard NLσ model with an order parameter in SO(3) [15-17]. Depending on P_μ, it describes all the symmetry-breaking patterns compatible with such an order parameter. The non-Lorentz-invariance of (1), i.e., P_0 not proportional to $P_\perp$, is really meaningful since it allows the three spin waves coming from the breaking of the O(3) rotation group to have different velocities, c_1, c_2, and c_3. We give another expression of action (1), convenient for the following:

$$S = \frac{1}{2}\int_0^\beta d\tau \int d^2x (\chi_{ab}\omega_0^a\omega_0^b + \rho_{ab}\omega_i^a\omega_i^b), \qquad (2)$$

where $g^{-1}\partial_\mu g = \omega_\mu^a T_a$, $T_a \in \mathrm{Lie}[SO(3)]$, and where $\chi_{ab} = -\mathrm{Tr}(P_0 T_a T_b)$ and $\rho_{ab} = -\mathrm{Tr}(P_\perp T_a T_b)$ are the susceptibility and spin-stiffness tensors. Note that (1) and (2) are nothing but the continuum limit of the Lagrangian of a system of quantum rigid bodies interacting on the lattice, where $(\chi^{-1})_{ab} = \chi^{ab}$ is the inertia tensor of the tops and ρ_{ab} is the coupling between nearest-neighbor tops on the lattice. This generalizes the analysis of CHN to frustrated quantum spin models with zero magnetization. In the following we shall restrict ourselves to the case where $\chi_{ab} = \chi_a\delta_{ab}$, $\chi_1 = \chi_2 \neq \chi_3$, and $\rho_{ab} = \rho_a\delta_{ab}$, $\rho_1 = \rho_2 \neq \rho_3$. The effective action (2) is then $O(3)\otimes O(2)/O(2)$ symmetric. It describes three interacting spin waves with velocities $c_1 = c_2 = (\rho_1/\chi_1)^{1/2}$ and $c_3 = (\rho_3/\chi_3)^{1/2}$. It is the action relevant for the AFT and other helical models.

Our program is as follows. We first briefly discuss the recursion relations associated to action (2) and show that there exists some scale at finite T where the quantum model becomes effectively classical. Then we calculate, by integrating over the quantum fluctuations, the action of this effective 2D classical $O(3)\otimes O(2)/O(2)$ NLσ model. Finally we compute the two-loop correlation length ξ of the classical model, from which will follow that of the quantum system. Computational details will be given in a forthcoming publication [18].

$D=2$ quantum model.—If not for the breaking of the Lorentz invariance, i.e., $\chi_{ab}\propto\rho_{ab}$, the action (2) would be that of the $D=3$, $O(3)\otimes O(2)/O(2)$ NLσ model which has been recently extensively studied [16,17]. We obtain, in the present non-Lorentz-invariant case, the one-loop recursion equations for the parameters entering in (2):

$$\begin{aligned}\frac{\partial c_1}{\partial l}&=-\frac{g}{8\pi}\frac{c_1^3}{c_3}(1-\alpha)\frac{c_1-c_3}{c_1+c_3},\\ \frac{\partial c_3}{\partial l}&=\frac{g}{16\pi}\frac{c_1}{c_3}(1-\alpha)(c_1^2-c_3^2),\\ \frac{\partial g}{\partial l}&=-g+\frac{g^2}{4\pi}c_1\left(\frac{c_3+\alpha c_1}{c_1+c_3}\right),\\ \frac{\partial \alpha}{\partial l}&=-\frac{g}{8\pi}\frac{c_1}{c_1+c_3}(1-\alpha)[c_3(1+\alpha)+(3\alpha-1)c_1],\end{aligned}\tag{3}$$

where $g=2/\rho_1$, $\alpha=1-\rho_3/\rho_1$, c_1 and c_3 are dimensionless couplings. From the field-theoretic point of view, these recursion relations stem only from the ultraviolet behavior of the theory and are not affected by the presence of the infrared cutoff β in the time direction. The renormalization group equation for β follows simply from the fact that it scales trivially since it is dimensionless:

$$\frac{\partial\beta}{\partial l}=-\beta.\tag{4}$$

Note that contrary to collinear antiferromagnets, the spin-wave velocities c_1 and c_3 do renormalize at this order. This is a consequence of the non-Lorentz-invariance of the theory. The flow diagram is sketched in Fig. 1 in a truncated space. Equations (3) and (4) admit a nontrivial fixed point with only one relevant direction which is obtained for $T=0$, $g^*=8\pi/c_3^*$, $c_1^*=c_3^*$, and $\rho_1^*=\rho_3^*$, i.e., $\alpha^*=0$. At this point the theory is Lorentz invariant and $O(3)\otimes O(3)/O(3)\sim O(4)/O(3)$ symmetric. As is the case for the classical $O(3)\otimes O(2)/O(2)$ NLσ model in $D=2+\epsilon$, the symmetry is dynamically enlarged at the fixed point [16,17]. Associated to the above fixed point, there exists a critical hypersurface which divides a disordered, possibly spin-liquid phase, for small S, from a Néel phase, for large S. Then, at $T=0$, when one varies, say, the spin S, the quantum model undergoes a second-order phase transition from Néel to disordered phase, the universality class of which is the usual Wilson-Fisher

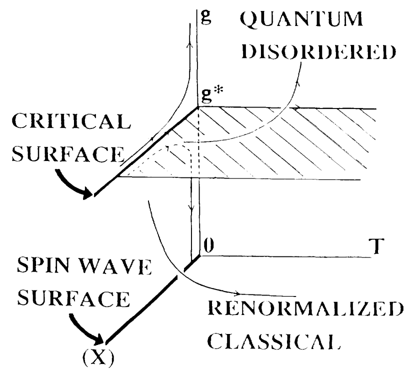

FIG. 1. Sketch of the RG flow in the five-dimensional space of coupling constants $(T,g,X=\{\alpha,c_1,c_3\})$.

$N=4$ one. There also exists a whole hypersurface of fixed points with $T=0$ and $g^*=0$, i.e., $S\rightarrow\infty$, which controls the long-distance physics of the spin waves in the Néel phase. In this phase, starting at low temperature, the RG flow, (3) and (4), drives the system towards a higher-temperature quasiclassical regime where the quantum fluctuations are negligible. Since Eq. (4) has a solution with $\beta(l)=\beta e^{-l}$, we expect that at some scale, $e^{l^*}=\bar{\lambda}\beta$, the quantum model is equivalent to a purely 2D classical $O(3)\otimes O(2)/O(2)$ NLσ model with effective parameters $\rho_{ab}(l^*)$. Once the matching scale is known, these parameters can be reexpressed in terms of the temperature T, the spin-stiffness tensor $\bar{\rho}_{ab}$, and the spin-wave velocities $\bar{c}_1,\bar{c}_3$ renormalized by the quantum fluctuations at $T=0$. However, as this precise matching scale e^{l^*} cannot be obtained from the recursion relations (3),(4), we shall have to integrate the quantum fluctuations in action (1) or (2) in order to obtain directly the effective $D=2$ classical NLσ model.

$D=2$ renormalized classical model.—To integrate out the time-dependent part of the fluctuations, we make the following decomposition of the field g in action (1), $g(x,t)=g_0(x)e^{\phi(x,t)}$, where $\phi(x,t)\in\text{Lie}[SO(3)]$. The above decomposition is unambiguously defined provided that $\int_0^\beta\phi(x,t)dt=0$, which ensures that the Fourier components of ϕ, $\tilde{\phi}(\mathbf{k},\omega_n)$, do not contain the classical $\omega=0$ mode. The effective action S_{eff} is defined by the following equation:

$$\exp\left[-\int d^2x\,L_{\text{eff}}(g_0(x))\right]=\int d\mu[\phi]\exp\left[-\int dt\int d^2x\,L(g_0e^{\phi})\right],\tag{5}$$

where $d\mu[\phi]$ is the Haar measure on SO(3). When expressed in terms of the ω_i^a fields, S_{eff} is found to be, at one-loop order and using a Pauli-Villars regularization scheme,

$$S_{\text{eff}}[\omega(x)]=\int d^2x\,(\rho_{\text{eff}})_{ab}(M)\,\omega_i^a(x)\,\omega_i^b(x),\tag{6}$$

with

$$(\rho_{\rm eff})_{ab}(M)=\beta\bar{\rho}_{ab}+\frac{1}{2\pi}\ln\beta M\,R_{ab}(\bar{\rho})+\frac{1}{2\pi}\zeta_{ab}(\bar{\rho},\bar{c})\,,\quad(7)$$

where M is the mass of the Pauli-Villars particle, and $\bar{\rho}_{ab}$ and $\bar{c}_a$ are the spin-stiffness tensor and the velocities at $T=0$ renormalized by the quantum fluctuations. The expressions for the diagonal tensors $R_{ab}(\bar{\rho})$ and $\zeta_{ab}(\bar{\rho},\bar{c})$ are

$$\begin{aligned}&R_{11}=R_{22}=\tfrac{1}{2}(1+\bar{\alpha}),\quad R_{33}=\tfrac{1}{2}(1-\bar{\alpha})^2,\\&\zeta_{11}=\zeta_{22}=\tfrac{1}{2}\bar{\alpha}\ln\bar{c}_1^2+\tfrac{1}{4}(1-\bar{\alpha})\\&\qquad\times\left\{1-\frac{\bar{c}_1^2\bar{c}_3^2}{\bar{c}_1^2-\bar{c}_3^2}\left[\frac{1}{\bar{c}_1^2}\ln\bar{c}_1^2-\frac{1}{\bar{c}_3^2}\ln\bar{c}_3^2\right]\right\},\\&\zeta_{33}=\tfrac{1}{4}(1-\bar{\alpha})^2\ln\bar{c}_1^2\,.\end{aligned}\quad(8)$$

$R_{ab}(\bar{\rho})$ is nothing but the Ricci tensor of the manifold $O(3)\otimes O(2)/O(2)$ which is the expected counter term one should obtain from a one-loop calculation of the 2D classical $O(3)\otimes O(2)/O(2)$ NLσ model with couplings $\bar{\rho}_{ab}$ [16,19]. Finally, $\zeta_{ab}(\bar{\rho},\bar{c})$ is the finite part which fixes the desired matching scale between quantum and classical renormalized regions. Equation (7) can be rewritten as

$$(\rho_{\rm eff})_{ab}(M)=(\rho_R)_{ab}(\beta^{-1})+\frac{1}{2\pi}\ln\beta M\,R_{ab}(\rho_R)+O(\beta^{-1})\,,\quad(9)$$

where

$$(\rho_R)_{ab}(\beta^{-1})=\beta\bar{\rho}_{ab}+(1/2\pi)\zeta_{ab}(\bar{\rho},\bar{c})\,.\quad(10)$$

The equation (9) is the one-loop renormalization group equation of the 2D-classical $O(3)\otimes O(2)/O(2)$ NLσ model which relates the bare couplings $\rho_{\rm eff}$, defined at scale M, to the renormalized ones ρ_R, at scale β^{-1}. This shows that Eq. (10) defines, at one-loop order, the desired relation between renormalized couplings of the effective classical model at scale β^{-1} to those of the quantum model, renormalized by the quantum fluctuations. It is this equation which enables us to obtain the properties of the quantum model from those of the corresponding classical ones. In the following, we shall be interested in the correlation length of the quantum model so that we need the expression for the classical one. To this end, we have calculated the two-loop β functions for the classical $O(3)\otimes O(2)/O(2)$ NLσ model and found for the correlation length at the corresponding leading order

$$\xi_{\rm cl}=\mu^{-1}C_\xi^{\rm PV}\sqrt{(1-\alpha)g}\,\exp\{-\tfrac{1}{8}[(1-\alpha)G(\alpha)-G(0)]\}\exp[2\pi G(\alpha)/g]\,,\quad(11)$$

where $g=2/\rho_1$ and $\alpha=1-\rho_3/\rho_1$ are renormalized couplings defined at the scale μ, and $G(\alpha)$ is given by

$$\begin{aligned}&G(\alpha)=2+2(1-\alpha)(\mathrm{arctanh}\sqrt{\alpha})/\sqrt{\alpha},\quad 0\le\alpha<1\,,\\&G(\alpha)=2+2(1-\alpha)(\arctan\sqrt{-\alpha})/\sqrt{-\alpha},\quad \alpha\le 0\,.\end{aligned}\quad(12)$$

Finally, $C_\xi^{\rm PV}$ is a constant, independent of α, which depends on the regularization scheme, Pauli-Villars (PV) in our case [20]. Using (8), (10), and (11) our final expression for the correlation length of the quantum $O(3)\otimes O(2)/O(2)$ NLσ model is

$$\xi_{\rm quant}=C_\xi^{\rm PV}\frac{\bar{\lambda}}{(k_BT)^{1/2}}\left(\frac{2(1-\bar{\alpha})}{\bar{\rho}_1}\right)^{1/2}\exp\{-\tfrac{1}{8}[(1-\bar{\alpha})G(\bar{\alpha})-G(0)]\}\exp\left(\frac{2\pi\bar{\rho}_1G(\bar{\alpha})}{2k_BT}\right),\quad(13)$$

where

$$\bar{\lambda}=\hbar\bar{c}_1\exp\left[-\frac{1-\bar{\alpha}}{16\bar{\alpha}}[(1-\bar{\alpha})G(\bar{\alpha})-G(0)]\left(1+\frac{\ln(\bar{c}_3/\bar{c}_1)^2}{1-(\bar{c}_3/\bar{c}_1)^2}\right)\right].\quad(14)$$

The renormalized spin-wave velocities and spin-stiffness constants are to be understood in natural units. Equation (13) is the *general* expression for the correlation length of frustrated quantum antiferromagnets with zero net magnetization. It contains no adjustable parameter and depends only on the renormalized couplings $\bar{c}_1$, $\bar{c}_3$, $\bar{\rho}_1$, and $\bar{\alpha}$. These constants have to be taken, as argued by CHN, as phenomenological input parameters, which can be obtained either from experiment or spin-wave calculation, for example. In Eq. (13), $\bar{\lambda}\beta$ is nothing but the matching scale between quantum and classical renormalized regimes.

In our expression for $\xi_{\rm quant}$, the preexponential factor behaves as $1/\sqrt{T}$ and differs from that of both Heisenberg, i.e., $N=3$, ferromagnets where it scales as $\sqrt{T}$ and of Heisenberg collinear antiferromagnets where it turns out to be temperature independent. Apart from the nontrivial α and c_3/c_1 dependence, Eq. (13) would be qualitatively that of a $N=4$ collinear quantum antiferromagnet, a result which is to be compared with the $N=4$ universality class found in $D=2+\epsilon$ for the magnetic phase transition of the corresponding classical model [16,17].

To gain a better insight on how (13) applies, we consider the problem of ^{3}He adsorbed on a graphite substrate which is thought to be modeled by a Heisenberg $S=\frac{1}{2}$ model on the triangular lattice. Depending on coverage, the interaction may be either ferromagnetic or antiferromagnetic [11]. Kopietz *et al.* [21] have recently calculated the correlation length in the ferromagnetic case, for which they found

$$\xi_F = a(0.039\pm0.013)S^{-1/2}(T/RJS^2)^{1/2} \times\exp(2\pi RJS^2/T), \quad R=\sqrt{3}. \tag{15}$$

Taking the bare values as given by Dombre and Read [15] for the spin-stiffness and spin-wave velocities in Eq. (13), we obtain $\bar{c}_1=3\sqrt{3}JSa/2$, $\bar{c}_3/\bar{c}_1=\sqrt{2}$, $\bar{\rho}_1=\sqrt{3}JS^2/4$, and $\bar{\alpha}=-1$. Using Eqs. (13) and (14) we find for the antiferromagnetic case

$$\xi_{\rm AF}=a(3.48)C_\xi^{\rm PV}\sqrt{RJ/T}\exp(0.64\,2\pi RJS^2/T)\,. \tag{16}$$

The precise value of $C_\xi^{\rm PV}$ is not known at the moment; however, it is reasonable to think that it is of the same order of magnitude as that of the ferromagnetic case. As it can be readily seen $\xi_{\rm AF}$ is considerably smaller than $\xi_{\rm F}$. This is the consequence of the frustration. This effect might be even stronger if instead of taking the bare values for the coupling constants, we take the renormalized ones as given by some improved spin-wave theory. Such a behavior could be, in principle, detected in experiments. Let us emphasize that the measurement of $\xi_{\rm quant}$ should provide a test for the existence of long-range order in quantum canted antiferromagnets.

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