How to Measure the QCD Transition Temperature

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I propose to use the transverse momentum distribution of lepton pairs in the ρ^0-ω peak to measure the QCD transition temperature in ultrarelativistic nuclear collisions. The signal-to-background ratio is approximately unity, and the transition temperature can be determined with roughly 15% accuracy.

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The production and study of quark-gluon plasma in ultrarelativistic nuclear collisions is an important subfield of nuclear physics. Fixed-target experiments have been performed at the Brookhaven Alternating Gradient Synchrotron and the CERN Super Proton Synchrotron with various nuclei, at beam energies from 15 to 200 GeV/nucleon. Future experiments are planned at Brookhaven's Relativistic Heavy Ion Collider and CERN's Large Hadron Collider, with heavy-ion beams at center-of-mass energies from 200 GeV to several TeV/nucleon. Because of these experiments, a large amount of theoretical attention has been devoted to the problems of (i) how to confirm the production of a quark-gluon plasma, and (ii) how to extract information about QCD from the collision data.

In this Letter, I propose to use the p_T distribution of lepton pairs in the ρ^0-ω peak to measure the QCD transition temperature T_c in ultrarelativistic nuclear collisions. Using the ρ^0 peak alone would produce the best results because the ρ lifetime, $\Gamma_\rho^{-1}=1.32$ fm/c, is by far the shortest of any copiously produced resonance. However, the masses of the ρ and ω mesons are nearly degenerate and the peaks cannot be separated at present, so the combined peak must be used.

The proposed experimental procedure is straightforward. First, use the transverse momenta of the lepton pairs in the ρ^0-ω peak to reconstruct the p_T distribution of the original mesons, $dN/p_T\,dp_T\,dy$. Then, fit by a thermal distribution:

$$\frac{dN}{p_T\,dp_T\,dy} \propto e^{-m_T/T_m}, \tag{1}$$

where $m_T=(p_T^2+m_\rho^2)^{1/2}$, and T_m is a free parameter. As the collision energy is increased, T_m should approach the same maximum value $T_{\max}$ over a large range of y. The transition temperature $T_c \approx T_{\max}$, with theoretical corrections of approximately 15%.

This proposal is similar to an earlier suggestion [1] to measure T_c using (continuum) photons with transverse momenta from 1 to 3 GeV. The technique is simply to fit the p_T distribution with a thermal distribution—the temperature from the fit is then approximately equal to the transition temperature. The main advantage of using the ρ^0-ω peak is that the signal-to-background ratio is much better than for the photons. The ρ^0-ω peak is approximately an order of magnitude higher than the continuum [2], while the number of thermal photons is approximately equal to the number of background photons. In addition, an even better signal can be achieved from the resonance peak by subtracting the measured contribution of the continuum. This subtraction is impossible for the photons, as both signal and background are from the continuum.

The proposal is a more sophisticated version of a suggestion by Siemens and Chin [3] to look for the extinction of the ρ^0-ω peak at high transverse momentum as a signal for quark-gluon plasma formation. It was later pointed out in Ref. [4], which discusses several diagnostic techniques for ultrarelativistic nuclear collisions using lepton pairs, that the signal proposed by Siemens and Chin would be reduced by transverse expansion [5]. However, while the presence of the ρ^0-ω peak could be just the result of transverse expansion, its absence is still a signal of quark-gluon plasma formation.

The dimuon peak from (relatively rare) J/ψ mesons has been measured with enough statistics to determine temperatures from the p_T distribution [6]. The J/ψ peak is much smaller than the ρ^0-ω peak, so statistics for the proposed measurement will be very good. The main background for the proposed measurement is the signal from ρ^0 and ω mesons at temperatures below T_c.

From a purely theoretical point of view, the measurement could be performed with either muons or electrons, as the theory is the same for the two cases. The goal is to measure a thermal distribution, at temperatures of order 150–200 MeV, so the best measurement would span a range of at least 200 MeV in m_T (the exact range needed depends on the statistics). As the goal is to study thermal leptons, the lowest values of m_T should be below about 1150 MeV, so that at least 10% of the thermal distribution is sampled; I would set the highest lower limit (for a marginally passable measurement) at 1500 MeV, sampling the highest 1% of the thermal distribution.

The rapidity distribution of the dilepton pairs can be adjusted freely, with some caveats. In the central region, the baryon density is low, and so the measurement there would yield (approximately) the value of T_c for a baryon-free plasma. As the baryon density increases, the value of T_c should change, and a measurement at any rapidity would yield the value of T_c at the baryon density present in that rapidity slice. The absolute normalization of densities is uncertain, so it will be difficult to measure the dependence of T_c on the baryon density. However, if T_c is measured in several regions of differing baryon density, it should be relatively simple to extrapolate these measurements and obtain a better estimate of the value at zero baryon density.

In the next section, I discuss the signal and background for the simplest case of an ultrarelativistic collision where the entire collision volume is initially in the deconfined phase, assuming that QCD has a strong first-order phase transition. In the next following section, I discuss the modifications to the calculations if the ρ^0-ω peak is used instead of the ρ^0 peak alone, or if only partial deconfinement is achieved in a collision. I discuss the effects of relaxing the assumptions about the dynamics, and expectations if the QCD phase transition is second order or there is no transition. I also discuss improvements in the measurement of T_c, and predict the behavior of temperatures measured with other meson peaks. Finally, I summarize the results.

Complete deconfinement.— In this section, I estimate the magnitude of the theoretical corrections to T_c. This is done in a very simple way, by calculating the ratio of signal (from mesons at T_c) to background (from mesons at lower temperatures or out of thermal equilibrium). More sophisticated calculations are in progress, in which the signal is calculated exactly as proposed and in which nonequilibrium effects are estimated and transverse flow is included. (For a general theoretical overview of the physics of ultrarelativistic nuclear collisions, on which this calculation is based, see Ref. [7].)

In an ultrarelativistic nuclear collision, the entire collision volume is initially filled with quark-gluon plasma. Virtually no resonance signals (from the light mesons) will come from this plasma. Heavy quark ($c\bar{c}$ and $b\bar{b}$) mesons, such as the J/ψ, can exist in the plasma, as they do not dissolve until the plasma reaches even higher temperatures [8]. Because the heavy mesons can exist in quark-gluon plasma, they have an additional high-temperature background to the signal at the transition temperature. As a result, measurements from the light-meson peaks provide a more accurate determination of T_c.

I assume that the hot matter expands longitudinally, neglecting transverse motion, from proper time τ_0 until the particles reach the detectors (effectively, $\tau=\infty$). I assume also, for the present, that there is a first-order QCD phase transition, where quark-gluon plasma is formed and then hadronizes via an equilibrium-mixed phase of plasma and hadronic (resonance) gas at the QCD transition temperature T_c. Because the equilibration times for common mesons are at least as short as all other relevant time scales, I assume that all mesons are always in thermal equilibrium. Finally, I assume that the properties of ρ and ω mesons do not change significantly from their $T=0$ values in the hadronic phase; this is supported by the results of Gale and Kapusta [9], who see only small thermal effects up to $T=150$ MeV. I use the standard high-energy physics units: $c=\hbar=k_B=1$.

The dilepton peak for species i is the product of the decay rate $\Gamma_{i\to l^+l^-}$ and the space-time integral of the density of species i:

$$N_i=\Gamma_{i\to l^+l^-}\int_{\tau_0}^{\infty}d\tau\, n_i(\tau)V(\tau)\,. \tag{2}$$

I calculate here the signal from the ρ^0 peak alone. In the next section, I discuss the (only slightly different) signal from the ρ^0-ω peak.

The transition begins at proper time τ_t and ends at approximately proper time $r\tau_t$, where $r\approx\frac{37}{3}$ is the ratio of massless degrees of freedom in the quark-gluon plasma and resonance gas. The volume occupied by the hadronic resonance gas at proper time τ (during the transition) is

$$V_h(\tau)=[r/(r-1)](\tau-\tau_t)\Delta Y A\,, \tag{3}$$

where ΔY is the rapidity range of the plasma and A is the cross section of the collision volume. The dilepton signal (per event) from decay of ρ^0 mesons in the mixed phase is

$$s_\rho=[r(r-1)/2]\tau_t^2\Gamma_{\rho\to l^+l^-}\Delta Y A n_\rho(T_c)\,, \tag{4}$$

where n_ρ is the equilibrium ρ^0 density.

The background from matter at lower temperatures is more difficult to calculate, but solid limits can be obtained. The volume of hadronic matter at proper time τ is

$$V_h(\tau)=\tau\Delta Y A\,. \tag{5}$$

For massless ρ mesons, $n_\rho\propto T^3$, and if all particles are massless then $T^3\propto 1/\tau$, so

$$n_\rho=n_\rho(T_c)r\tau_t/\tau \tag{6}$$

after the completion of the hadronization transition. If physical ρ and π masses are used, the ρ^0 density drops off even faster, so Eq. (6) gives an upper limit on this part of the background of

$$b_\rho^{\rm hg}=(f-1)r^2\tau_t^2\Gamma_{\rho\to l^+l^-}\Delta Y A n_\rho(T_c)\,, \tag{7}$$

where f is the ratio of the freeze-out time τ_f to the time of completion of the hadronization transition $r\tau_t$. Thus, the signal-to-background ratio is at least

$$s/b=(r-1)/2r(f-1)\,, \tag{8}$$

so if the system freezes out in less than half the time required for the transition then the signal-to-background ratio will be greater than unity.

If freeze-out occurs very late (at a very low temperature), the background is still limited. Assuming that entropy is conserved, I calculate the temperature as a function of proper time:

$$\tau\sigma(T)=r\tau_t\sigma(T_c)\,, \tag{9}$$

where σ is the entropy density of the gas. For $T\sim 150$ MeV, pions are the only massless degrees of freedom, so

$$\sigma(T)=(2\pi^2/15)T^3, \tag{10}$$

and because the ρ mass $m_\rho \gg T$ we obtain

$$n_\rho=3(m_\rho T/2\pi)^{3/2}e^{-m_\rho/T}. \tag{11}$$

The signal from the equilibrium hadronic gas after the mixed phase is

$$b_\rho^{\rm hg}=9r^2\tau_l^2\Gamma_{\rho\to l^+l^-}\Delta Y A T_c^6\left(\frac{m_\rho}{2\pi}\right)^{3/2} \times\int_{T_f}^{T_c}dT\,T^{-11/2}e^{-m_\rho/T}, \tag{12}$$

where T_f is the freeze-out temperature for the ρ mesons. Setting $T_f=0$, we find

$$b_\rho^{\rm hg}\cong 9r^2\tau_l^2\Gamma_{\rho\to l^+l^-}\Delta Y A(T_c/m_\rho)(m_\rho T_c/2\pi)^{3/2} \times e^{-m_\rho/T_c}[1+O(T_c/m_\rho)]. \tag{13}$$

The estimated signal-to-background ratio is then

$$s/b=m_\rho/6T_c\sim 0.65\text{–}0.86 \tag{14}$$

for $T_c=140$–200 MeV. This signal-to-background ratio is not high enough to make it trivial to extract the transition temperature accurately, especially if $T_c\sim 200$ MeV. However, it should be possible to correct for $b^{\rm hg}$ because it comes mainly from the region $1-T_c/m_\rho<T/T_c\leq 1$, so the range of temperatures that contributes is within about 25% of T_c. In addition, because the range of contributing temperatures is relatively small, the error in T_c should only be about 15% without corrections.

The background from ρ^0 mesons created in the initial collision will be small, as most of these become part of the plasma very quickly. This part of the calculation is the least understood, but the contribution here appears to be small. Assuming that the ρ mesons are essentially massless, the number of ρ mesons is approximately conserved, so

$$V_h(\tau)n_\rho(\tau)\approx r\tau_l\Delta Y A n_\rho(T_c). \tag{15}$$

Using Eq. (2), ignoring the time for the ρ^0 mesons to come on mass shell, and cutting off the integral at time $\tau_0\approx 1$ fm/c when the mesons are absorbed in the plasma, we estimate the signal from the initial ρ^0 mesons:

$$b_\rho^i=r\tau_l\tau_0\Gamma_{\rho\to l^+l^-}\Delta Y A n_\rho(T_c). \tag{16}$$

For ultrarelativistic collisions, the ratio

$$b_\rho^i/s_\rho=2\tau_0/r\tau_l \tag{17}$$

is small, so the initial ρ^0 mesons contribute very little to the background.

Finally, there is a dilepton signal from the ρ^0 mesons that freeze out and leave the system. These mesons decay in proper time $1/\Gamma_\rho$, contributing approximately

$$b_\rho^f=\tau_f\Gamma_\rho^{-1}\Gamma_{\rho\to l^+l^-}\Delta Y A n_\rho(T_f) \tag{18}$$

to the background. Keeping just $b^{\rm hg}$ and b^f, the signal-to-background ratio is at least

$$\frac{s}{b}=\frac{(r-1)m_\rho}{6rT_c+2m_\rho/\Gamma_\rho\tau_l}=0.6\text{–}0.8. \tag{19}$$

Typically, $\tau_l\cong 1/\Gamma_\rho=1.32$ fm/c, so b^f is small and the signal-to-background ratio is not changed much from Eq. (14).

Further considerations.— In practice, all of the above calculations are complicated by the fact that the resonance peaks for the ρ^0 and ω mesons cannot be separated. Mixing these two peaks, the signal-to-background ratio is

$$\frac{s}{b}=\frac{(r-1)m_\rho}{6rT_c+2m_\rho/\Gamma_{\rho+\omega}\tau_l} \tag{20}$$

for the case of complete deconfinement, where

$$\Gamma_{\rho+\omega}=\frac{\Gamma_{\rho\to X}\Gamma_{\omega\to X}(\Gamma_{\rho\to l^+l^-}+\Gamma_{\omega\to l^+l^-})}{\Gamma_{\omega\to X}\Gamma_{\rho\to l^+l^-}+\Gamma_{\rho\to X}\Gamma_{\omega\to l^+l^-}}. \tag{21}$$

Putting in constraints on the decay rates (to dimuons) from data [10], $\Gamma_{\rho+\omega}\gtrsim(5\text{ fm}/c)^{-1}$, and $s/b=0.45$–0.6. Thus, T_c can still be determined with about 15% accuracy using the ρ^0-ω peak in the dimuon spectrum.

For dielectrons, the branching ratios are equal, so $\Gamma_{\rho+\omega}=(2.5\text{ fm}/c)^{-1}$, and $s/b=0.5$–0.7. This is likely the case for dimuons also, but experiments so far have obtained only an upper limit on $\Gamma_{\omega\to\mu^+\mu^-}$. Thus, while the most likely correction for mixing of the ρ and ω peaks is the same for dimuons and dielectrons, the theoretical basis of the correction will not be as sound for dimuons until the ω branching ratio is measured. This should be achievable in the near future, barring possible technical difficulties, as the current upper limit is only about a factor of 3 greater than the dielectron branching ratio.

If only partial deconfinement is achieved, and the system begins with a fraction q of the volume in the deconfined phase, the calculations do not change very much. Basically, all times are approximately multiplied by q, so that s and $b^{\rm hg}$ are roughly proportional to q^2, and b^i and b^f are proportional to q. Thus, unless $q\ll 1$, s/b is not changed much.

In general, relaxing the assumptions about the collision dynamics increases the signal-to-background ratio. The meson densities probably have their equilibrium values at temperatures near T_c. In any case, if the mesons depart from equilibrium at a temperature near T_c, more of the signal comes from temperatures near T_c. If transverse expansion is included, freeze-out occurs sooner, and even more of the signal comes from the mesons at T_c.

Probably the most important caveat is that transverse expansion may alter the m_T distribution enough to destroy the sought-after temperature dependence. This is very sensitive to the amount of transverse expansion. In simulations where the plasma hadronizes via droplet formation, this does not appear to be a problem. However, if hadronization proceeds by detonation waves, or some other sort of explosive process, the signal would reflect the hadronization process rather than the transition temperature. However, the utility of the signal is enough to suggest detailed studies of the effects of transverse expansion, which have not yet been done.

It is also possible that the QCD transition is not first order. If there is a second-order phase transition, and the entropy density is much higher in the deconfined phase, the time to go through the transition (during which the temperature changes very little) is still approximately $r\tau_t$. If the entropy density is not very much different, the signal falls off exponentially with $1/T$, and so it is still dominated by the signal from the highest temperatures at which the mesons can exist. Thus, the signal from the ρ^0-ω peak is always dominated by temperatures near T_c, and thus provides a good measure of the QCD transition temperature.

If there is no phase transition at all, T_c is not well defined. In this case, however, there is probably a maximum temperature at which ρ and ω mesons can survive. This temperature is probably not much higher than 200 MeV, so the dominant contribution to the signal comes from mesons within about 25% below this temperature and the p_T distribution serves as a good measure of this temperature.

Improvements in the accuracy of the temperature measurement depend mainly on the accuracy of the calculation of $b^{\rm hg}$. The other background components can be reduced by going to higher collision energies, as $b^i + b^f \propto \tau_t$ while $s + b^{\rm hg} \propto \tau_t^2$ (and hence grow faster with increasing collision energy). However, the ratio $s/b^{\rm hg}$ is essentially independent of collision energy, so increasing collision energies does not help in separating these, and the contribution $b^{\rm hg}$ cannot be measured directly. Therefore, an accurate calculation of $b^{\rm hg}$ is needed to reduce the error in the measurement of T_c below 15%.

Finally, it is possible to predict experimental results in more detail. First, the temperature determined from the ρ^0-ω peak will approach T_c from below as the collision energy increases. Because $b^{\rm hg} \propto s$, the variation of the signal with collision energy is due only to b^f. The ratio b^f/s is inversely proportional to τ_t, or q in the case of incomplete deconfinement. Both of these are approximately proportional to the entropy density $\sigma \propto (dN/dy)/A$, and so the measured temperature $T_m = T_\infty - \alpha A/(dN/dy)$, where T_∞ is the temperature as determined from $s + b^{\rm hg}$ and α is an undetermined constant.

For other mesons that cannot exist in the deconfined phase, the results will be the same, except that T_∞ will be closer to T_c. More specifically, if the meson lifetime $\Gamma_m^{-1} \ll \tau_f$, the $T_\infty = T_c - \beta/M$ if the heavy meson has a short lifetime, where M is the meson mass and β is another undetermined constant. For mesons that can exist in the deconfined phase, T_m will continue rising until the maximum temperature where the meson can survive is reached.

Because this technique is expected to measure some limiting temperature in all three cases discussed above—a first-order QCD phase transition, a second-order phase transition, and a smooth transition (mesons "melting," but no phase transition)—the nature of the temperature measured must be determined by other means. It is clear that T_m will reflect the temperature at which the ρ and ω disappear from the meson spectrum, but the meaning of this disappearance cannot be determined without further theoretical or experimental work. However, I believe that the information obtained will provide valuable constraints on models of ultrarelativistic nuclear collisions and strong interactions.

In conclusion the p_T distribution of lepton pairs in the ρ^0-ω peak gives an accurate measure of the QCD transition temperature in ultrarelativistic nuclear collisions. In the case that QCD has a strong first-order phase transition, this measurement is accurate to about 15% without correction. The accuracy can be improved by calculations of the contribution from the hadron gas between T_c and the freeze-out temperature.

If QCD has a second-order phase transition, the p_T distribution still gives an accurate measure of the transition temperature. If QCD has no phase transition at all, then T_c is not well defined, and the p_T distribution measures the maximum temperature at which ρ and ω mesons can exist.

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