Intermittency in the Multifragmentation of Hot Nuclei?

D. H. E. Gross,[(1),(2)] A. R. DeAngelis,[(1),(2)] H. R. Jaqaman,[(1),(3)] Pan Jicai,[(2)] and R. Heck[(1)]
[(1)]*Hahn-Meitner-Institut, Bereich Kern- und Strahlenphysik, D-1000 Berlin 39, Germany*
[(2)]*Fachbereich Physik der Freien Universität, Berlin, Germany*
[(3)]*Physics Department, Birzeit University, Birzeit, West Bank*
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The statistical decay of an equilibrated hot nucleus is modeled by sampling the microcanonical phase space for nuclear fragmentation. It is shown to reproduce the measured fluctuations and correlations (intermittency) in data for 1 GeV/nucleon $^{197}_{79}$Au on emulsion. Within our model multifragmentation is linked to the nuclear liquid-to-gas phase transition. The model is used to realistically study the conditions leading to the phenomenon of intermittency in critical heavy-ion reactions.

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Intermittency is a concept that was originally developed in the study of the fluctuations that occur in turbulent flow [1-4]. It corresponds to the existence of large nonstatistical fluctuations that possess self-similarity at all scales.

Bialas and Peschanski [5,6] have proposed to extend this concept to the study of dynamical fluctuations in the rapidity (η) distributions of particles from high-multiplicity events produced in high-energy collisions. They suggested the calculation of the factorial moments

$$F_i = M^{i-1}\frac{\langle \sum_{m=1}^{M} n(\eta_m)[n(\eta_m)-1]\cdots[n(\eta_m)-i+1]\rangle}{\langle N(N-1)\cdots(N-i+1)\rangle} \tag{1}$$

of the rapidity distribution $n(\eta)$ of N particles over M bins of width $\delta\eta$. Under certain conditions (e.g., second-order phase transition, self-similar cascading mechanism, see below) one expects the moments F_i to follow a power law as a function of the size (resolution) $\delta\eta$ of the rapidity bins:

$$F_i \propto (\delta\eta)^{-f_i}. \tag{2}$$

That is, the moments scale or show self-similarity at various resolutions, $F_i(a\delta\eta)=a^{-f_i}F_i(\delta\eta)$. This behavior is called intermittency; f_i is the intermittency exponent. It is intimately linked to the multifractal properties of the underlying physical process and/or to the power laws in its critical behavior (see below). Using normal moments instead of the factorial ones would of course lead to a similar scaling. It is believed, however, that the factorial moments considerably suppress statistical fluctuations due to the finite multiplicity per event [5,6]; see, however, [7]. In this Letter we use the factorial moments.

Using a simple mathematical model, the α model, Bialas and Peschanski were able to demonstrate that self-similar cascading processes are a possible source for intermittency [6,8]. In a further development it was shown [7,9] that the fluctuations associated with critical phenomena in a two-dimensional Ising model lead to an intermittent pattern of the cluster size distribution. By renormalization techniques (self-similarity) it was possible to evaluate the intermittency exponents f_i for the fluctuations at the critical point.

The rapidity distribution $n(\eta)$ shows a characteristic fluctuating "fingering" over a broad range of resolutions. The set of η values which contribute to the ith moment (the support) is a typical multifractal. The reduction of the fractal dimension from 1, corresponding to the fingering, is called the anomalous fractal dimension d_i. It is related to the intermittency exponent by [7,9,10]

$$d_i = f_i/(i-1). \tag{3}$$

For further literature on the relation of intermittency to chaos, scaling, and fractal geometry see also [11,12] and references therein. For an illuminating review on the mathematics of (multi)fractal geometry, critical scaling, and thermodynamics, see [13,14].

The interesting and novel point is that different processes (cascading or phase transitions of second order) seem to give *different dependences of d_i on i* [15]:

$$d_i = \text{const} \tag{4a}$$

(for monofractal, second-order phase transitions [9]), and

$$d_i \propto i \tag{4b}$$

(for multifractal, cascading, α model [6]). Thus, an intermittency analysis of the fluctuations and correlations may carry important information on the dynamics of the decaying system.

More recently, Ploszajczak and Tucholski [11,16] suggested looking for intermittent behavior in the fragment size distributions in *nuclear* multifragmentation at intermediate energies. They analyzed the factorial moments of the charge distributions from the fragmentation of $^{197}_{79}$Au nuclei on emulsion at 1 GeV/nucleon [17], and were able to see evidence for intermittent behavior in the nuclear fragmentation data.

Up to now we are aware of only this one limited set of 415 events that is suitable for the study of intermittency in nuclear multifragmentation. As the physical conditions leading to intermittency are not yet well understood, it is essential to have sufficiently realistic models which permit study of the system under many conditions. Moreover, a theoretical model allows one to generate a large enough number of events to get sufficient statistics.

In analogy to the Ising model, a possible source for intermittent behavior in the nuclear fragmentation case might be the critical region associated with the liquid-gas phase transition in hot nuclei [18–22]. In some recent work [23,24] this phase transition has been investigated using the Metropolis sampling of the microcanonical multifragmentation decay of equilibrated hot nuclei developed by Gross and co-workers [25–27]. Two critical regions of the fragmenting system were found. One is related to the liquid-gas transition in hot nuclear matter, and the other could be identified with a sudden and macroscopic opening of hot fission decay channels.

In Ref. [16] the authors expressed serious doubts that the liquid-gas phase transition within a thermodynamical model could be the source of the intermittent behavior seen in the experimental data. Therefore it is important to see whether microcanonical multifragmentation explains the observations. As suggested in Ref. [16], one divides the range ΔZ of the distribution of fragment charges Z into M bins each of size $\delta Z = \Delta Z/M$, and then calculates the scaled factorial moments [5,6,28]

$$F_i = \frac{\langle \sum_{m=1}^{M} n_m(n_m-1)\cdots(n_m-i+1)\rangle}{\sum_{m=1}^{M}\langle n_m\rangle^i}. \qquad (5)$$

n_m is the multiplicity, or number of particles or fragments, in the mth bin, $(m-1)\delta Z < Z < m\,\delta Z$. The angular brackets denote the average over many events. Often F_i as defined above is called the horizontal averaged factorial moment [29] of the distribution of the *number* of fragments versus charge bins δZ. It should not be confused with the moments $\sum_m (Z_m)^i$ of the charge distribution, which were used by Campi [30,31] and also by us [23,24].

We have calculated the scaled factorial moments for the microcanonical fragmentation of gold nuclei with an excitation energy spectrum that falls off exponentially and becomes uniform at energies $E^* > 1.2$ GeV. This distribution is derived [32] from the experimental results of Ref. [17]. Some results for the moments are shown in Fig. 1. In analogy to Ref. [16], the events contributing to these moments are selected from events generated by our Metropolis sampling by the constraint of having at least three intermediate mass fragments (IMF's) with charge $Z \geq 3$.

In order to reduce possible correlations coming from the Metropolis sampling method, we accumulated every 5075th event to get five equivalent samples of 412 events each, as compared to the experimental sample of 415 events. We found that our results varied only slightly (on the order of less than 5%) from sample to sample, and that they were well represented by the combination of all five sets. Our discussion then is based on this combined sample, though the results would not change if we used any one of the smaller samples. The errors shown in the plots of the experimental data are as given in [16], which we approximated by assuming an error of $\pm 5.5\%$ in the moments.

In Figs. 1, 3, and 4 we plot $\ln(F_i)$ versus the resolution (bin size), $-\ln(\delta Z)$. The open circles connected by a dashed line are the moments calculated from our sample data set of 2059 events. They fall somewhat below the experimental points. This is due at least in part to the fact that our model produces more fragments per event on average than the experimental data show. This may have a simple explanation. In the experiment one records only those events in which the sum of the detected fragment charges is exactly 79. As there is a greater probability to miss some light fragments in higher-multiplicity events, this biases the experiment. We simulated this bias by rerunning our sample, and allowing for the possibility that a light fragment $(Z < 26)$ might not be recorded. We then rejected all events with total charge less than 79, as in the experiment. We found that even an inefficiency as small as 2.75% for protons, linearly decreasing with charge, *reduces the average multiplicity and considerably enhances the moments without changing the slopes much.* This can be seen quite clearly in the figures, in which the open diamonds (connected by a solid line) give our results after adjusting for the bias. In a subsequent paper [33] we will investigate this further.

Both with and without the bias, the moments show a pronounced linear rise with decreasing bin size δZ. This is just the "intermittency" observed in the experiment [16]. In our analysis we have excluded the point $\delta Z = 1$, to conform with the analysis in Ref. [16]. Note that in the data the factorial moments at $\delta Z = 1$ are considerably larger than at the other bin sizes. This is presumably due to the fact that $\delta Z = 1$ is the physical limit of the resolution, and thus marks the breakdown of scaling versus δZ.

Following Ploszajczak, Tucholski, and Bozek [34] the

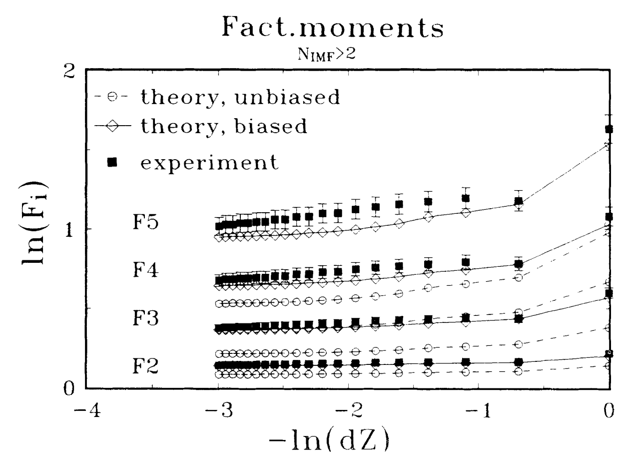

FIG. 1. The dependence of the factorial moment $\ln(F_i)$ of order i on the bin size $-\ln(\delta Z)$, for the microcanonical multifragmentation of hot $^{197}_{79}$Au nuclei. Results are shown for moments of orders 2–5, using those events containing at least three IMF's. For the experiment (solid squares) this filter selected 144 of 415 events, for the unbiased theory (open circles connected by dashed lines), 1114 of 2059 events, and for the theory after allowing for experimental bias (open diamonds connected by solid lines), 365 of 1066 events. Errors bars are shown for the experimental data; corresponding errors exist but are not shown for the theory.

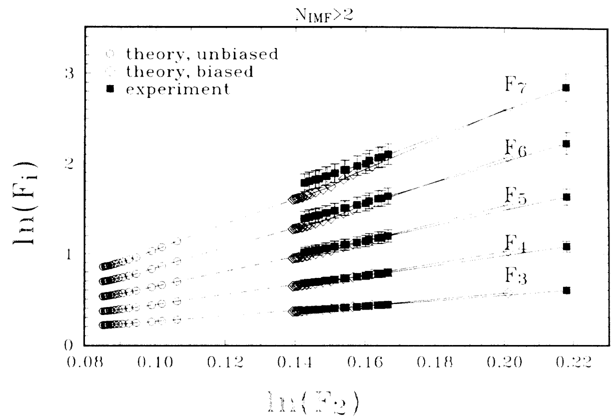

FIG. 2. $\ln(F_i)$ vs $\ln(F_2)$ for the same data sets as in Fig. 1. Again error bars are given only for the experimental data but exist also for the theory.

intermittency can even be made more evident by plotting $\ln(F_i)$ vs $\ln(F_2)$. In Fig. 2 we do this for the same events shown in Fig. 1, and we see that the model data scale with the same slope as the experimental data. It is quite interesting that here the point $\delta Z=1$ also follows the general systematics. Apparently, the limiting behavior in δZ does not affect the $\ln(F_i)$ vs $\ln(F_2)$ scaling. In a subsequent detailed publication [33] we show that *this kind of scaling may follow from very general and simple features of the fragmentation process which are not limited to nuclear fragmentation.* In fact a nearly identical scaling with very similar slopes was recently found in high-energy hadron production processes [35]. Note again that simulating the experimental bias brings the experiment and theory into much better agreement.

A critical comment should be made here. Because of the finite size of nuclei, statistical, so-called nondynamical, fluctuations cannot be avoided [7], even by the use of factorial moments. Therefore, not all of the linear dependence of $\ln(F_i)$ on $\ln(\delta Z)$ is linked to intermittency in a corresponding infinite system. As the two types of fluctuations are inseparable one should not make a distinction between them.

Figure 3 displays the scaled factorial moments F_i for events satisfying the criterion of having at least four IMF's. Figure 4 gives the moments F_i with no filter, i.e., without any constraint. Apparently, the moments F_i are smaller and the slopes f_i (dimensions d_i) are larger when events with less than three IMF's are filtered out than when all events are used (Fig. 4).

Figure 5 shows the anomalous fractal dimension $d_i = f_i/(i-1)$ for the filtered data. They are calculated from linear regressions to the data of Figs. 1 and 3 (excluding the point $\delta Z=1$). The errors in the dimensions are obtained by adding the variance in the slope due to the individual errors in the $\ln(F_i)$, and the variance due to deviations from linearity of the mean values of $\ln(F_i)$. Again we see that adjusting for bias brings the theory

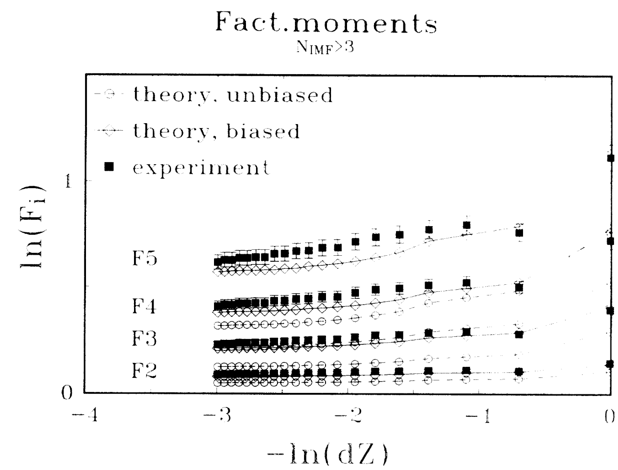

FIG. 3. The same as in Fig. 1, but for events with at least four IMF's. The numbers of events which pass this filter are 80 for the experiment, 905 for the unbiased theory, and 272 for the theory with biasing.

into much better agreement with the experiment (compare the solid and dashed lines). Although the adjusted results closely follow the experimental data, the behavior of the dimension is not clearly either that of a cascade process (linear with i) or a phase transition (constant with i) in an infinite system. The fact that our model yields results not clearly consistent with a second-order phase transition in infinite systems may be due to finite-size effects or the influence of the long-range Coulomb force [23,24,27], and it might also be that a filter using the number of IMF's picks out events in a broader region than that covered by the second-order phase transition.

To summarize, we have shown that the scaled factorial moments calculated from data on the fragmentation of gold nuclei can be reproduced using a microcanonical model of the *thermal* breakup of the nucleus. Our model describes the height and slopes of the moments, and

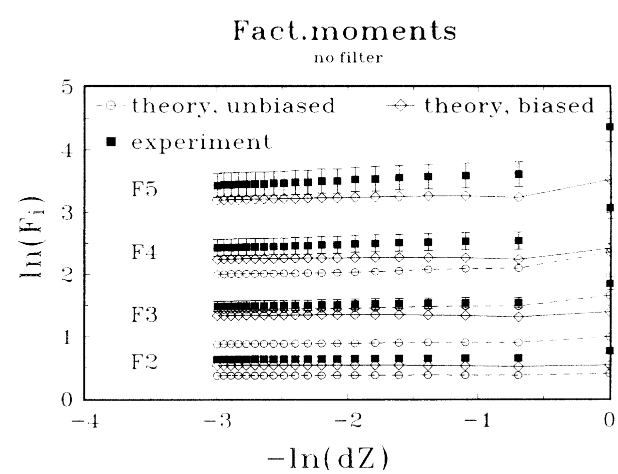

FIG. 4. The same as Figs. 1 and 3, but for all events without any filter.

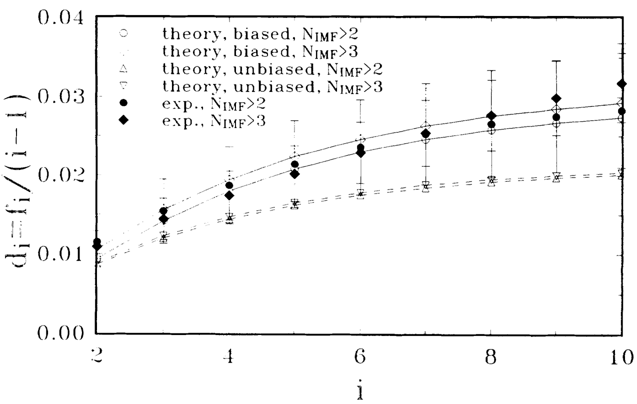

FIG. 5. Anomalous fractal dimension d_i for both sets of filters. The dashed lines connect the theory-generated points without biasing; the solid lines connect the theory-generated points with biasing. Error bars are shown for the experimental data; corresponding errors exist but are not shown for the theory.

reproduces the $\ln(F_i)$ vs $\ln(F_2)$ scaling. The moments may show evidence for an experimental bias towards events of lower multiplicity.

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