Melting of the Abrikosov Flux Lattice in Anisotropic Superconductors

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It has been proposed that the Abrikosov flux lattice in high-T_c superconductors is melted over a significant fraction of the phase diagram. We provide a thermodynamic argument which establishes that the angular dependence of the melting temperature is controlled by the superconducting mass anisotropy. Using a low-frequency torsional-oscillator technique, this relationship has been tested in untwinned single-crystal $YBa_2Cu_3O_{7-\delta}$. The results offer decisive support for the melting proposal.

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It has been proposed [1] that melting of the Abrikosov flux lattice may occur in high-T_c superconductors. The first results presented as evidence for such melting were obtained in a vibrating-reed ($\sim 10^3$ Hz) experiment [2]. However, the samples used exhibited significant pinning, and an explanation in terms of thermally assisted depinning was subsequently found to be more satisfactory [3-5]. It is clear that a weak pinning material offers the best opportunity for observing the melting transition free of the complications associated with pinning. In addition, the straightforward considerations outlined below indicate the desirability of using a low-frequency probe. A low-frequency ($\sim 10^{-1}$ Hz) torsional-oscillator experiment was therefore recently performed [6] on an untwinned single crystal of $YBa_2Cu_3O_{7-\delta}$. The magnetic field was applied along the **c** axis of the crystal, and the damping of the oscillator was measured as a function of temperature. A sharp damping peak was located, whose temperature T_m was found to be related to the applied field B according to

$$B \propto (T_c - T_m)^2 , \tag{1}$$

where T_c is the transition temperature. This was the first report of a transition field varying quadratically with $T_c - T_m$, the dependence predicted [7-9] for flux-lattice melting close to T_c. However, it has been suggested that thermal depinning may eventually also be able to account for these results [10]. The work reported here directly addresses this central issue of depinning versus melting.

Untwinned single-crystal $YBa_2Cu_3O_{7-\delta}$ is thought to be the cleanest available high-T_c material. Previous torque measurements [11] in a field of 1 T exhibited completely reversible behavior in a $\sim$10-K-wide region below T_c. In such a field, the estimated [7-9] melting temperature is approximately 4 K below T_c; i.e., it lies within the reversible region. However, it is known that irreversibility tends to set in at progressively higher temperatures as the time scale of the measurement is decreased [10]. This suggests that the probe time scale should not be significantly less than that employed in the reversible torque measurements (10^1 sec). In addition, on theoretical grounds, long-wavelength displacement fluctuations are expected to dominate the melting process. Both of these considerations indicate that one should use a low-frequency probe when investigating flux-lattice melting. As noted above, using a 10^{-1}-Hz mechanical oscillator a very sharp transition has been reported [6] in untwinned $YBa_2Cu_3O_{7-\delta}$. Our goal in this work was to determine whether that transition is controlled by the equilibrium mean-field free energy.

Let us therefore consider the free energy density F of an extreme type-II superconductor, i.e., one in which the penetration depth λ is much greater than ξ, the coherence length. In intermediate fields, $\phi/\lambda^2 \ll B \ll \phi/\xi^2$, one has [12]

$$F - \frac{B^2}{8\pi} = \frac{\phi_0 B}{32\pi^2\lambda^2} \ln \frac{\phi_0 \eta}{2\pi\xi^2 B} , \tag{2}$$

where ϕ_0 is the flux quantum, B is the magnetic induction, and η is a constant of the order of unity. For an anisotropic material (Campbell, Doria, and Kogan [13]) with the same geometric average λ and ξ (e.g., $\lambda^3 = \lambda_a \lambda_b \lambda_c$ with $\lambda_m = \lambda m_\mu^{1/2}$, m_μ being the eigenvalues of the mass tensor m_{ik}), the superconducting part of F differs from the right-hand side of Eq. (2) by the replacement

$$B \rightarrow B^* = (m_{ik} B_i B_k)^{1/2} , \tag{3}$$

where summation over repeated indices is implied. The *invariant* quantity B^* can also be written as $Bm_{zz}^{1/2}$ with z directed along **B**, or as $B(m_a \sin^2\theta + m_c \cos^2\theta)^{1/2}$ for a uniaxial crystal, θ being the angle between the **c** axis and **B**. This result is hardly surprising: The energy cannot depend on the choice of coordinate system, and $m_{ik} B_i B_k$

is the simplest invariant one can form out of m_{ik} and B_i. There are other possible invariants; however, B^* is distinguished by the fact that it appears in the free energy. As an example, the upper critical field in the **b** direction in the anisotropic case is obtained from the isotropic formula $B_{c2}=\phi_0/2\pi\xi^2$ by the replacement shown in Eq. (3): $B_{c2}(m_{ik}b_ib_k)^{1/2}=\phi_0/2\pi\xi^2$, or, for the uniaxial case, $B_{c2}(m_a\sin^2\theta+m_c\cos^2\theta)^{1/2}=\phi_0/2\pi\xi^2$ (see Ref. [14]). In principle, given any field-dependent macroscopic equilibrium property of the mixed phase of an isotropic superconductor in high fields, one may directly translate it to the anisotropic situation by means of the replacement in Eq. (3).

The particular property of interest here is "Lindemann melting" of the flux lattice, in which the mean-square amplitude $\langle u^2\rangle$ of the displacement of the vortex from its equilibrium position reaches a certain fraction of the intervortex distance. Disregarding the possible influence of fluctuations on the mean-field elastic moduli, the temperature of this transition is given by $k_BT_m=\phi_0^{5/2}c_L^2/4\pi^2\lambda^2B^{1/2}$, where the Lindemann fraction $c_L\sim0.1$ [7-9]. We then obtain for the anisotropic case,

$$k_BT_m=\frac{\phi_0^2c_L^2}{4\pi^2\lambda^2(T_m)}\left(\frac{\phi_0}{B^*}\right)^{1/2}. \tag{4}$$

Using the "two-fluid" approximation for the penetration depth $\lambda^2=\lambda^2(0)T_c/4(T_c-T)$, we obtain near T_c,

$$T_c-T_m=AB^{1/2}\varepsilon^{1/4}(\theta),$$

where

$$A=\frac{\pi^2k_BT_c^2\lambda_{ab}^2\gamma^{1/2}}{\phi_0^{5/2}c_L^2},\quad \varepsilon=\sin^2\theta+\gamma^2\cos^2\theta, \tag{5}$$

and where we have introduced the anisotropy parameter $\gamma^2=m_c/m_a$ and the in-plane penetration depth at zero temperature $\lambda_{ab}^2=\lambda^2(0)m_a=\lambda^2(0)\gamma^{-2/3}$. The same result for the anisotropy of the melting temperature has recently been obtained on more general *scaling* grounds by Blatter, Geshkenbein, and Larkin [15].

It is important to note that γ is the only parameter that enters the predicted angular dependence, Eq. (5). Furthermore, torque magnetometry—a completely different equilibrium measurement—has provided [11] the value $\gamma=7.7\pm0.2$ for the sample studied in this work. Thus, as far as the angular variation of T_m is concerned, there are no free parameters. This circumstance allows the predicted variation to be given a very sharp test.

To check Eq. (5), the untwinned single crystal of $YBa_2Cu_3O_{7-\delta}$ used previously [6,11] was mounted on a low-frequency ($\sim10^{-1}$ Hz) torsional oscillator. The magnetic field was applied at an angle θ to the **c** axis, where θ could be varied from 0° to 90°. Damping of the oscillator was measured by monitoring the free decay of the angular oscillations, starting from an initial displacement of $\sim10^{-1}$ deg. Further details of the sample and the experimental setup may be found in Ref. [6]. As in that study, the "dissipation" reported here represents the quantity $I/\tau V$, where I ($=7.1$ g cm^2) is the suspension moment of inertia, V ($=6\times10^{-7}$ cm^3) is the sample volume, and τ is the decay time. Figure 1 shows the temperature-dependent dissipation observed for three different angles in a field of 2 T. As is clarified in the caption, the magnitude of the dissipation changes markedly with angle, but the sharpness of the peak is relatively constant. For the transition field, it was found that a $(T_c-T_m)^n$ dependence, with $n=2$, was obeyed at all angles to within our experimental accuracy. Up to $\sim60°$ the experimental uncertainty in the exponent n was less than 0.2. In the vicinity of 90°, the uncertainty was quite large, ~1, because the differences (T_c-T_m) were small. With this caveat, our data confirm the parabolic $B(T)$ flux-melting signature predicted by Eq. (5) for all angles.

Since the field dependence of the vortex transition temperature is independent of angle to within our experimental uncertainty, the angular dependence at fixed field contains all the available anisotropy information. Uncertainties are smaller at higher fields, and the highest available field for the present experiments was 2 T. Figure 2 displays the observed transition temperature as a function of angle in this field. The data were fitted by Eq. (5) using a least-squares-fitting program, treating A and γ as fitting parameters, and setting $T_c=90.2$ K, as determined from the c-axis phase boundary [6]. The fit is excellent, and gives the value $\gamma=7.6\pm0.4$. The central result of our study is the remarkable agreement between this value

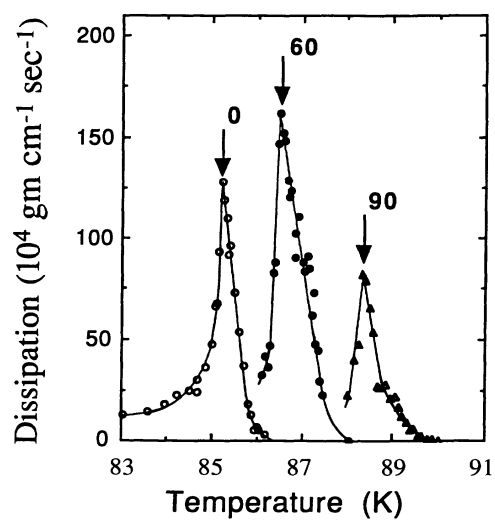

FIG. 1. Temperature dependence of the mechanical dissipation associated with the angular motion of an untwinned single crystal of $YBa_2Cu_3O_{7-\delta}$ with respect to a magnetic field of 2 T. The three different data sets are labeled by the angle made by the magnetic field with the **c** axis of the crystal. To accommodate these data on the same graph, the dissipation results at 0° have been scaled up by a factor of 10. The line drawn through each data set is to guide the eye.

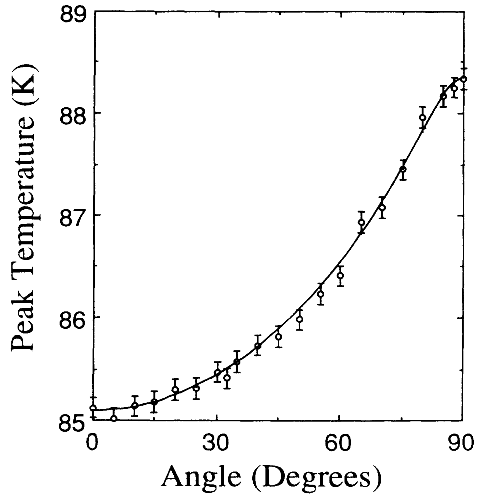

FIG. 2. The temperature of the dissipation peak (see Fig. 1) plotted as a function of the angle of the (2 T) magnetic field to the **c** axis. The line through the data is a least-mean-squares fit by Eq. (5), giving the parameters $\gamma=7.6$ and $A=1.3$ K/T$^{1/2}$.

and the equilibrium torque result (7.7 ± 0.2), convincingly establishing that the transition is controlled by equilibrium properties. One can also use the experimental value of A ($=1.3$ K/T$^{1/2}$) to check the Lindemann melting fraction, c_L. Using the value $\lambda_{ab}=1400$ Å, one obtains the physically reasonable value of 0.16 for the melting fraction.

The depinning temperature defined (for individual vortices) via $\langle u^2\rangle \sim \xi^2$ leads to $k_B T_{\rm dp}=\phi_0^{3/2}B^{1/2}/4\pi^2\kappa^2$ (κ is the Ginzburg-Landau parameter). When "translated" to an anisotropic situation [15] the result is qualitatively inconsistent with our data. Other models of thermal depinning have been discussed in the literature [5], but no specific anisotropy predictions are available. Experimentally, the angular variation of the pinning in $YBa_2Cu_3O_{7-\delta}$ is known [16] to exhibit a sharp peak about 1° or so wide near $\theta=90°$, ascribed to the intrinsic pinning of vortices when the cores fit between the CuO planes. (Consistent with that finding, pinning confined to a similar narrow angular range has been detected below 80 K in the sample used here [11].) If the angular variation shown in Fig. 2 were somehow due to an anisotropy in the weak residual pinning, a sharp increase would therefore be expected at 90°. None is observed.

In conclusion, Eq. (5) contains quantitative melting theory predictions for four observables: the exponent governing the temperature dependence of the transition field, the angular independence of that exponent, the magnitude of the melting field, and the angular variation of the melting temperature. Experimentally, the transition observed in twin-free single-crystal $YBa_2Cu_3O_{7-\delta}$ obeys all these predictions. In our view, the most compelling result is the angular variation of the peak temperature. Our work has firmly established that this variation is controlled by the equilibrium anisotropy, strongly supporting the flux-lattice-melting proposal.

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