Negative Magnetoconductance in an Anderson Insulator with Strong Spin-Orbit Scattering

P. Hernandez and M. Sanquer

Service de Physique de l'Etat Condensé, Departement de Recherche sur l'Etat Condensé, les Atomes et les Molécules, Centre d'Etudes de Saclay, 91191 Gif-sur-Yvette CEDEX, France

(Received 31 July 1991; revised manuscript received 26 December 1991)

We have measured the magnetoconductance of amorphous Y_xSi_{1-x} samples in the diffusive, barely, and deeply insulating regimes. Application of a magnetic field decreases the localization length ξ in the barely insulating regime where spin-orbit scattering (SOS) is important. In an insulator without SOS, we see an increase of ξ, indicating that spin-reversal symmetry and time-reversal symmetry control the magnetoconductance in the barely insulating regime, in contrast with predictions obtained by directed-path approximations.

PACS numbers: 71.55.Jv, 72.15.Rn, 72.20.My

Many recent theoretical works [1-6] have reached contradictory conclusions about the effect of a magnetic field on the localization of electrons in disordered insulators. Theories which neglect the returning loops (forward-directed-path approach [7]) obtain a positive magnetoconductance (MC), regardless of the strength of the spin-orbit scattering (SOS) [2,3]. This contrasts with a predicted negative MC in disordered insulators with strong SOS, obtained either from a random-matrix-theory approach [1] or from an heuristic picture taking explicitly the returning loops within the localization radius ξ [4]. Shapir and Ovadyahu [8] have first shown experimentally that the MC changes its sign from positive to negative as SOS increases in the barely insulating regime of Au-doped In_2O_{3-x} samples. They conclude that interferences among time-reversal-conjugated closed loops are important in this regime. In fact, as in the weakly localized diffusive regime, the sign of these interferences, and the MC associated with their cancellation by magnetic fluxes, is reversed by SOS, as observed. A random-matrix-theory approach [1] obtains the MC as the result of changes of ξ itself depending on the invariance of the system under time-reversal and spin-reversal symmetries. A halving of ξ (respectively, doubling) was obtained when approximately a quantum of flux passes through ξ^2 if there is strong SOS (respectively, no SOS). Those results have been shown at least in the quasi-1D limit assuming that the removal of Kramers' degeneracy can be neglected in strong disorder. In higher dimensions, it has been argued [9] that simple changes of ξ remain valid.

In the variable-range-hopping regime, the conductance is given by

$$g(T) \propto \exp\left\{-\left(\frac{T_0}{T}\right)^{1/(d+1)}\right\} \propto \exp\left(-\frac{2r_m}{\xi}\right), \quad (1)$$

where T_0 is related to ξ by $k_B T_0 \sim d/g(E_F)\xi^d$, $g(E_F)$, d, and r_m being respectively the density of states at the Fermi surface, the dimensionality, and the Mott hopping length. As a consequence of the predicted changes for ξ, T_0 decreases and saturates when one applies a magnetic field in a system without SOS and

$$\ln[g(H)/g(0)] \sim T^{-1/(d+1)} \times [\xi(H)^{-d/(d+1)} - \xi(0)^{-d/(d+1)}]$$

is positive and diverges at low temperature. In contrast, in a system with strong SOS, T_0 increases and $\ln[g(H)/g(0)]$ is negative at low temperature. $\ln[g(H)/g(0)]$ should saturate when many flux quanta are put inside ξ.

Indeed, increases of ξ with a magnetic field have been reported in samples without SOS [1,10]. For instance, In_2O_{3-x} films (without SOS) studied down to $T=12$ mK exhibit an increase of ξ by a factor ~ 1.3 when the magnetic field grows from 1 to 7 T [10]. On the other hand, negative MC in the variable-range-hopping regime of amorphous Mo_xGe_{1-x} alloys [11], amorphous W-Re alloys [12], and Au-doped In_2O_{3-x} [8], where SOS is large, has been reported. But, as the temperature was above $T=1$ K, it is not possible to deduce if it results from a decrease of ξ.

It is the aim of this work to investigate the case of a barely disordered insulator with strong SOS down to very low temperature. A previous experiment on the same amorphous Y_xSi_{1-x} alloys indicates a multiplication of $\xi(H=0)$ by a factor of 0.6 for fields above 3 T [1]. In this paper we report new measurements in the a-Y_xSi_{1-x} series on samples with different T_0 parameters, as well as MC in a disordered insulator without SOS (a carbon-composite SPEER temperature sensor). The MC of a metallic sample of the same a-Y_xSi_{1-x} series is also presented to estimate the strength of the SOS by weak localization fits. The MC of strongly insulating a-Y_xSi_{1-x} alloys is also investigated to discuss the connection with forward-directed-path approaches.

Sample fabrication is described in Ref. [13], where it has also been reported that the metal-insulator transition takes place when the room-temperature conductivity falls below the critical Mott value [about 20000 $(\Omega\,\mathrm{m})^{-1}$ for an interatomic distance of 0.35 nm]. The two samples called sample 1 and sample 2 hereafter are made on the same run and the microscopic changes are probably small. Nevertheless, sample 1 is a metal and sample 2 is

TABLE I. Characteristics of the a-Y_xSi_{1-x} samples. At $T=4.2$ K it is formally possible to fit MC data for sample 2 by weak localization fits.

	Sample 1 (metal)	Sample 2 (insulator)
$\sigma_{300\text{ K}}$; $\sigma_{300\text{ K}}/\sigma_{4.2\text{ K}}$	27000 Ω^{-1}m^{-1}; 5.33	18000 Ω^{-1}m^{-1}; 15
$n(E_F)$	10×10^{45} J^{-1}m^{-3}	7×10^{45} J^{-1}m^{-3}
Phase-breaking length	$L_\phi=180$ nm at $T=12$ mK	$2r_m=82$ nm at $T=12$ mK
	130 nm at 98 mK	48 nm at 98 mK
	64 nm at 450 mK	33 nm at 0.45 K
	9.5 nm at 4.2 K	19 nm at 4.2 K
		$L_\phi=7.8$ nm at $T=4.2$ K
SO length	$L_{SO}=7.4$ nm at $T=12$, 98, 450 mK	
	$L_{SO}=5.2$ nm at 4.2 K	$L_{SO}=4.4$ nm at 4.2 K
Localization length ξ; T_0	···	$\xi=16.4$ nm; $T_0=7.4$ K

an insulator. Useful parameters are summarized in Table I. Sample 1 has a conductivity given between 0.012 and 4.2 K by

$$\sigma(T)=\sigma(0)+A\sqrt{T}$$

with

$$\sigma(0)=4135\ \Omega^{-1}\text{m}^{-1},\quad A\sim450\ \Omega^{-1}\text{m}^{-1}\text{K}^{-1/2}. \qquad (2)$$

Figure 1 displays the MC of sample 1 and the weak localization fits [14] with the parameters $L_\phi=(D\tau_\phi)^{1/2}$ and $L_{SO}=(D\tau_{SO})^{1/2}$ given in Table I. (The fits include a prefactor 4 in front of the theoretical magnetoconductivity, D is the diffusion constant, τ_ϕ and τ_{SO} are respectively the phase-breaking and the SO relaxation times.) Weak localization fits give $L_{SO}<L_\phi$ below $T=4.2$ K, and between $T=4.2$ and 0.098 K, $L_\phi(\text{m})\simeq4\times10^{-8}[T(\text{K})]^{-1/2}$. With these considerations we can approximate the factor A in (2) by [14]

$$A\simeq\frac{e^2}{4\pi^2\hbar}\left[0.915\left(\frac{4}{3}-\frac{3F}{2}\right)\left(\frac{k_B}{\hbar D}\right)^{1/2}-\frac{1}{4\times10^{-8}}\right]. \qquad (3)$$

The first term in brackets corresponds to the electron-electron interaction contribution in the diffusion channel

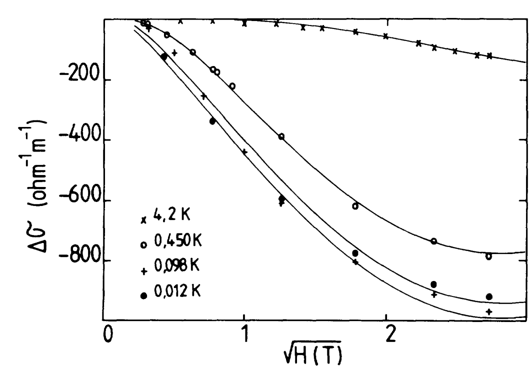

FIG. 1. Magnetoconductance for sample 1 and the weak localization fits [14] (continuous lines); the parameters L_ϕ and L_{SO} are listed in Table I.

(F is the screening parameter) and the second—including the sign—to the weak localization in the case of strong SOS. Strong SOS suppresses the triplet part in the diffusion channel such that $F=0$. We estimate that $k_Fl\sim1$ because of the proximity of the metal-insulator transition, and deduce that $D\sim4\times10^{-5}$ m^{2}s^{-1} as a rough estimation; with this value of D, we calculate $A\sim280$ Ω^{-1}m^{-1}K$^{-1/2}$ in good agreement with (2). Moreover for large magnetic fields, the weak localization contribution disappears and A must increase by a factor of 1.75, which corresponds to the observed factor: $A(7.5\ \text{T})\sim1.9A(B=0)$. So, weak localization with strong SOS explains the observed MC, as electron interactions in the singlet part of the diffusion channel, which is about 3 times larger but insensitive to the magnetic field, explain the main part of the temperature dependence of $\sigma(T)$.

Between $T=4.2$ and 0.08 K, sample 2 corresponds to a variable-range-hopping (VRH) activation law (1) with $d=3$ and $T_0\simeq7.4$ K. We estimate that $g(E_F)\sim7\times10^{45}$ J^{-1}m^{-3} by putting into the room-temperature conductivity the value of the diffusion constant D at the metal-insulator transition. From (1) we obtain $\xi\simeq16.4$ nm. Below $T=0.08$ K, $g(T)$ diverges slightly from (1) [Fig. 2(a)] and current-voltage nonlinearities emerge. This divergence can be attributed to the appearance of a Coulomb gap of about 0.3 K at the Fermi level [15]. The main effect of the magnetic field is to change the T_0 parameter in (1) (when T_0 is well defined, i.e., between $T=4.2$ and 0.08 K). If we suppose that $g(E_F)$ does not vary with H, variations of T_0 correspond to variations of ξ as plotted in Fig. 3. The field H_ξ such that

$$H_\xi\xi^2=\hbar/4e \qquad (4)$$

is indicated in Fig. 3. H_ξ is the typical field at which the time-reversal symmetry is broken on the scale ξ. Therefore, H_ξ is the field above which changes of ξ are calculated in Ref. [1]. Because L_{SO} is smaller than ξ (Table I), we are in the situation of strong SOS where Pichard *et al.* [1] predict a decrease of ξ. We indeed observe such a decrease by a factor of 0.675. The decrease of ξ implies a divergence of the magnetoresistance when the tempera-

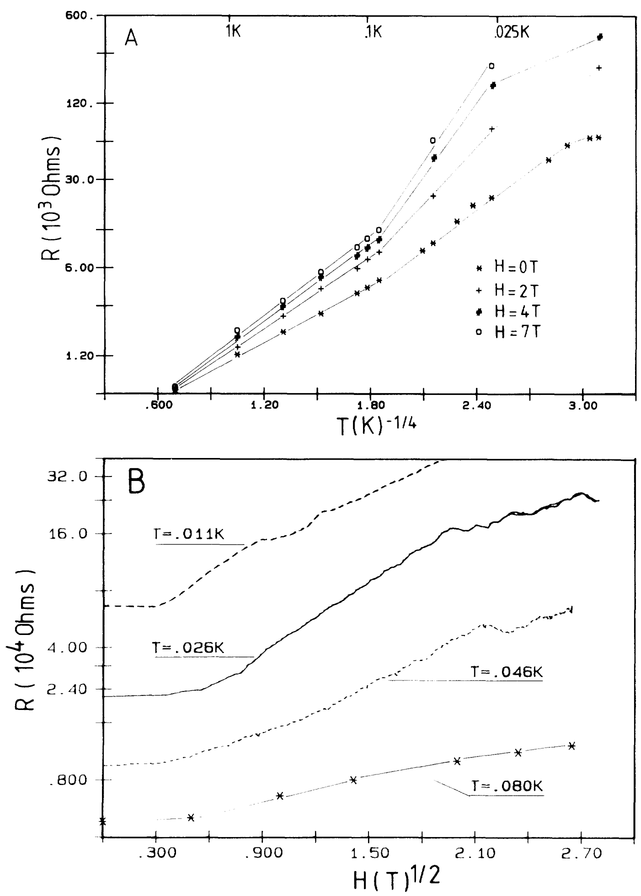

FIG. 2. (a) $\log(R)$ vs $T^{-1/4}$ at various magnetic fields for sample 2; the lines are guides for the eyes. Below $T=0.1$ K, $\log(R)$ diverges from the VRH law (see the text). (b) $\log(R)$ vs $\sqrt{H}$ at various temperatures for sample 2. The fluctuations seen at high field and low temperature are reproducible.

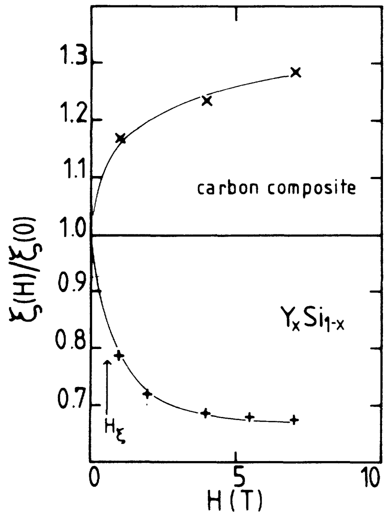

FIG. 3. Variation of the localization length with the magnetic field in our sample 2 (strong SOS) and in our SPEER carbon-composite sensor (no SOS). H_ξ is indicated by an arrow for sample 2.

ture decreases; below $T=0.08$ K, $\ln[R(H)]$ continues to diverge and saturates at high fields [Fig. 2(b)].

This negative MC is in striking contrast with the large observed positive MC of systems with no SOS, such as our carbon-composite sample, which obeys (1) below $T\simeq 1$ K ($d=3$, $T_0\sim 16$ K at $H=0$ T). Between 0.009 and 0.120 K the magnetic field decreases T_0; the corresponding variations of ξ are shown in Fig. 3. Surely no SOS is associated with the disordered configuration of carbon atoms.

In sample 2 we detect no sign of positive MC even at small magnetic fields [see Fig. 2(b)]. However, deep enough in the insulating regime and for small enough magnetic field ($H \ll H_\xi$), it is probably valid to neglect returning loops [7]. Consider $H_m r_m^{3/2} \xi^{1/2} \sim \hbar/4e$; $H_m(T)$ is the field necessary to put a flux quantum through the coherent "cigar-shaped" domain [7]. From (1) and (4) we have $H_m = H_\xi (T_0/T)^{-3/8}$ such that for $T_0/T \gg 1$ a small magnetic field changes dominantly the interferences among forward-directed paths inside the cigar-shaped domain, and a positive MC should appear (whatever the SOS strength). For T_0 of a few kelvin, H_ξ hardly exceeds a few H_m even in dilution refrigerators, such that a magnetic field which changes interferences among forward-directed paths also modifies interferences between the largest time-reversal-conjugated closed loops of diameter $\gtrsim \xi$. To test how deep it is necessary to go in the insulating regime (with strong SOS) to restore positive MC at low fields, we perform conductance measurements in fairly insulating a-Y_xSi_{1-x} samples (with large T_0), with a three-terminal capacitance bridge technique implying two probes only on the sample. Preliminary comparison between two- and four-probe measurements on our insulating samples with a standard resistance bridge indicates that the two-probe conductance also follows the variable-range-hopping law (1) but with a larger T_0 than the four-probe conductance (two-probe resistance is of the order of four-probe resistance at $T=4.2$ K). In a two-probe measurement characterized by a variable-range-hopping law (1) with $T_0\sim 300$ K between $T=4.2$ and 0.09 K, we observe a positive MC of about 8% for $T=0.15$ K and $H=0.1$ T before a large negative MC at higher fields. Figure 4 shows the MC at $T\simeq 1$ K of a set of samples (3-5) of increasing resistivity; $\sigma_{300\,\mathrm{K}}/\sigma_{4.2\,\mathrm{K}}$ is 24.8, 4650, and 46670 and $\sigma_{4.2\,\mathrm{K}}/\sigma_{1\,\mathrm{K}}$ is 3.9, 1520, and 4600, respectively [16]. As sample 2, sample 3, for which we estimate $T_0\sim 400$ K, shows only negative MC at least down to $T=1$ K. But samples 4 and 5 exhibit a positive MC at small fields which grows when T decreases and reaches about 6% at $T=1$ K in sample 5 (not represented). The field scale for this effect is T dependent such that we believe that we reach the regime where interferences between forward-directed paths are important. In samples 4 and 5 we certainly are unable to

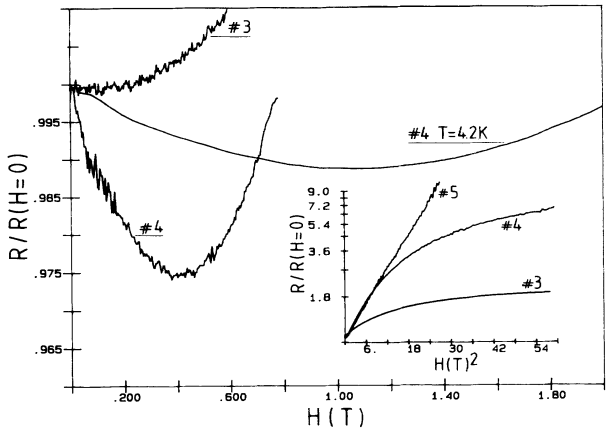

FIG. 4. $R/R(H=0)$ vs H for samples 3 and 4. The temperature is about 1 K except for one curve. Inset: $\log[R/R(H=0)]$ vs H^2 at $T\sim 1$ K for samples 3–5. $R(H=0)$ is respectively 727.3 Ω, 158 MΩ, and 56.5 GΩ at $T=1$ K for samples 3–5 and 103.9 kΩ for sample 4 at $T=4.2$ K.

reach H_ξ, because of the decrease of ξ, and we detect no saturation of $\ln[R(H)]$ at the highest fields. On the contrary we are more sensitive to the field dependence of the MC for $H \ll H_\xi$; we find that—after the initial small positive MC—this regime is well described by $\ln R \sim H^2$ (see the inset of Fig. 4).

Our data and a few previous experiments indicate that in the barely insulating regime, there is at low temperature a very large negative MC in the presence of strong SOS and a very large positive MC in the absence of SOS. Below 1 K the data are consistent with changes of ξ when a few flux quanta are put inside the localization domain. The comparison between samples such as In_2O_{3-x} films, carbon composites, and our series, which exhibit similar $g(E_F)$ and ξ but opposite MC behavior, is helpful to discriminate SO effects from others, such as Zeeman and many-electron effects.

In summary, our studies illustrate the rich variety of MC behaviors occurring in disordered insulators, depending on the value of ξ and on the magnetic field, and show the limitations of the different theoretical approximations.

We thank J. L. Pichard for motivating this work and numerous fruitful discussions, B. Boucher and R. Tourbot for their contribution to the amorphous samples fabrication, and P. Pari for his cryogenic support.

[1] J. L. Pichard, M. Sanquer, K. Slevin, and P. Debray, Phys. Rev. Lett. **65**, 1812 (1990).
[2] Y. Meir, N. S. Wingreen, O. Entin-Wohlman, and B. L. Altshuler, Phys. Rev. Lett. **66**, 1517 (1991).
[3] E. Medina and M. Kardar, Phys. Rev. Lett. **66**, 3187 (1991).
[4] J. P. Bouchaud, J. Phys. I (France) **1**, 985 (1991).
[5] E. Medina, M. Kardar, Y. Shapir, and X. R. Wang, Phys. Rev. Lett. **64**, 1816 (1990).
[6] H. L. Zhao, B. Z. Spivak, M. P. Gelfand, and S. S. Feng (to be published).
[7] V. L. Nguen, B. Z. Spivak, and B. I. Shklovskii, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 35 (1986) [JETP Lett. **43**, 44 (1986)].
[8] Y. Shapir and Z. Ovadyahu, Phys. Rev. B **40**, 12441 (1989).
[9] J. L. Pichard, in *Proceedings of Localization 1990,* IOP Conf. Proc. No. 108 (Institute of Physics and Physical Society, London, 1990).
[10] F. P. Milliken and Z. Ovadyahu, Phys. Rev. Lett. **65**, 911 (1990).
[11] S. Yoshizumi, T. H. Geballe, M. Kunchur, and W. L. McLean, Phys. Rev. B **37**, 7094 (1988).
[12] P. Chaudhari and H. U. Habermeier, Phys. Rev. Lett. **44**, 40 (1980).
[13] M. Sanquer, R. Tourbot, and B. Boucher, Europhys. Lett. **7**, 635 (1988).
[14] B. L. Altshuler, A. G. Aronov, M. E. Gershenson, and Yu. V. Sharvin, Sov. Sci. Rev. Sect. A **9**, 233 (1987).
[15] B. I. Shklovskii and A. L. Efros, in *Electronic Properties of Doped Semiconductors,* edited by M. Cardona (Springer-Verlag, Berlin, 1984).
[16] In this restricted temperature range 1 K < T < 4.2 K, it is hard to distinguish the exponent of T in (1): $-\frac{1}{4}$ or $-\frac{1}{2}$.