S-Matrix Bootstrap of a Scalar Higgs Boson

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We use the N/D formalism to perform a bootstrap calculation of a neutral, scalar resonance coupled to longitudinal W,Z gauge bosons. We find that a consistent solution only exists for a mass value very close to 2.01 gauge-boson masses. The solution does not constrain the couplings and requires large scattering lengths.

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Because of complications associated with the renormalization of scalar fields, theoretical speculation often returns to the possibility that the Higgs scalar of the standard model is a composite system rather than an elementary particle [1]. This possibility has also been raised in discussions of the scattering of gauge bosons. Using language borrowed from S-matrix studies of pion-pion scattering, it was found that unitarization of amplitudes for gauge-boson scattering in the heavy Higgs boson limit produces amplitudes without sharp peaks [2] which are difficult to interpret. Based upon the tools of S-matrix theory, the question of whether or not a particle is generated dynamically involves the "bootstrap hypothesis" [3,4]. The S-matrix bootstrap has never, to our knowledge, successfully calculated the mass and width of any known particle. Nevertheless, it is interesting to explore the conditions under which a Higgs scalar arises from an N/D bootstrap calculation.

The question of the Higgs boson mass arises in the context of the symmetry-breaking mechanism responsible for the weak interactions. The authors of Ref. [2] discussed the strength of gauge-boson interactions as a function of the Higgs boson mass. If the Higgs boson mass is close to W,Z threshold then arguments based upon perturbative field theory suggested that the weak interactions would remain weak at energies approaching 1 TeV. If, conversely, the Higgs boson mass is above a few hundred GeV then the gauge-boson interaction becomes strong at energies above the threshold for scattering W,Z pairs. Large enough Higgs boson masses could in fact induce additional low-energy, broad resonances, or even bound states, in the Higgs channel. Using an approximation to an N/D bootstrap equation [3] Lee, Quigg, and Thacker found no restriction on the possible range of Higgs boson masses, but did find a relationship between Higgs boson mass and width.

We have accordingly performed a detailed bootstrap calculation as suggested in Ref. [2]. Our results are at variance with the estimates made in that reference. We conclude that the S-matrix bootstrap predicts a unique Higgs boson mass slightly above the W,Z scattering threshold with no constraint on the width.

We follow the authors of Ref. [2] by working in the approximation that ignores channels involving transversely polarized vector particles. This is a high-energy approximation that requires energies large compared to the W,Z masses for validity. Our result, that the bootstrap works only near threshold, cannot therefore be taken as a bootstrap prediction of the mass of a composite Higgs scalar. We only assert that the bootstrap calculation excludes solutions at energies high enough for the transverse channels to be decoupled.

We sketch the assumptions, somewhat different from those in Ref. [2], that underlie the calculation and then discuss the calculation and the results. A more complete version of the work reported here will be published elsewhere.

The longitudinal components of the W and Z particles are treated as a triplet of scalar particles with a common mass of about 85 GeV. The particles are taken to be members of a weak-isospin triplet. The zeroth approximation to the scattering amplitude is a set of scalar, isoscalar poles, one in each of the three possible elastic channels (called s, t, and u channels). The common mass and residue of the poles are to be determined by a "bootstrap" condition. We assume that the mass is above the scattering threshold so that it corresponds to a resonance rather than a bound state.

We write a once-subtracted dispersion relation for the resonating channel (isospin 0, angular momentum 0) partial-wave scattering amplitude. The subtraction constant is the isospin zero scattering length (multiplied by the common mass of the triplet). The dispersion relation accordingly assumes that the scattering amplitude is analytic in a complex energy plane that is cut along the real energy axis. There is a gap in the cut between zero energy and the scattering threshold. Elastic unitarity give the discontinuity across the positive energy cut. Partial-wave projections of the t- and u-channel poles give the discontinuity across the negative energy cut. The dispersion relation is a nonlinear integral equation for the partial-wave scattering amplitude. The solution, if it exists, is necessarily unitary.

We solve the integral equation using methods developed long ago by one of us [4]. We adjust the input pole mass and residue until it agrees with the mass and width of any resonance in the solution. When agreement is reached then the bootstrap condition is satisfied. We find that the existence and width of a solution satisfying the bootstrap condition is critically dependent upon the scattering length. The mass, however, is narrowly constrained.

We let s be the barycentric total energy and

$$\nu=(s-4)/4 \tag{1}$$

the squared barycentric momentum. The W,Z mass is taken as unity [5]. The barycentric energies in the two crossed channels are

$$t=-2\nu(1-z) \tag{2a}$$

and

$$u=-2\nu(1+z)\,. \tag{2b}$$

The starting point of the bootstrap is to create a crossing-symmetric amplitude consisting only of poles at the Higgs boson mass M. The zero-order isospin amplitudes A^I are then [6]

$$A^0=-\alpha M^2[3/(s-M^2)+1/(t-M^2)+1/(u-M^2)]\,, \tag{3a}$$

$$A^1=-\alpha M^2[1/(t-M^2)-1/(u-M^2)]\,, \tag{3b}$$

$$A^2=-\alpha M^2[1/(t-M^2)+1/(u-M^2)]\,, \tag{3c}$$

with α taken to be an arbitrary parameter of order unity.

The next step is to construct unitary partial-wave elastic scattering amplitudes $f^I(\nu)$. Because we are primarily interested in the zero-angular-momentum state we shall generally omit other labels on the f's. The f^I's are analytic in the ν plane with cuts along the real axis except for a gap $-1<\nu<0$. They have the same imaginary parts for negative $\nu\leq -1$ as the partial-wave projections of the A^I. The f^I are normalized to

$$f^I(\nu)=\rho\exp(i\delta^I)\sin(\delta^I),\quad \rho=\sqrt{(\nu+1)/\nu},\quad \nu\geq 0\,, \tag{4}$$

so that according to Eq. (4) the discontinuity across the cut along the positive real ν axis ("the physical region") is simply

$$\mathrm{Im}f^I(\nu)^{-1}=-\rho^{-1}\,. \tag{5}$$

It is well known [7] that arbitrary poles may be added to the f^{-1} functions. Such Castillejo-Dalitz-Dyson (CDD) poles probably represent the addition of more "elementary particles" to the S matrix and therefore should be avoided in a bootstrap calculation.

The partial-wave projections of the t and u poles are analytic functions in the complex ν plane with a cut for $\nu\leq -1$. These projections provide the "driving potential" for unitary partial-wave scattering amplitudes [1,4]. Notice that the I-spin 0 and 2 amplitudes have identical driving potentials.

The f^I are assumed to satisfy once-subtracted dispersion relations. The partial-wave projections of the A^I are defined by

$$B_l^I=\frac{2l+1}{32\pi}\int_{-1}^{1}dz\,P_l(z)A^I(s,t,u)\,, \tag{6}$$

where P_l is the ordinary Legendre polynomial. An important caveat is that the s-channel pole from Eq. (3a) is excluded from the definition of the B's unless M represents a true bound state ($M<2$). This is because the discontinuity across the physical-region cut is uniquely prescribed by Eq. (5). The $l=0$ amplitudes then satisfy the equations ($I=0,2$)

$$f^I(\nu)=a^I+B^I(\nu)-B^I(0)+(\nu/\pi)\int_0^\infty dx\,\rho\sin^2(\delta^I)x^{-1}(x-\nu-i\epsilon)^{-1}\,, \tag{7}$$

where the subtraction constant a^I is the scattering length.

The scattering length, because it is a subtraction constant, encodes the high-energy behavior of the partial-wave amplitude. We find that this constant plays a key role in the bootstrap process. The key role is in fact implicit in Eqs. (3) and (7) which show that the only distinction between the I-spin 0 and 2 amplitudes is in the scattering lengths.

The nonlinear equation (7) is solved by the N/D method of Ref. [1] using the N integral equation of the first paper of Ref. [3]. The result is

$$N^I(\nu)=a^I+B^I(\nu)-B^I(0)+(\nu/\pi)\int_0^\infty dx\,N^I(x)K^I(x,\nu)\,, \tag{8a}$$

$$\mathrm{Re}D^I(\nu)=1-(\nu/\pi)\int_0^\infty dx\,N^I(x)L(x,\nu)\,, \tag{8b}$$

$$\mathrm{Re}f^I(\nu)^{-1}=\mathrm{Re}D^I/N^I\,, \tag{8c}$$

with the definitions

$$K^I(x,\nu)=\left[\frac{x}{x+1}\right]^{1/2}\frac{[B^I(x)-B^I(\nu)]}{(x-\nu)}\,, \tag{9a}$$

$$L(x,\nu)=[x(x+1)]^{-1/2}P/(x-\nu)\,, \tag{9b}$$

where P/x denotes principle value.

Equations (8) were programmed in VAX FORTRAN and run on a MicroVAX 3800. Two programs were written independently and checked against each other. The programs were also checked by substituting a separable kernel obtained by using $1/(x+b)$ in place of $B(x)$ in Eq. (9). Use of the separable kernel permits analytic solutions of the equations for checking the computer solutions. The parameter α was initially taken to be $[M/(246\ \mathrm{GeV})]^2$ to facilitate comparison with Ref. [2].

A successful bootstrap is obtained when $\mathrm{Re}f^{-1}$ (as a function of s) has a simple zero at the input mass with coefficient $1/(3M^2\alpha)$, according to Eq. (3a). Three arbitrary parameters also appear to be available, namely, M^2, $M^2\alpha$, and the scattering length. Our principal result is that there appears to be a unique bootstrap solution at a mass value of about 2.01 W,Z masses.

The bootstrap solution is sensitive only to the scattering length. No resonance appears in the output amplitude unless the scattering length exceeds about 13.5, in units of the W,Z Compton wavelength. A successful bootstrap is obtained using the α parameter of Ref. [2] with a scattering length of about 13.7. We also find solutions for larger values of the α parameter with larger scattering lengths, but with the mass value practically unchanged. The low-energy part of the phase shift is shown in Fig. 1.

Our calculation shows that a singlet spin-zero Higgs boson bootstraps itself in the corresponding spin-singlet elastic scattering amplitude of W,Z's. The bootstrap mass is very close to the scattering threshold, and is insensitive to the assumed coupling strength. The residues of the input and output poles can be made consistent by appropriate choice of the scattering length. If the scattering length is chosen to be sufficiently small then there is no output resonance [8].

The last remark describes the mechanism for keeping the resonance in the I-spin 0 amplitude without also having one in the corresponding I-spin 2 amplitude. The I-spin 2 scattering length must be less than the threshold value for a resonance.

Our result disagrees with the estimates made in Ref. [2] where any Higgs boson mass bootstraped itself. It is, however, quite in accord with intuitive arguments based upon nonrelativistic quantum mechanics, as follows. Particle exchange poles in the t and u channels correspond to Yukawa-like potentials; the inverse mass corresponds to the range of the potential and the residue of the pole corresponds to the strength. Strong, attractive potentials are required for resonances. The s-wave shift, as a function of momentum, must pass through $\pi/2$, the resonance position with a steep slope. The steep slope at $\pi/2$ provides the requisite large residue for the output resonance. The phase shift starts from zero at zero momentum, and it must rise steeply in order to pass through $\pi/2$ with steep slope. The slope at zero momentum is the scattering length which must accordingly be large and the resonance must accordingly be close to threshold [9].

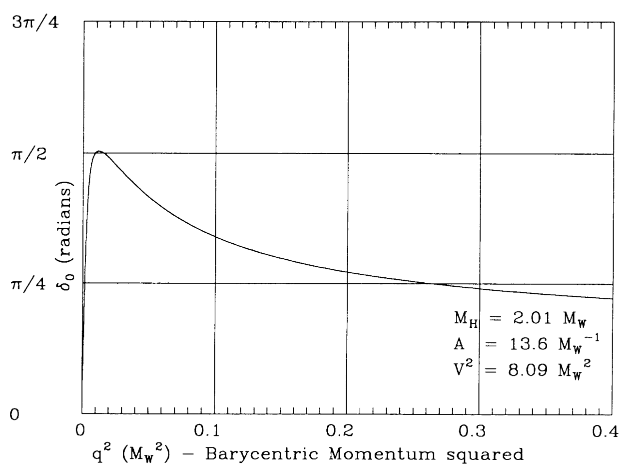

FIG. 1. Isospin 0, angular momentum 0, bootstrap-solution phase shift vs square of barycentric energy, for scattering of longitudinal W,Z's. Residues of t- and u-channel poles are the same as in Ref. [2].

Our calculation uses the weak-isospin formalism in order to facilitate comparison with Ref. [2]. The assumption of isospin invariance does not, however, influence the generality of our conclusions because we are only interested in the zero-charge channels. There are three such channels, two symmetric and one antisymmetric. We assume that symmetry is a good s-channel quantum number. Equation (3) shows that the two symmetric channels have the same driving potential, so the assumed I-spin invariance determines the residue to the s-channel output resonance. This residue influences the value of the scattering length over some small range but has little influence upon the bootstrap mass value.

The use of once-subtracted dispersion relations raises a more delicate question, whether the scattering amplitude might satisfy an unsubtracted dispersion relation. If it does, as would seem to be the case in potential scattering [10], then the scattering length is no longer an arbitrary parameter. The bootstrap equations determine not only the resonance parameters but also the scattering length. This issue remains to be investigated.

Also remaining to be investigated is another consistency condition: Does the scalar input pole demand that a p-wave resonance exists in the antisymmetric channel? At first blush it would appear not; the angular-momentum barrier permits use of a twice-subtracted dispersion relation with an arbitrary "scattering length" to control the existence of an I-spin 1 resonance. This answer may, however, ignore some hidden subtleties.

Our calculation is quite different from that of Hikasa and Igi [11]. Those authors consider the scattering of massless longitudinal W,Z's and write down an N integral equation similar to our Eq. (8a). Their equation for the D function differs essentially from our Eq. (8b) by the addition of a CDD pole [7] which provides two additional parameters for fitting output resonances to input poles. Such a computation corresponds to the input of an elementary Higgs boson and from the bootstrap point of view is devoid of predictive power.

We are also troubled by Hikasa and Igi's application of the N/D formalism to the scattering of massless particles. The elastic scattering amplitude for massless particles does not have a gap between the right- and left-hand cuts along the real axis of the complex s plane (or ν plane, which is equivalent). It is therefore not obvious that one can write a dispersion relation such as our Eq. (7) for the partial-wave amplitudes. The meaning of the N/D decomposition is consequently obscure in that case.

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[1] S. Weinberg, Phys. Rev. D **13**, 974 (1976); **19**, 1277 (1979); L. Susskind, Phys. Rev. D **20**, 2619 (1979).

[2] B. W. Lee, C. Quigg, and H. B. Thacker, Phys. Rev. D **16**, 1519 (1977); further explained in C. Quigg, "Gauge Boson Dynamics," Fermilab Report No. Fermilab-Pub-91/58-T (unpublished). The word "bootstrap" reflects our characterization of this work and does not appear in these references.

[3] G. F. Chew and S. Mandelstam, Phys. Rev. **119**, 467 (1960), and see generally, G. F. Chew, *S-Matrix Theory of Strong Interactions* (Benjamin, New York, 1962). For a later review, see P. D. B. Collins and E. J. Squires, *Regge Poles in Particle Physics*, Springer Tracts in Modern Physics Vol. 45 (Springer, Berlin, 1968), Chap. VI.

[4] J. L. Uretsky, Phys. Rev. **123**, 1459 (1961), corrected in A. W. Martin and J. L. Uretsky, Phys. Rev. **135**, B803 (1964); K. Smith and J. L. Uretsky, Phys. Rev. **131**, 861 (1963); A. M. Saperstein and J. L. Uretsky, Phys. Rev. **133**, B1340 (1964); **140**, B352 (1965).

[5] $\hbar = c = 1$.

[6] These amplitudes differ by inconsequential constant terms from the amplitudes in Ref. [2], Eq. (3.11).

[7] L. Castillejo, R. H. Dalitz, and F. J. Dyson, Phys. Rev. **101**, 456 (1956); F. J. Dyson, Phys. Rev. **106**, 157 (1957); and the last article of Ref. [3].

[8] We are struck by the fact that the bootstrap resonance occurs at a very nonrelativistic energy. Our manifestly relativistic calculations constitute egregious overkill. The same results may be obtainable from a Schrödinger equation with Yukawa potentials [the Fourier transforms of the t and u terms in Eq. (3)] and an attractive core adjusted to give the correct scattering length. The Schrödinger equation, in the spirit of the present calculation, is after all only one more method for generating unitary scattering amplitudes.

[9] More precisely, and nonrelativistically, $\tan\delta/\sqrt{\nu}$ can be represented near the resonance by a simple pole $g/(\nu_0 - \nu)$. Then $\partial\delta/\partial\nu$ at ν_0, the resonance position, has the value $1/g\nu_0^{1/2}$. If g is large enough to make a zero-energy resonance, for example, then δ must pass vertically through $\pi/2$ at zero momentum. The scattering length must accordingly be infinite.

[10] See, e.g., L. I. Schiff, *Quantum Mechanics* (McGraw-Hill, New York, 1968), 3rd ed., p. 344ff.

[11] K. Hikasa and K. Igi, Phys. Lett. B **261**, 285 (1991). We are indebted to Professor Vernon Barger for bringing this work to our attention prior to publication while we were still checking our own results.