Double Bremsstrahlung

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The cross section for double bremsstrahlung differential in the radiated photon energies and angles has been measured for 70-keV electrons on targets of Al, Cu, Ag, Tb, and U for photons radiated at $\pm 45°$ to the incident beam for photon energies in windows from 10 to 30 keV. In contrast with previous experiments at $\pm 90°$, the results are in reasonable agreement with the relativistic first Born approximation at lower Z. However, the results exhibit a Z dependence which disagrees with the first Born Z^2 dependence, suggesting the need for consideration of a second Born approximation.

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Double bremsstrahlung is a quantum electrodynamic process in which two photons are radiated simultaneously in the scattering of an electron by an atom. This process was first mentioned by Heitler and Nordheim [1] who estimated that the cross section would be about 137 times smaller than that of single bremsstrahlung, or essentially smaller by a factor of the fine-structure constant due to the emission of the second photon. While the two-photon process is too small to make much contribution to the production of radiation, it is nevertheless especially interesting as an example of a quantum radiative process for which, unlike single bremsstrahlung, there seems to be no prescription for a classical calculation of the cross section. Although recently there has been much interest in two-photon and multiphoton processes, the radiative two-photon process has been studied in only two experiments [2,3], both in a $\pm 90°$ geometry. In both experiments, a significant discrepancy, as large as 2 orders of magnitude, with theories has been observed.

The first measurement of the cross section was made in 1985 by Altman and Quarles [2]. They measured the cross section for two-photon emission for 75-keV electrons on thin targets of silver, terbium, gold, and uranium. The electron was not observed, so a differential cross section integrated over the unobserved electron was measured. Altman and Quarles also evaluated the theoretical cross section by numerical integration of the very complicated formula for the cross-section differential in the two photon energies and angles and the electron angles worked out by Smirnov [4] in the relativistic first Born approximation. The experimental result for gold was about 300 times the computed theoretical value. It was not clear whether the discrepancy was due to an error in the theoretical formula, the numerical integration of the formula, or the experimental data. An independent evaluation of the theory in the nonrelativistic Coulomb approximation was then provided by Veniard, Gavrila, and Maquet [5] and also by Florescu and Djamo [6]. The result for the particular geometry of Ref. [2] was larger by about a factor of 3 than the relativistic Born approximation but was still about a factor of 100 smaller than the observed cross section. These nonrelativistic calculations when evaluated in the Born rather than the Coulomb approximation were close to the relativistic result computed in Ref. [2], suggesting that neither the formula of Smirnov nor the integration over the unobserved electron was the problem.

Because of the large measured cross section, it was imperative to look for alternative processes that could produce two coincident photons and that might dominate the double bremsstrahlung process. This was undertaken by Lehtihet and Quarles [7–9]. A wide variety of background processes were identified and considered; finally, it was suggested that the large experimental cross section could have arisen from a rather complicated process. Incident electrons could elastically scatter in the thin target into one of the Mylar vacuum windows in front of one of the photon detectors. The electron would lose energy in the Mylar window which was a thick target for 75-keV electrons. The electron could produce double bremsstrahlung in the thick target. Because of the particular geometry, with the two detectors at $\pm 90°$ to the incident beam, each detector could see each Mylar window. Thus, it was argued that the experiment of Ref. [2] measured the yield from double bremsstrahlung for 75-keV electrons in the thick Mylar window. The Z^2 dependence reported in Ref. [2] was due to the initial elastic scattering in the target.

In a recent Letter, Hippler [3] has reported two-photon cross sections for 8.82- to 12.5-keV electrons on argon, krypton, and xenon. There is a significant enhancement over the prediction of the nonrelativistic Coulomb cross section of Ref. [5] ranging from about a factor of 2 for 8.82-keV electrons and photon energies at 2.8 and 3.2 keV to about a factor of 100 for xenon for photon energies of 2.8 and 1.2 keV. Generally, the data are much higher than theory for lower photon energy and are near to theory (and in two cases even lower) when one photon has a higher energy. The geometry was the same as that of Ref. [2] and thus it seems likely that the results are subject to much of the same background processes. Using a somewhat different thick-target model, Hippler has estimated that the thick-target effect suggested by Quarles and Lehtihet [7] is not the explanation for the observed enhancement in this case. Since the thick-target effect depends critically on the particulars of the geometry, and these details have not yet been published, it is not possible to do an independent calculation of the effect. Still, it is not unreasonable that the effect would be different for a different energy range, and unlike the data of Ref. [2], some of the points lie rather close to the theory or even below it, suggesting a smaller thick-target effect.

The uncertainty in the contribution of various background processes which can clearly be serious in a $\pm 90°$ geometry led us to undertake a new experiment in which the two photon detectors could not see each other. Here, we report the results of a new measurement of the two-photon emission cross section for the emission of photons at $\pm 45°$ to the incident electron beam. This new experiment is about 2 orders of magnitude more sensitive than the experiment of Ref. [2], thus permitting the measurement of much smaller cross sections. This new geometry still has some potential background problems which are discussed below, but essentially eliminates the thick-target process in the detector window and several other background processes which have plagued the earlier experiments.

Bombarding electrons are provided by an electron accelerator tuned to a nominal energy of 70 keV. Targets are positioned normal to the incident beam within a small scattering chamber which doubles as a Faraday cup for charge collection. The targets used are thin films of Al, Cu, Ag, TbF_3, and UF_4 of approximately 50 $\mu g/cm^2$ thickness, which is thin enough in each case to ensure single-collision conditions and renders photon attenuation and electron energy loss negligible. All of the targets have a 15-$\mu g/cm^2$ carbon backing and the thicknesses are known to roughly 10%. The intensity of the electron beam is monitored by a current integrator and is held at approximately 0.1 nA. This is high enough to yield a coincidence rate which is adequate to provide reasonable data collection times and low enough to furnish an acceptable real-to-accidental ratio.

The experimental coincidence setup has been described in detail previously [10]. Photons are detected at $\pm 45°$ to the incident beam in two collimated and planar HpGe detectors. This geometry was chosen to optimize the solid angles and to eliminate cross talk between the detectors. Individual events consist of the delay time Δt between the two detected photons and their respective energies k_1 and k_2. Software was developed for processing the data to obtain a two-dimensional energy array of the net coincidence probability with the statistical error. To do this, the total events from a run are sorted to produce two energy arrays corresponding to events whose delay times fall inside and outside of the real coincidence timing peak region. A net coincidence energy array is computed by subtraction of the two energy arrays, appropriately normalized.

Energy calibration and testing of the coincidence system were performed using two different sources of well-known two-photon production. First, a calibrated ^{133}Ba radioactive source was used to provide an energy calibration and to determine solid angles; and, second, the $K\alpha$-L coincidence cascade from the single K-shell ionization of Tb was used as an energy calibration and to test the accuracy of the system.

The ^{133}Ba source is well suited as a calibration mechanism since it produces a high rate of coincident photons over a wide range of energies [11]. The source is placed at the target location, and data are collected for the appropriate energy window. The product of the detector solid angles and efficiencies is then determined using the net rate for a particular photon energy combination along with the tabulated probability per disintegration of producing the two photons in coincidence. The solid-angle product is then computed using previously determined detector efficiencies. In this case the 31-31-keV coincidence was utilized.

An *in situ* measurement of the $K\alpha$-L cascade of Tb provides an excellent means of both calibrating and testing the system. The advantage is that the $K\alpha$-L data are taken under identical experimental conditions as the data for double bremsstrahlung, providing an absolute scale for the measured cross sections and reducing the major uncertainty to a statistical one. To test the system, the cross section has been determined for the $K\alpha$-L coincidence. For the present setup, a result of 5.6 ± 1.1 mb was obtained, which agrees well with the theoretical value of 5.5 ± 0.8 mb.

The differential cross section for double bremsstrahlung can be determined from

$$\frac{d^4\sigma}{dk_1\,dk_2\,d\Omega_1\,d\Omega_2} = \frac{N_c}{N_0 t\,\Delta k_1 \Delta\Omega_1 \varepsilon_1(k_1) \Delta k_2 \Delta\Omega_2 \varepsilon_2(k_2)},$$

where N_c is the number of coincidences, N_0 is the number of incident electrons, t is the target thickness, $\Delta k_{1,2}$ are the detector energy windows, $\Delta\Omega_{1,2}$ are the detector solid angles, and $\varepsilon_{1,2}$ are the energy-dependent detector efficiencies. The product $\Delta\Omega_1 \Delta\Omega_2$ is determined using a measured coincidence rate from the ^{133}Ba source. Δk_1 and Δk_2 are selected to define an energy window $\Delta k_1 \Delta k_2$ from which a two-photon rate is obtained for determining the absolute cross section. Detector efficiencies have been determined in a separate experiment.

The results are summarized in Fig. 1 where the two-photon emission cross section is plotted versus atomic number. The cross section is divided by Z^2 and is in units of $\mu b/(keV^2 sr^2)$. Each data point gives the result for an energy window of width $\Delta k_1 = \Delta k_2 = 10$–$30$ keV centered at the average value of $\langle k_1 \rangle = \langle k_2 \rangle = 20$ keV. The 10–30-keV region was chosen in order to obtain acceptable statistical errors, and because the cross section is not expected to vary significantly over this energy range. The error bars shown represent the 1 standard deviation statistical error in the number of true coincidences. The systematic errors in target thickness, charge collection, solid angle, and detector efficiency are small compared to the statistical error in the experiment. Subtraction of a

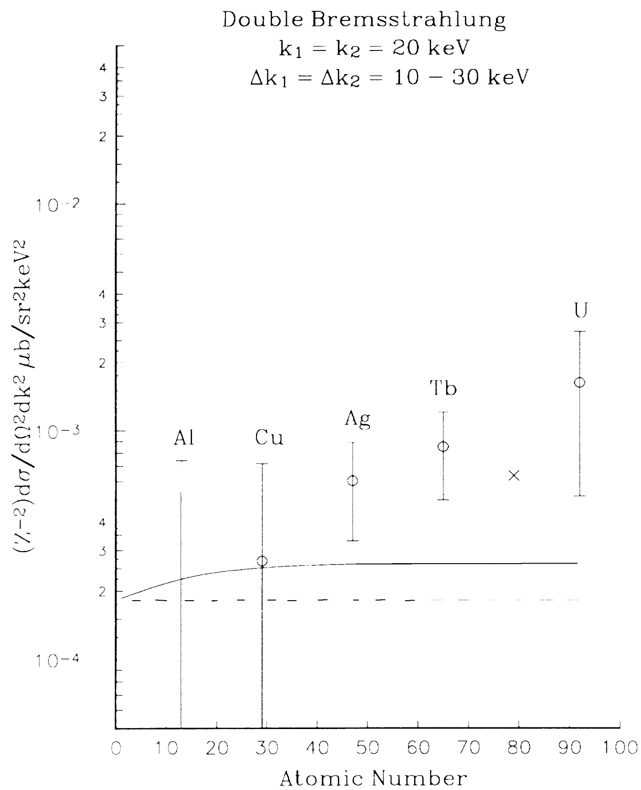

FIG. 1. The cross section for double bremsstrahlung divided by Z^2 in units of μb/(sr^2keV^2) for 70-keV electrons for two photons radiated at $\pm45°$ within an energy range of 10 to 30 keV. Each data point is the result for an energy window of width $\Delta k_1 = \Delta k_2 =$10–30 keV centered at the average value of $k_1 = k_2 =$20 keV. The data points are shown plotted on a scale of cross section vs Z. The relativistic first Born approximation is shown as the dashed line; the relativistic first Born approximation including the Elwert factor is the solid line; and the nonrelativistic Coulomb approximation for the conditions of Ref. [2] ($\pm90°$ geometry, 75-keV electrons on Au) is the "×" at $Z=79$.

target-out background was found unnecessary since spectra collected for the target backing of 15 μg/cm^2 C displayed no measurable two-photon effect.

The dashed line is a numerical integration of the fully differential formula of Ref. [4] and is the relativistic first Born approximation for 70-keV electrons and the geometry of $\pm45°$ to the incident beam direction. The solid line has the Elwert correction factor included [12]. The Elwert factor has been useful in single bremsstrahlung in correcting the Born approximation (the Bethe-Heitler equation) especially at larger radiated photon energy. It was originally derived by comparison of the nonrelativistic Born and Coulomb approximations for the one-photon bremsstrahlung cross section. As can be seen, inclusion of the Elwert factor introduces a small additional Z dependence. The "×" at $Z=79$ is the nonrelativistic Coulomb approximation of Ref. [5] for 75-keV electrons on Au and the original $\pm90°$ geometry. The nonrelativistic approximation is therefore expected to differ from the corresponding 70-keV prediction for the present geometry of $\pm45°$ and is shown mainly to suggest the order of magnitude of the nonrelativistic prediction.

The experimental result shown for Al is essentially zero with an error of 7.4×10^{-4}. Within 1 standard deviation error, both Al and Cu agree with the first Born approximation. The data show an increasing disagreement with the Born approximation as Z increases, which is expected since the approximation becomes less valid for higher atomic numbers. A linear fit of the data points according to a power law gives a Z dependence of $Z^{3.2\pm1.4}$. Thus while a Z^2 dependence cannot be rule out, the data indicate a Z dependence that suggests that a second Born approximation may be needed to describe the process adequately. We have seen preliminary results of a nonrelativistic calculation for this new geometry [13] which tend to follow the observed Z dependence but are lower in magnitude than the data.

Although the $\pm45°$ geometry was especially chosen to eliminate cross talk between the two detectors and to allow for maximum solid angle, it was realized after the experiment was nearly complete that this geometry may also suffer from a special background which it was feared could dominate the double bremsstrahlung process. An incident electron can elastically scatter (Möller scattering) from a target electron producing two correlated electrons at about half the incident energy preferentially at $\pm45°$. Each electron could then produce thick-target bremsstrahlung in the Mylar windows which would be indistinguishable from the correlated photons from double bremsstrahlung in the target.

Before interpreting the current data as double bremsstrahlung, it was decided to measure this effect directly by enhancing the production of thick-target bremsstrahlung in the windows by replacing the Mylar with a high-Z window. Since the thick-target bremsstrahlung spectrum (single photon) is well known and scales as Z, the two-photon coincidence yield should increase by the ratio of Z^2 of the high-Z window to that of Mylar ($Z\approx6$). Tantalum foils of approximately 0.38 mil thickness were placed in front of each Mylar window, and were collimated to eliminate any effect that might be produced by electrons scattered into the windows from the chamber walls. The measured real two-photon rate with the Ta windows was corrected for the significant difference in photon attenuation and scaled to that expected from Mylar. It was concluded that real coincidences produced in the Mylar due to this effect could account for no more than 10% of the observed rate.

It is difficult to compare the results of the present experiment directly with those of Ref. [3] because of the different geometry and energy range. However, two points are worth noting. First, the present data with both photons radiated with an average of 20 keV are for the case where 30% to 80% of the available energy is radiated. The data of Ref. [3] cover a similar radiated energy range. Second, although we have presented the data averaged over the 10- to 30-keV range, we examined the lower radiated energy range separately and did not see any evidence for an enhancement in cross section reported by Ref. [3]. Thus, while we do not report photon energy dependence per se, our data are not consistent with the kind of energy dependence reported by Ref. [3]. On the other hand, if one averaged the data of Ref. [3] over the reported energy range, the average would be dominated by the lowest energy point and the average of the data would disagree significantly with the theory.

We attempted to measure the cross section at a beam energy of 150 keV to investigate the electron energy dependence of the double bremsstrahlung cross section. However, with the present target chamber, this effort was unsuccessful. A large real coincidence rate, about 50 times larger than that observed at 70 keV, was detected, and it was found to be independent of the target and even present with no target in place. This was in marked contrast with the 70-keV case where no background was measurable. It is believed that at a higher beam energy, there is a much greater probability for correlated photons to arise from multiple interactions of the beam stopping in the target chamber, which also acts as a Faraday cup. This implies that if the detected photon energies are much less than the beam energy, this type of background could create problems even when the detectors are shielded from each other.

In conclusion, absolute cross sections have been determined for the production of double bremsstrahlung by 70-keV electrons on thin targets for a range of Z values, in which the photons were detected at $\pm 45°$ to the incident beam. In contrast to earlier experiments at $\pm 90°$ geometry, the results are in reasonable agreement with the relativistic first Born approximation for the lower Z targets. However, the measurements tend to deviate from the first Born Z^2 behavior and appear to vary as $Z^{3.2 \pm 1.4}$. This deviation suggests the need for consideration of the second Born approximation in the theory in order to describe the process.

A further study of the behavior of the cross section as photon angle and incident beam energy are varied is continuing. The target chamber is being redesigned to allow for the addition of two more detectors and an external Faraday cup. This will increase the real data collection rate by about 6 times, allow for the simultaneous study of different photon angles and a broader range of photon energies, and permit measurements at a higher beam energy.

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