High-Energy Nuclear Quasielastic Reactions: Decisive Tests of Nuclear-Binding/Pion Models of the European Muon Collaboration Effect

L. Frankfurt,[(a),(b)] G. A. Miller, and M. Strikman[(a),(c)]

Institute for Nuclear Theory, HN-12, University of Washington, Seattle, Washington 98195

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The light-cone nucleon momentum distributions obtained from nonrelativistic spectral functions or given by nuclear-binding/pion models are often used to analyze high-Q^2 quasielastic and deep-inelastic (e,e') reactions. We demonstrate that in such models the presence of non-nucleonic components causes the scattering from forward and backward moving target protons to be significantly different. Other models do not have this property. The sensitivity of current $(e,e'p)$ and (p,pp) color transparency experiments is sufficient to observe these differences.

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The forthcoming high-Q^2 experiments designed to investigate color transparency in $(p,2p)$ [1] and $(e,e'p)$ [2] reactions will also measure the nucleon momentum distribution at much higher energies than any previous experiment. Naively they allow [3] the measurement of the nucleon light-cone density matrix $\rho_A(y,k_t)$ and closely related quantities needed to describe various high-energy phenomena, including the European Muon Collaboration (EMC) effect and high-Q^2 (e,e') reactions. Our notation is that y/A is the fraction of the nuclear momentum (energy minus the z component of momentum) carried by a single nucleon.

The aim of this Letter is to determine experimental observables sensitive to the qualitative features of the light-cone density matrix. In particular, we focus on those relevant for understanding the nuclear dependence of deep-inelastic lepton scattering discovered by the EMC: The structure function (per nucleon) of a bound nucleon is some 15% or so smaller than that of a free nucleon for $x_{Bj}\sim 0.5$. For such kinematics valence quarks provide the dominant contribution to the cross section. Since the structure function is a measure of the light-cone momenta carried by partons, the EMC measurement indicates that momenta carried by valence quarks is depleted. A model of the nuclear light-cone density matrix is required to interpret this effect. One class of such models is the binding/pion model in which the depletion phenomena is interpreted in terms of energy and momentum lost by the nucleon. In such models, the nuclear binding and correlation effects cause the energy of a nucleon to be less than its mass. We show that this effect is measurable in $(p,2p)$ and $(e,e'p)$ reactions as a kinematic shift in ρ_A.

We now discuss the data analysis. At high energies the cross section of the $hA\to h'N_f+(A-1)^*$ reaction can be written as [4]

$$\frac{d\sigma}{(dp_{h'}/E_{h'})(d^3p_{N_f}/E_{N_f})}=\rho_A(p_{\rm res})\frac{1}{2\pi}\frac{d\sigma}{dt}(s'=(p_{h'}+p_{N_f})^2,t)\frac{[s'-(m_{h'}+m_N)^2]^{1/2}[s'-(m_{h'}-m_N)^2]^{1/2}}{2p_h m_A/A}, \qquad (1)$$

where $\rho_A(p_{\rm res})$ is the nucleus spectral function, and $p_{\rm res}$ is the four-momentum of the residual nucleus. Introducing light-cone variables $v_\pm=v_0\pm v_3$, v_t for the momenta involved (the projectile direction is taken as the 3 axis), and defining $p\equiv p_A-p_{\rm res}$ we can write

$$s'=m_h^2+m_N^2+(p^2-m_N^2)+p_{h+}p_-+p_{h-}p_+$$
$$\approx m_h^2+m_N^2+p_{h+}p_-\left[1+\frac{p_+}{p_-}\frac{m_N^2}{p_{h+}^2}+\frac{p^2-m_N^2}{p_{h+}p_-}\right]. \qquad (2)$$

The ratio $p_+/p_-\approx 1$ so the second term in the bracket can be neglected. We also find, using the independent-pair approximation, that the small term $(p^2-m_N^2)/p_{h+}p_-$ is the same for forward and backward moving nucleons. Thus at large energies $s'\approx m_h^2+m_N^2+p_{h+}p_-$ so that this reaction probes the dependence of the spectral function on $p_{{\rm res}_-}$ and $p_{{\rm res}_t}$ but not on $p_{{\rm res}_+}$. Moreover, in a realistic experimental setup (two-arm spectrometer) the resolution in $(p_A-p_{\rm res})_-$, $p_{{\rm res}_t}$, remains high at large energies, while the resolution in $p_{{\rm res}_+}$ becomes poor [5]. Thus the measurement of the light-cone projection of $P_A(p_{\rm res})$,

$$\rho_A(y,k_t)=\int P_A(p_{\rm res})\delta(p_{{\rm res}_t}+k_t)\times\delta(y-A(p_A-p_{\rm res})_-/m_A)d^4p_{\rm res}, \qquad (3)$$

is possible. Here y is the light-cone fraction carried by the interacting nucleon. The scale is chosen so that y varies between 0 and A.

The quantity $\rho_A(y,k_t)$ is determined from the cross section of Eq. (1) (or from a similar expression for a lepton beam) only if it is a good approximation to neglect final-state interactions (FSI). We therefore discuss ways to minimize the effects of FSI. The high-Q^2 experiments of Refs. [1] and [2] have the specific kinematic advantage that the projectile and final hadron(s) can have only elastic reaction with nucleons in the target. At high energies, forward scattering dominates. Thus elastic interactions can change the transverse component of p_t of the momentum of the projectile and emerging particle(s), but not the plus component. One can estimate the change in the plus-momentum fraction z of the fast particle of momentum p_h in elastic collisions. Applying conservation of momentum and taking the angular distribution to vary as $e^{2/3t\langle r_N^2\rangle}$ lead to the result

$$\delta z \simeq -\langle t\rangle/2p_h m_N \simeq (\tfrac{2}{3}\langle r_N^2\rangle 2p_h m_n)^{-1} \simeq (5\times 10^{-2}\,\mathrm{GeV})/p_h\,, \qquad (4)$$

so that δz is proportional to the positive energy loss. Using $\langle r_N^2\rangle$ as the square of the electromagnetic radius of the nucleon accounts for the observed forward angular distribution of pp scattering. Thus the change in longitudinal momentum caused by rescattering seems fairly small. If the effects of color transparency were to be observed, δz would be even smaller. Note that the relevant values of p_h can be varied experimentally, so that one can reduce the effects of δz in a controlled manner. Another experimental technique to restrict the effects of final-state interactions in hadronic reactions is to constrain the sum of the transverse momenta of the emerging projectile and knocked-out proton to be less than the Fermi momentum. This does not reduce the statistical accuracy of the experiment very much since the vast majority of the events involve particles of low transverse momenta. In any case, one expects that the effects of final-state interactions are smallest for the lightest nuclear targets. This is not a limitation, since the effects we discuss below are large even for the deuteron.

Even if a particle experiences no change in z as it interacts with the nuclear medium, one generally expects a loss of flux due to absorption into other channels. This absorption effect modifies Eq. (1) by multiplying the quantity $\rho_A(y,k_t)$ by a factor that is essentially independent of y and k_t [6]. We are concerned here with examining a ratio [$A(\delta)$ of Eq. (12)] of linear combinations of cross sections, and the influence of absorption disappears in such ratios.

We now turn to the prescription for obtaining ρ_A used in many papers on the EMC effect, and in most papers on high-Q^2 (e,e') reactions. This is to identify $P_A(p_{\rm res})$ with the nonrelativistic spectral function

$$P_A(p_{\rm res}) = \langle A|a^\dagger(\mathbf{p})\delta(H-p^0_{\rm res})a(\mathbf{p})|A\rangle \qquad (5)$$

normalized as

$$\int P_A(p_{\rm res})d^4p_{\rm res} = A\,. \qquad (6)$$

On the other hand, the baryon charge of the nucleus is given by [4]

$$\int yP_A(p_{\rm res})d^4p_{\rm res}\,\theta(y)\theta(A-y) = A\,, \qquad (7)$$

with $y \equiv A(p_A - p_{\rm res})_-/m_A$. Equation (7) can be derived by considering either the Adler sum rule for deep-inelastic scattering off the nucleus or the matrix element of the baryon current at $q=0$. The difference between Eqs. (6) and (7) arises from relativistic effects. We use Eq. (7) here since it is a necessary sum rule.

The cross section of deep-inelastic scattering off the nucleus is expressed through

$$f_N(y) \equiv \frac{1}{A}\int yP_A(p_{\rm res})\delta(y - A(p_A - p_{\rm res})_-/m_A)d^4p_{\rm res} \qquad (8)$$

simply as

$$F_{2A}(x) = A\int f_N(y)F_{2N}\left(\frac{x}{y}\right)dy\,. \qquad (9)$$

In binding/pion models of F_{2A}, the depletion of the F_{2A}/F_{2N} ratio at $x\sim 0.5$ (the EMC effect) is proportional to

$$\eta \equiv 1 - \int_0^A yf_N(y)dy\,. \qquad (10)$$

The quantity η is the light-cone fraction of momentum carried by objects other than nucleons [if the normalization condition (7) is imposed]. Calculations [7] using the spectral functions of Ref. [8] lead to $\eta=0.034$ which would explain a large fraction of the EMC effect. Note that a nonzero value of η causes the function $f_N(y)$ to be peaked at a smaller value than $y=1$.

The new experimental opportunity to measure ρ_A allows one to make the first direct observation of a nonzero value of η from the change in $f_N(y)$ that occurs by replacing the argument y by $2-y$. Since a nonzero value of η causes $f_N(y)$ to be asymmetrical about $y=1$, it is reasonable to compare $f_N(y=1-\delta)$ with $f_N(2-y=1+\delta)$. Thus we rewrite Eq. (10) using baryon charge conservation [Eq. (7)] and taking the upper limit A to be either ∞ or 2. We find

$$\eta = \int_0^\infty (1-y)f_N(y)dy \simeq \int_0^1 \delta[f_N(1-\delta) - f_N(1+\delta)]d\delta\,. \qquad (11)$$

The upper limit of the integral is safely taken as 2 since contributions to η from values of y greater than 2 are completely negligible in any known model.

Equation (11) shows that the dominant contribution to η must arise from nonzero values of δ. Since the measurements of $hA \to h'N + (A-1)^*$ allow the measurement of $f_N(y)$ directly, we calculate the asymmetry

$$A(\delta) = \frac{f_N(1-\delta) - f_N(1+\delta)}{[f_N(1+\delta) + f_N(1-\delta)]/2}\,. \qquad (12)$$

Note that the loss of nucleon momentum to non-nucleonic components is responsible for the nonvanishing of $A(\delta)$. One may use Eqs. (8) and (10) to show that if $\eta=0$, then $f_N(1+\delta) = f_N(1-\delta)$.

We evaluate two limiting cases, $A=2$ and $A=\infty$ (nuclear matter). For $A=2$ we use the Paris wave function

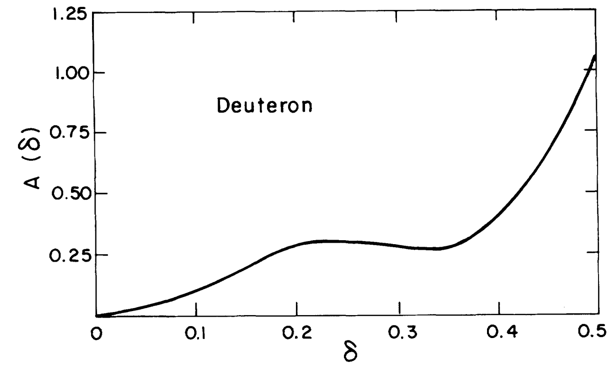

FIG. 1. $A(\delta)$ deuteron target.

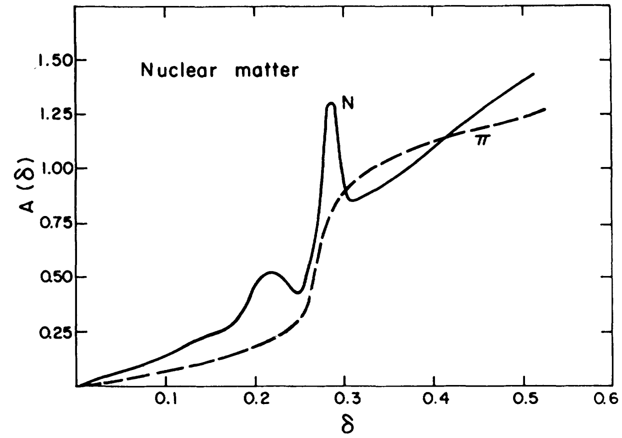

FIG. 2. $A(\delta)$ for nuclear matter. Spectral functions of Ref. [8], solid curve; nuclear matter pion model, dashed curve.

and for nuclear matter we use the spectral function calculated in Ref. [8] and used in the (e,e') reaction [9]. Since the realistic spectral functions of nuclei have about the same dependence on $\mathbf{p}_{\text{res}}$ and p^0_{res} for $|\mathbf{p}_{\text{res}}| \geq 0.3$ GeV/c [10], the results for $A(\delta)$ depend weakly on A for $A > 2$ and for $\delta \gtrsim 0.3$ We checked that renormalization of nonrelativistic functions $P_A(p_{\text{res}})$, $\psi_D^2(k)$ by the factor $m/(m_A - p^0_{\text{res}})$ to satisfy the normalization condition (7) leads to a small change of $A(\delta)$ only [though it significantly increases $f_N(1 \pm \delta)$].

For the deuteron the prescription of Eq. (3) amounts to the assumption that the spectator nucleon is on the mass shell, but the struck nucleon is not. The large asymmetry for the deuteron target shown in Fig. 1 is a consequence of that assumption and the use of the Paris wave function with its value of $\eta = 0.0032$. The results for nuclear matter are shown in Fig. 2. The models we use predict a large asymmetry which would be easy to observe with measurements performed in the appropriate kinematical regime. [The accuracy of a measurement of $A(\delta)$ is about the same as for the relative measurement of $f(1 \pm \delta)$.] The wiggles displayed in Fig. 2 arise from the discontinuity of the spectral function at the Fermi surface.

We emphasize that a deviation of $A(\delta)$ from 0 is not trivial. The binding/pion models are not the only way to construct the nuclear light-cone density matrix. An alternate approach is to use the relativistic light-cone quantum mechanics (LCQM) [11,12] in which the nucleon is taken as on the mass shell. Then the nucleon energy is always larger than its mass, even though the nucleus is a bound state. In particular, it is straightforward to demonstrate that $A(\delta) = 0$ if the light-cone wave function of the deuteron contains a two-nucleon component only [11]. For heavy nuclei, the application of light-cone quantum mechanics [4] leads to a very small but nonzero $A(\delta) < 0$ for $\delta \geq 0.5$ due to contributions of short-range correlations between three or more nucleons. Similarly, one obtains a very small positive $A(\delta)$ for small δ. Thus, at present, models derived from LCQM predict no substantial asymmetry.

We note that the asymmetry of Figs. 1 and 2 is of the same sign as that expected from the influence of final-state interactions shown in Eq. (4). This could cause a misinterpretation of the results. However, one can check that an experimental measurement of $A(\delta)$ is meaningful by seeing if it changes with Q^2, choosing a lighter nuclear target, or making a cut on the transverse momentum of the outgoing system.

Another model using conventional nuclear dynamics, closely related to the spectral function model, is the pion model [13,14]. One assumes that the NN interaction can be described by meson exchanges, and that the nucleons and pions carry the entire momentum of the nucleus. Thus the light-cone fraction carried by nucleons is degraded by the pionic effects and an asymmetry $A(\delta)$ arises.

To estimate $A(\delta)$ in this model one can consider emission of pions as a perturbation, so that in a first approximation nucleons carry the entire momentum of the nucleus

$$f_N^{(0)}(y) = \int d^3k\, n_A(k)\,\delta\left(y - 1 + \frac{k_3}{m_N}\right). \tag{13}$$

The nuclear density $n_A(k)$ is that of Ref. [8]. We define $f_\pi(y)$, the pion light-cone distribution, so that

$$n_\pi = \int_0^\infty f_\pi(y)\,dy \tag{14}$$

is the number of pions, and

$$\eta_\pi = \int_0^\infty y f_\pi(y)\,dy \tag{15}$$

is the light-cone momentum carried by pions. In this model the pion accounts for all of the non-nucleonic degrees of freedom so that $\eta = \eta_\pi$.

Then one can use the notion that a nucleon carrying a momentum fraction $z + y$ emits a pion of momentum z and ends with y to obtain the distribution function $f_N(y)$.

The result is

$$f_N(y)=f_N^{(0)}(y)(1-n_\pi)+\int_y^\infty f_N^{(0)}(z+y)f_\pi(z)dz\,. \quad (16)$$

The sum rules for the total momentum and baryon charge are satisfied by Eq. (16).

To compare Eq. (16) with results of the calculation in the first approach we fixed the parameters of the pion model [14] ($R=0.9$ fm and $g'=0.6$) to reproduce the result of the first calculation for $\eta_\pi=0.034$. One can see from Fig. 2 that this model leads to values of $A(\delta)$ comparable to those of the spectral-function model. Thus, $A(\delta)$ is computed to be measurable in two distinct versions of the binding/pion model.

It is worth remembering that fitting of the EMC effect requires $\eta_\pi\simeq0.05$ for Fe [4]. If, however, one requires consistency of the model with the Drell-Yan data [15] for the ratio of antiquark distributions in nuclei and deuteron, $\eta_\pi\lesssim0.015$. Values of η_π in this range can be obtained by changing g' [14,16].

Our calculation indicates that an experiment that could measure $A(\delta)$ with an accuracy of a few percent [5] would be sensitive to values of η_π as low as 0.01.

We have demonstrated that forthcoming high-Q^2 color transparency experiments have several uses. These experiments are also able to discriminate between several models of the EMC effect, place limits on mesonic components of nuclear wave functions, and, more generally, shed light on relativistic treatments of nuclei.

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[a]On leave from Petersburg Nuclear Physics Institute, Tomsk 634 050, Russia.

[b]Present address: Physics Department, Michigan State University, East Lansing, MI 48824.

[c]Present address: Physics Department, Pennsylvania State University, University Park, PA 16802.

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