πNN and Axial-Vector Form Factors and the NN One-Boson-Exchange Model

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An explanation is proposed to the problem why the πNN form factor known from NN analysis is by far harder than the one obtained in theoretical subhadronic models for nucleon structure. The soft πNN form factor compatible with the models is allowed for the NN interaction by considering the additional contribution due to the exchange of the correlated three-pion state of $I=1$, $J^P=0^-$ between the two nucleons. The same mechanism explains consistently the axial-vector form factor.

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Recently Thomas and Holinde raised the problem that the πNN form factor (FF) determined from the analysis of the NN data contradicts the one known in theoretical studies due to the subhadronic models for the nucleon structure: The former is by far harder than the latter [1]. They summarize the FF parameter known from the NN studies as $\Lambda_{NN}^{(1)} \gtrsim 1300$ MeV [2], and the one from the theoretical studies as $\Lambda_F^{(1)}=500$–800 MeV [3] (730 MeV is preferred). In these studies the axial-vector FF is closely related to the πNN FF, $\Lambda_F^{(1)} \approx \Lambda_A^{(1)}$ or $\Lambda_F^{(1)} < \Lambda_A^{(1)}$, where $\Lambda_A^{(1)}$ is the axial-vector FF parameter. The experimental data from charged-current ν scattering indicate that $\Lambda_A^{(1)} \approx 730$ MeV or $\Lambda_A^{(2)}=1.03 \pm 0.04$ GeV [4].

The FF functions in the above argument are defined by

$$F^{(N)}(q^2)=\left[1+\left(\frac{q}{\Lambda^{(N)}}\right)^2\right]^{-N} \quad (N=\tfrac{1}{2},1,2). \qquad (1)$$

One notes that an approximately equivalent effect is made by different FF functions if $\Lambda^{(2)}=\sqrt{2}\Lambda^{(1)}$ or $\Lambda^{(1/2)}=\Lambda^{(1)}/\sqrt{2}$ holds.

In this paper we propose an answer to remove the contradiction between $\Lambda_F^{(1)}$ and $\Lambda_{NN}^{(1)}$. Let us begin with a historical survey of the problem. As early as in 1968 the πNN FF was introduced into the NN one-boson-exchange (OBE) model which represents the nucleon structure [5–7]. By adjusting the FF parameter, all the NN data below $E_L=350$ MeV of incident proton laboratory energy could be almost quantitatively described, where the FF works sensitively and very well in describing the $^3S_1$ and ϵ_1 phase parameters.

The FF parameter was found to be [5]

$$\Lambda_{NN}^{(1)}=2300\text{–}2500 \text{ MeV}. \qquad (2)$$

Thus the FF was recognized [6] to be by far harder than the electromagnetic FF, $\Lambda_{\rm em}^{(1)} \approx 600$ MeV (or $\Lambda_{\rm em}^{(2)} \approx 840$ MeV), as well as the $\pi N\Delta$ vertex FF, $\Lambda_{\pi N\Delta}^{(1)} \approx 590$–$840$ MeV (or $\Lambda_{\pi N\Delta}^{(1/2)} \approx 420$–$600$ MeV [6,8]).

In 1984 the NN OBE model was combined with the πNN three-body calculation to describe $pp \to pp$, $pp \to \pi NN$, and $pp \to \pi d$ processes in a unified way [9,10]. This was a development from the standard πNN theory [11,12] . In Ueda's study the following was found [9,10]. The contribution from the box diagram depicted in Fig. 1(a) is much too large for the $^3P_2$ phase shift of pp scattering below $E_L=1$ GeV if the πNN FF parameter is used as is in Eq. (2). The proper value for the $^3P_2$ phase shift was found to be [9] $\Lambda_{NN}^{(1)}=640$–1000 MeV (or $\Lambda_{NN}^{(2)}=900$–1400 MeV). Later a preferred value was found as [10]

$$\Lambda_{NN}^{(1)}=640\text{–}720 \text{ MeV} \quad (\text{or } \Lambda_{NN}^{(2)}=900\text{–}1000 \text{ MeV}). \qquad (3)$$

However, this value failed to describe the ϵ_1 parameter and the $^3S_1$ phase shift, since the tensor force is cut off too much. To recover from this discrepancy it was necessary to introduce the additional contribution of the exchange of a boson which has the same quantum number as the pion and a mass of 900–1000 MeV. This boson was required to be decoupled from Δ and to make no contribution to the box diagram of Fig. 1(a). The boson was called Π and interpreted as an effective representation of the correlated three-pion ($C3\pi$) state.

A soft πNN FF as in Eq. (3) is required also for the two-pion exchange (2πex) via Δ three-body force, depicted in Fig. 1(b), in order to explain the remaining part of the triton binding energy that is unexplainable by two-body forces [13,14]. A value as low as $\Lambda_{NN}^{(1)} \approx 800$ MeV is appropriate for the mechanism of Fig. 1(b).

Now let us address our viewpoint in this paper. We take the t-channel view for the πNN vertex [see Fig. 1(c)]. The major source for the soft πNN FF is considered to be that the correlated three-pion ($C3\pi$) state of channel $I=1$, $J^P=0^-$ mediates between the nucleon and the pion. Simultaneously, this $C3\pi$ state mediates between the two nucleons. This is considered as one-Π exchange (1Πex) in effect [see Fig. 1(d)]. For the $C3\pi$ state of channel $I=1$, $J^P=0^-$ one considers, first, the states of the π-ρ with $l=1$, the π-σ with $l=0$, and the resonance $\pi(1300)$.

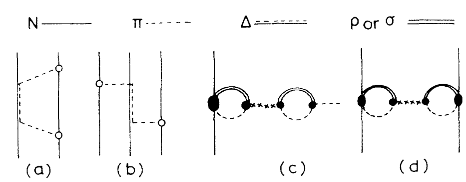

FIG. 1. (a) The box diagram of the three-body calculation where the Δ contributes. (b) 2πex via Δ three-body force. (c) The t-channel view for the πNN vertex where the $C3\pi$ state mediates between the nucleon and the pion. The series of crosses indicates the chain of loops. (d) The exchange of the $C3\pi$ state between the two nucleons.

Quite analogously, the major source for the axial-vector FF is considered to be that the $C3\pi$ state of channel $I=1$ or 0, $J^P=1^+$ mediates between the nucleon and the weak boson. For the $C3\pi$ state of channel $J^P=1^+$ one considers the states of the π-ρ with $l=0$, the π-σ with $l=1$, and the axial-vector bosons of mass ~ 1200 MeV [for example, $a_1(1260)$].

First, consider the π-ρ scattering amplitude. This is given by the sum of the resonant and the nonresonant terms, T_R and T_P, respectively. While the former can be described by the quark model, the latter allows the particle-exchange picture and constitutes a more peripheral part than the former in the whole amplitude $T(W)=T_P(W)+T_R(W)$, where W is the invariant mass of the system. The resonance term is given by the Breit-Wigner formula modified by initial and final distortions due to T_P and a cutoff factor at high energies off resonance. The resonance parameters are given in the particle table [15].

T_P is obtained by solving the Lippmann-Schwinger equation with relativistic kinematics. The potential for the equation is $V(p',p)=-g(p')g(p)$, where p and p' are the initial and final c.m. relative momenta, respectively, and

$$g(p)=G\left(\frac{p}{\beta}\right)^l\frac{1}{(p^2+\beta^2)^{1+l/2}}\,. \tag{4}$$

Then, using the spectral function $\rho_{\pi\rho}(W)$, the effective mass of the π-ρ system is obtained by

$$\bar{M}=\int_{3m_\pi}^{\infty}dW\,W\rho_{\pi\rho}(W)\,, \tag{5}$$

$$\rho_{\pi\rho}(W)=N\int_{2m_\pi}^{\infty}dm_\rho D_\rho(m_\rho)\left|\frac{T(m_\rho,W)}{p\mu(p)}\right|^2, \tag{6}$$

where N is a normalization factor, $T(m_\rho,W)$ is the total amplitude, $D_\rho(m_\rho)$ represents the mass distribution of m_ρ, and $\mu(p)$ is the reduced mass.

The adjustable parameters used to calculate $\bar{M}$ are the potential parameters of G and β in Eq. (4). Since the lightest boson exchanged between π and ρ is π, the range parameter β is constrained by $\beta\gtrsim m_\pi$. We set it as $\beta=200$ MeV. Furthermore, we constrain the strength G so as not to make any bound or resonance state in the channels. Then we find that $\bar{M}\gtrsim 870$ MeV for $J^P=0^-$, and $\bar{M}\gtrsim 880$ MeV for $J^P=1^+$. We could not obtain a value as low as $\bar{M}\sim 730$ MeV, since the most probable value for m_ρ is higher than this.

Thus we seek the possibility of a lower effective mass next in the π-σ interaction, where σ is the $I=0$, $J^P=0^+$ $\pi\pi$ correlated state. In the realistic OBE models the mass of σ ranges from about 350 to about 600 MeV. We set it as $m_\sigma=475$ MeV and the width as 250 MeV and calculate the value of an effective mass in a similar treatment as for the π-ρ calculation. We find a lower value for the effective mass than for the π-ρ case. The most proper value of 730 MeV for $\bar{M}$ is obtained with $G=0.98$ fm^{-2} and $G=0.436$ fm^{-1} for the $J^P=1^+$ and 0^- channels, respectively, with $\beta=200$ MeV in common. The spectral functions in these cases are shown in Fig. 2. We have confirmed also that $\bar{M}=730$ MeV is reproducible with $\beta\lesssim 500$ MeV for both channels.

Thus, the soft πNN vertex FF is reproduced in the model where the $C3\pi$ state mediates between the nucleon and the pion [see Fig. 1(c)]. The $C3\pi$ state is generated by the π-ρ and π-σ interactions and $\pi(1300)$. However, the π-σ should be the major component. The $C3\pi$ state of $I=1$ and $J^P=0^-$ with an effective mass of 730 MeV also mediates between the two nucleons [see Fig. 1(d)]. This effective boson is now regarded as "the heavy pion Π" considered before [9,10]. Furthermore, quite analogously, the axial-vector FF with $\Lambda_A^{(1)}\sim 730$ MeV is reproduced in the model.

The standard 1πex potential with the hard πNN FF with [16] $g_\pi^2=14.17$, $m_\pi=138.7$, and $\Lambda_{NN}^{(2)}=3602$ MeV is effectively equivalent to the sum of the 1πex potential with the soft πNN FF and the 1Πex potential with

$$g_\pi^2=14.17,\quad m_\pi=138.7,\quad \Lambda_{NN,\pi}^{(2)}=1000\ \text{MeV}\,, \tag{7}$$

$$g_\Pi^2=23.2,\quad m_\Pi=730.0,\quad \Lambda_{NN,\Pi}^{(2)}=3000\ \text{MeV}\,. \tag{8}$$

The sum of the above 1πex and 1Πex potentials creates approximately the same effect as the 1πex potential with the hard FF for momentum transfers below 5 GeV/c. Therefore the tensor component is correctly reproduced. Then the $^3S_1$ and ϵ_1 phase parameters are correctly described. On the other hand, the Π coupling with Δ is suppressed, as is demonstrated later. Thus the Π contribution to mechanisms like Figs. 1(a) and 1(b) is quite suppressed. Therefore the difficulty of too large a contribution from mechanisms with the π with the hard FF is removed. Then the $^3P_2$ phase parameter, which has a large contribution from the mechanism of Fig. 1(a) through the LS component, is also correctly described in the new scheme.

So far we have discussed the dressed part of the πNN FF which is composed of the π-σ, π-ρ, and $\pi(1300)$ intermediate states. On the other hand, the core part could be considered as follows. This should be composed of the

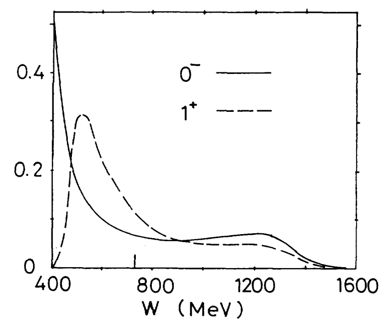

FIG. 2. The spectral functions $\rho_{\pi\sigma}^{J=0}(W)$ and $\rho_{\pi\sigma}^{J=1}(W)$ for channels 0^- and 1^+, respectively, are represented by the solid and dashed curves. Both reproduce $\bar{M}=730$ MeV.

correlated states of particles heavier than the π-σ, π-ρ, or $\pi(1300)$, such as ρ-ω, N-$\bar{N}$, and other heavier particle states with the quantum numbers $I=1$ and $J^P=0^-$. It is reasonable to assume that the average mass of the core part is $\Lambda_c \gtrsim 2$ GeV. On the other hand, we have assigned to the average mass of the dressed part the value $\Lambda_d \sim 730$ MeV. Then the ratio of the dressed part to the core part contribution to the πNN FF is $(q^2+\Lambda_d^2)^{-1}/(q^2+\Lambda_c^2)^{-1}$, assuming equal strengths of the couplings to the nucleon. This ratio is less than $\sim 20\%$ for the momentum transfer $q \lesssim \Lambda_d$. Thus the contribution from the core part is minor in nature.

Another way to see the soft FF defined by Eq. (7) is as follows. The sum of the dressed and core parts of the πNN FF could be parametrized as

$$F(q^2)=[\bar{\Lambda}_d^2/(q^2+\bar{\Lambda}_d^2)][\Lambda_c^2/(q^2+\Lambda_c^2)]\,, \qquad (9)$$

where the first factor in Eq. (9) is a πNN FF for the dressed part with $\bar{\Lambda}_d$ a little less than 730 MeV, while the second factor represents the effect of the core part with $\Lambda_c \gtrsim 2$ GeV. One remarks that a value a little smaller than 730 MeV is possible for $\bar{\Lambda}_d$: $\bar{\Lambda}_d = 652$–730 MeV is reproduced with the π-σ interaction of $G=0.5$–0.436 fm^{-1} and $\beta=200$ MeV in Eq. (4) under the same condition as before. Then one notes that, in the relevant region of the momentum transfer of $q \lesssim 1$ GeV/c, the πNN FF of Eq. (7) is approximately equivalent to $F(q^2)$. In this sense the FF in Eq. (7) is the valid parametrization which could involve implicitly the core part effect and hence be better than the 20% approximation in a practical point of view.

On the other hand, the core part effect must be explicit in the 1Πex amplitude as the vertex FF of Π, since without this the resulting potential is too singular, like r^{-3} at $r=0$. One may assign the second factor, $\Lambda_c^2/(q^2+\Lambda_c^2)$, in Eq. (9) to it. This is approximately equivalent to the FF of Π in Eq. (8) in the dipole parametrization.

Since the Π is the $C3\pi$ state of the π-σ and π-ρ as well as the dressed part of the πNN FF, the spectral function for Π is the same as that of the πNN FF depicted in Fig. 2. In this sense one may replace the m_Π in Eq. (8) by $\bar{\Lambda}_d$. In this case one must use a smaller coupling constant than that in Eq. (8) to make an equivalent effect to the Π contribution defined by Eq. (8).

The contribution of the axial-vector boson ϕ which is the $C3\pi$ state of $I=1$ or 0, $J^P=1^+$ with effective mass ~ 730 MeV should be negligible for the exchange between the two nucleons. It is known [17] that the 1ϕex amplitude is quite different from the 1Πex. In any case the ϕ cannot substitute for the Π. Therefore the 1ϕex contribution is not favorable for the NN interaction, since the NN interaction is already described well enough without the ϕ contribution. A negligible coupling of the ϕ with the nucleon is necessary. This is consistent with $\bar{p}$-p annihilation at rest, where the annihilation from the $I=1$ and 0, $J^P=1^+$ $\bar{p}$-p state is known to be minor [18].

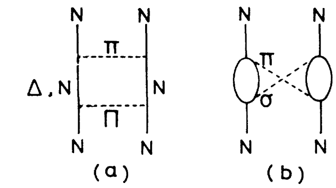

FIG. 3. (a) The box diagram for the π and Π exchanges with the ΔN or NN intermediate states. (b) The π-σ crossed exchange.

Now let us demonstrate that the effective ratio of the $N\Delta\Pi$ vertex strength to the $NN\Pi$ one is less than $\frac{1}{4}$ in the box diagrams of the π and Π exchanges with the ΔN or NN intermediate states [Fig. 3(a)]. Since the major component of Π is considered to be the π-σ correlated state, let us adopt the approximation that the Π is this state. In the t-channel view the ratio of the $N\Delta\Pi$ vertex strength to the $NN\Pi$ one is equal to the ratio of the $\bar{N}\Delta$-$\pi\sigma$ amplitude to the $\bar{N}N$-$\pi\sigma$ amplitude. In turn, in the s-channel view the amplitudes are expanded into partial waves with $j^p=\frac{1}{2}^\pm, \frac{3}{2}^\pm, \ldots$. Each partial wave can be approximated by the dominant resonances or the nucleon. Note that a crossing term in the $\bar{N}\Delta$- (or $\bar{N}N$-) $\pi\sigma$ amplitude is also expanded as the sum of s-channel resonance terms. In this way, the π-σ crossed exchange between the two nucleons can be effectively involved in the present manipulation [Fig. 3(b)].

One limits the resonances to mass $M_* < 1600$ MeV, since these resonances are predominantly relevant in the phenomena with momentum transfer $k \lesssim 730$ MeV/c. This is known from the fact that the Green's function of the intermediate resonance and nucleon state, $[E - E_*(k) - E_N(k)]^{-1}$, is proportional to $\{1+[k/(744 \text{ MeV}/c)]^2\}^{-1}$ with $M_*=1600$ MeV and E corresponding to $E_L=140$ MeV. Thus the following resonances with $j^p=\frac{1}{2}^\pm$ and $\frac{3}{2}^\pm$ are taken into account: $N(940)$, $\Delta(1232)$, $N(1440)$, $N(1520)$, and $N(1535)$.

On the basis of the established couplings, $NN\pi$, $NN\sigma$, and $N\Delta\pi$, the diagrams of Figs. 4(a)–4(c) are depicted for the case of $j^p=\frac{1}{2}^+$ [$N(940)$]. Then the effective ratio of the $N\Delta\Pi$ to the $NN\Pi$ for $j^p=\frac{1}{2}^+$ [$N(940)$] is approximately given by

$$R[N(940)] = \frac{1}{2}\frac{f_{N\Delta\pi}(e-M_\Delta-m_\pi)^{-1}}{f_{NN\pi}(e-M_N-m_\pi)^{-1}}\,, \qquad (10)$$

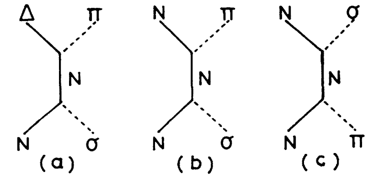

FIG. 4. The diagrams for the partial-wave amplitude with $j^p=\frac{1}{2}^+$: (a) $N\sigma$-$\Delta\pi$, (b) $N\sigma$-$N\pi$, and (c) $N\pi$-$N\sigma$.

where the ratio of the amplitude of Fig. 4(a) to that of Fig. 4(b) is evaluated with the coupling constants and the Green's functions in the $\Delta\pi$ and $N\pi$ channels. Furthermore, the factor of $\frac{1}{2}$ in Eq. (10) takes into account the fact that the amplitude of Fig. 4(c) makes an equal contribution as that of Fig. 4(b). One uses that $f_{N\Delta\pi}^2=0.28$ and $f_{NN\pi}^2=0.08$ and puts $e=M_N+\frac{1}{4}m_\pi$, assuming that e shares one-half of the two-nucleon system energy at $E_L=140$ MeV. Then one finds that $R[N(940)]\sim\frac{1}{4}$.

For the case of $j^P=\frac{3}{2}^+$ [$\Delta(1232)$] no diagram exists, since no $N\Delta\sigma$ coupling exists for the isospin invariance and no $\Delta\Delta\sigma$ coupling is allowed for no positive evidence for this coupling. Note that the Δ-σ state coming from the coupling is suppressed, even if the $\Delta\Delta\sigma$ coupling is used, since it is relatively very massive.

Next the ratios for the cases of $N(1440)$, $N(1520)$, and $N(1535)$ are obtained by replacing the ratio $f_{N\Delta\pi}/f_{NN\pi}$ in Eq. (10) by $\Gamma_{\Delta\pi}^{1/2}/\Gamma_{N\pi}^{1/2}$, where $\Gamma_{\Delta\pi}$ and $\Gamma_{N\pi}$ are the decay widths of the respective resonances [15]. Then one finds that the ratios are 0.068, 0.091, and 0.035 for the $N(1440)$, $N(1520)$, and $N(1535)$, respectively. An average of the ratios over the nucleon and the other resonances is ~ 0.11. Summarized, the effective ratio of the $N\Delta\Pi$ vertex strength to the $NN\Pi$ one is less than $\frac{1}{4}$.

In conclusion, based on the t-channel view, we have presented a consistent explanation for the soft πNN and axial-vector FF's and the short-range part of the NN tensor force in the OBE model.

This t-channel view is *not* exclusive of that for the subhadronic models which succeed in explaining the FF's. At present the interplay between the two views is unknown. However, some dual nature between them might exist.

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[1] A. W. Thomas and K. Holinde, Phys. Rev. Lett. **63**, 2026 (1989).
[2] R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. **149**, 1 (1987); T. E. O. Ericson and M. Rosa-Clot, Nucl. Phys. **A405**, 497 (1983).
[3] A. Chodos *et al.*, Phys. Rev. D. **10**, 2599 (1974); G. E. Brown and M. Rose, Phys. Lett. **82B**, 177 (1979); S. Theberge, A. W. Thomas, and G. A. Miller, Phys. Rev. D **22**, 2838 (1980); **23**, 2106(E) (1981).
[4] N. Baker *et al.*, Phys. Rev. D **23**, 2499 (1981); K. Miller *et al.*, *ibid.* **26**, 537 (1982); T. Kitagaki *et al.*, *ibid.* **28**, 436 (1983).
[5] T. Ueda and A. E. S. Green, Phys. Rev. **174**, 1304 (1968).
[6] T. Ueda and A. E. S. Green, Nucl. Phys. **B10**, 289 (1969).
[7] R. Bryan and B. L. Scott, Phys. Rev. **177**, 1435 (1969).
[8] F. Selleri, Phys. Rev. Lett. **6**, 64 (1961).
[9] T. Ueda, Phys. Lett. **141B**, 157 (1984).
[10] T. Ueda, Nucl. Phys. **A463**, 69c (1987); Prog. Theor. Phys. **76**, 729 (1986).
[11] B. Blankleider and I. R. Afnan, Phys. Rev. C **24**, 1572 (1981).
[12] H. Garcilazo and T. Mizutani, *πNN System* (World Scientific, Singapore, 1990).
[13] T. Ueda, T. Sawada, and S. Takagi, Nucl. Phys. **A285**, 429 (1977); T. Ueda, T. Sawada, T. Sasakawa, and S. Ishikawa, Prog. Theory. Phys. **72**, 860 (1984).
[14] S. A. Coon, M. D. Scadron, P. C. Mcnamee, B. R. Barrett, D. W. E. Blatt, and B. H. J. McKeller, Nucl. Phys. **A317**, 242 (1979).
[15] Particle Data Group, M. Aguilar-Benitez *et al.*, Phys. Lett. B **204** (1988).
[16] T. Ueda and A. E. S. Green, Phys. Rev. **18**, 337 (1978).
[17] S. Ogawa, S. Sawada, T. Ueda, W. Watari, and Y. Yonezawa, Prog. Theor. Phys. Suppl. **39**, 140 (1967).
[18] M. Maruyama and T. Ueda, Nucl. Phys. **A364**, 297 (1981).