New Mass Formula for Octet and Decuplet Baryons

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The general parametrization method [Phys. Rev. D **40**, 2997 (1989)] (an exact consequence of a QCD-like relativistic field theory) is used to parametrize the masses of **8** and **10** baryons to second order in flavor breaking. Because at first order in flavor breaking, three-quark contributions to octet masses are very small, we neglect such contributions at second order and derive a "corrected" Gell-Mann–Okubo formula; when written in a way free from electromagnetic corrections, it is $\frac{1}{2}(n+\Xi^0)+T=\frac{1}{4}(3\Lambda+\hat{\Sigma}^+)$ with $T=\Xi^{*-}-\frac{1}{2}(\Omega+\Sigma^{*-})$ and $\hat{\Sigma}^+=2\Sigma^+-\Sigma^0+2(n-p)$. The two sides of the formula are respectively 1132.4 ± 0.8 and 1133.9 ± 0.1 MeV.

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As is well known the Gell-Mann–Okubo mass formula for the baryon octet and the equal spacing rule for the decuplet,

$$(n+\Xi)/2=(3\Lambda+\Sigma)/4,\quad \Omega-\Xi^*=\Xi^*-\Sigma^*=\Sigma^*-\Delta\,, \qquad (1)$$

are true to first order in flavor breaking. To second order, they do not hold; only one relationship remains, $\Omega-\Delta=3(\Xi^*-\Sigma^*)$, as shown by Okubo [1] long ago. Of course the same conclusion is reached using, instead of the standard tensor or group procedure, our general parametrization [2] in the spin-flavor space and keeping all terms to second order in flavor breaking. But the terms in the spin-flavor parametrization are not classified only by their order in flavor breaking; thus, using such parametrization, one is led to consider other possible approximations besides just the order in flavor breaking.

We will show that if in the spin-flavor parametrization, which is an exact consequence of any QCD-like relativistic field theory, we keep all second-order flavor-breaking terms except those containing the variables of three quarks (we will show why such terms are expected to be small), one obtains a mass formula [Eq. (2)] that improves the conventional Gell-Mann–Okubo formula:

$$\frac{1}{2}(n+\Xi^0)+T=\frac{1}{4}(3\Lambda+\hat{\Sigma}^+)\,, \qquad (2)$$

$$(1132.4\pm0.8)\quad(1133.9\pm0.1)$$

where

$$T=\Xi^{*-}-\frac{1}{2}(\Omega+\Sigma^{*-})=5.2\pm0.7\text{ MeV} \qquad (3)$$

and $\hat{\Sigma}^+$ means $\hat{\Sigma}^+=2\Sigma^+-\Sigma^0+2(n-p)$. The charge specifications and the replacement of Σ by $\hat{\Sigma}$ are due to the fact that Eq. (2) must take into account, at this accuracy, the electromagnetic contributions. The combinations of masses in Eq. (2) are such that Eq. (2) is unaffected by the electromagnetic corrections (calculated at zero order in flavor breaking) [3]. The standard Gell-Mann–Okubo formula (made free from electromagnetic effects) is the same Eq. (2) without the T term; its two sides are left$=1127.2\pm0.3$ and right$=1133.9\pm0.1$ MeV.

We recall a few results of the general parametrization method. If the underlying theory is QCD-like, that is, (a) the only flavor λ matrix in the strong field Lagrangian is λ_8 (from the mass terms), and (b) the electromagnetic and weak currents are carried only by the quarks (that is, the Lagrangian of the theory is the basic one, constructed only in terms of quark and gluon fields without, e.g., explicit pion fields), the parametrization of various baryon properties (Refs. [2(a)–2(d)]) turns out to be very simple. It looks similar to that in a naive constituent nonrelativistic quark model [4] (NRQM); but it is exact and thus fully relativistic although noncovariant. The importance of the above properties (a) and (b) is due to the fact that when calculating any strong or electromagnetic quantity only λ_8 and λ_3 enter; because they commute no λ (flavor) different from λ_8 and λ_3 can appear in the end result, and thus no Casimir flavor operator. This simplifies the parametrization, reducing the variety of terms that can appear.

For the masses of the **8**,**10** lowest baryon states the parametrization correct to first order in flavor breaking is (Ref. [2(a)], Sec. XII)

$$M(1)=M_0+B\sum_i P_i^\lambda+C\sum_{i>k}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)+D\sum_{i>k}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)(P_i^\lambda+P_k^\lambda)+E\sum_{\substack{i\neq k\neq j\\(i>k)}}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)P_j^\lambda\,. \qquad (4)$$

The 1 in $M(1)$ recalls that the mass is correct to first order; M_0,B,C,D,E are five real parameters, the $\boldsymbol{\sigma}_i$'s are Pauli spin matrices. The projection operator on the strange quark $P^\lambda=\frac{1}{3}(1-\lambda_8)$ arises from the flavor-breaking mass term in the Lagrangian; this is assumed [2] to be expressed in terms of the small q^2 mass-renormalized quark fields—the constituent quark fields. The indices in (4) and the sums extend from 1 to 3; to obtain the mass of a baryon one takes the expectation value of (4) in its NRQM (or SU_6) spin-flavor state. We repeat that in spite of its nonrelativistic, NRQM-like, appearance, Eq. (4) is an exact formula (to first order in flavor breaking). Because in (4) there are five parameters and eight masses to fit, we deduce (Ref. [2(a)]) three mass formulas; they are of course the Gell-Mann–Okubo formula and the decuplet equal spacing rule of Eqs. (1). The values of the parameters are (in MeV) $M_0=1086$, $B=188.4$, $C=49.2$, $D=-15.4$, and $E=4.0\pm0.1$ [5]; the term E, the smallest one, is the only three-quark term. Equation (82) of Ref. [2(a)] (where $\epsilon=-E/2$) shows that it contributes less than 5 parts per 10^3 to the masses of the octet baryons.

Extending the general spin-flavor parametrization to second order in flavor breaking, the masses $M(2)$ become

$$M(2)=M(1)+a\sum_{i>k}P_i^\lambda P_k^\lambda+b\sum_{i>k}(\sigma_i\cdot\sigma_k)P_i^\lambda P_k^\lambda + \tfrac{1}{2}c\sum_{i\neq k\neq j}(\sigma_i\cdot\sigma_k)(P_i^\lambda+P_k^\lambda)P_j^\lambda . \qquad (5)$$

The three new terms in (5) with coefficients a,b,c clearly affect only the baryons with strangeness 2 or 3 (Ξ in the octet and Ξ^*,Ω in the decuplet); the others are unmodified. For Ω,Ξ^*,Ξ the second-order corrections due to the terms a,b,c in (4) are as follows: Ω, $3(a+b+c)$; Ξ^*, $a+b+c$; and Ξ, $a+b-2c$. It is clear that the equal spacing for the decuplet no longer holds but that the Okubo relationship $\Omega-\Delta=3(\Xi^*-\Sigma^*)$ remains true. In fact with 8 masses and 7 parameters (note that a and b enter only in the combination $a+b$) there can be only one relationship holding to second order. If we included third-order flavor breaking (affecting only the Ω), no relationship would remain, obviously.

Assume now that the three-quark term in Eq. (5) is absent ($c=0$); that this approximation may be reasonable is suggested by the fact stated above, that the three-quark term contribution is already very small [6] at first order in flavor breaking and the c term in (5) is further depressed, being of next order in $\Delta m/m_\lambda\simeq0.34$, the flavor-breaking parameter. In this approximation only the parameter $a+b$ remains at second order in flavor breaking; all **8** and **10** baryon masses are then summarized by Eq. (6) below (S represents strangeness) [7]:

$$M(2)=M(1)+\tfrac{1}{2}(a+b)(S^2+S)\equiv M(1)-T(S^2+S) . \qquad (6)$$

Because we have altogether 6 parameters, two relationships exist; they are that of Eq. (2) with $T\equiv-(a+b)/2$ [expressed in terms of the masses by Eq. (3)] and the second-order Okubo equation $\Omega-\Delta^-=3(\Xi^{*-}-\Sigma^{*-})$, now rewritten with explicit charges so as to be free of electromagnetic effects—see below. Using the latter equation we can also write T as

$$T=\Sigma^{*-}-\tfrac{1}{2}(\Delta^-+\Xi^{*-}) . \qquad (7)$$

We prefer to evaluate T from (3) because the experimental errors on the Δ's are larger than those of the other masses of the decuplet. For $\Delta^-=1233\pm\sigma$, Eq. (7) would give $T=3.2\pm0.4\pm\sigma/2$ instead of 5.2 ± 0.7 of Eq. (3); but σ is at least 3 and a more precise test of Okubo's second-order formula, the equality of the T's of Eqs. (3) and (7), needs a good knowledge of the Δ masses.

As another remark, we called "new" the mass formula (2); indeed this formula including the electromagnetic effects is new, as far as we know. But leaving aside the electromagnetic corrections, Eq. (2) is in fact very old. A set of mass relationships that, when combined linearly, led to Eq. (2) were derived in 1966 by Federman, Rubinstein, and Talmi [8] using the nonrelativistic quark model, neglecting three-quark interactions. The new feature here is that Eq. (2) is now derived by the general parametrization; thus its validity does not depend on the potential model description. In fact we have here a new case, in addition to those of Refs. [2(a)-2(d)], where the general parametrization explains the quantitative success of the NRQM; the motivation [2] for the parametrization method was indeed to understand these quantitative successes.

Finally, we discuss electromagnetic effects in Eq. (2) and in the other relationships. Clearly the spin-flavor parametrizations (4), (5), (6) refer to the eigenvalues of the strong Hamiltonian, the "strong" masses. The observed masses include, however, an electromagnetic contribution δM; to deduce the strong masses one must calculate and subtract from the observed masses the electromagnetic contribution. This can be done again by the parametrization method. On calculating the electromagnetic contribution δB to baryon B at zero order in flavor breaking, one can check [9] that the sum of the electromagnetic corrections $\frac{1}{2}(\delta n+\delta\Xi^0)$ on the left-hand side of Eq. (2) is equal to the sum of the electromagnetic corrections on the right-hand side $\frac{1}{4}(3\delta\Lambda+\delta\hat{\Sigma}^+)$; also (with the charge values appearing in the definition of T) it is $\delta T=0$. Thus Eq. (2) as written, with the charges indicated and $\hat{\Sigma}^+$ defined above, is independent of electromagnetic corrections at zero order in flavor breaking.

As a final remark, this study has its basis in the remarkable decrease during the years of the experimental errors on the masses of the decuplet baryons; it becomes of some interest to reduce them further, if possible [10].

[1] S. Okubo, Phys. Lett. **4**, 14 (1963).

[2] (a) G. Morpurgo, Phys. Rev. D **40**, 2997 (1989); (b) **40**, 3111 (1989); (c) **41**, 2865 (1990); (d) **42**, 1497 (1991). In (a) the method is formulated and applied to the magnetic moments, electromagnetic transition matrix elements, and masses of the baryons, in (b) it is applied to the semileptonic baryon decays, in (c) to the meson masses, and in (d) to the $V\to p\gamma$ meson decays. Note the following misprints in (a): in G, Eq. (45), read $\frac{1}{2}\Lambda$ instead of $\frac{1}{4}\Lambda$; the combination $\delta-\beta-2\gamma$ (not $\delta-\beta+2\gamma$) should appear in Eqs. (64) and (66). We used the correct formulas in the calculations.

[3] Of course Eq. (2) might also be written $\frac{1}{2}(p+\Xi^0)+T=\frac{1}{4}(3\Lambda+2\Sigma^+-\Sigma^0)$.

[4] (a) G. Morpurgo, Physics (N.Y.) **2**, 95 (1965) [also reproduced in J. J. Kokkedee, *The Quark Model* (Benjamin, New York, 1969), p. 132]; (b) *Theory and Phenomenology in Particle Physics,* Proceedings of the International School of Physics "E. Majorana," Erice, Italy, 1968, edited by A. Zichichi (Academic, New York, 1969), pp. 83–217; (c) Annu. Rev. Nucl. Sci. **20**, 105–146 (1970).

[5] All masses and mass parameters are in MeV. For their values see Particle Data Group [J. J. Hernández *et al.*, Phys. Lett. B **239**, 1 (1990)]. The best determination of E is from $\Xi^{*-}-\Xi^--(\Sigma^{*-}-\Sigma^-)=6E$; this equation is not affected by electromagnetic corrections nor by the second-order flavor corrections considered here. We note an additional point on the three-quark terms in the parametrization: The correlations among quark variables implied by the fixed angular momentum $2\mathbf{J}=\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2+\boldsymbol{\sigma}_3$ allow us to simplify the three-quark terms transforming them into terms with less than three quarks. It is understood that three-quark terms are those that appear before the exploitation of such correlations.

[6] Though the E term contributes less than 5 per 10^3 to the octet masses, it contributes up to 5% to some decuplet ones [Eq. (83) of Ref. [2(a)]]; however, the decuplet combination T [Eq. (3)] is independent of E.

[7] For convenience we list the masses in Eqs. (82) and (83) of Ref. [2(a)] (correct to first-order flavor breaking) completed by Eq. (6) to include second-order flavor breaking (except three-quark terms): $\Xi(\text{new})=\Xi[\text{Eq. (82)}]-2T$, $\Xi^*(\text{new})=\Xi^*[\text{Eq. (83)}]-2T$, $\Omega(\text{new})=\Omega[\text{Eq. (83)}]-6T$; all other masses remain unchanged as in Eqs. (82) and (83).

[8] P. Federman, H. Rubinstein, and I. Talmi, Phys. Lett. **22**, 208 (1966); see also Ref. [4(b)], p. 163.

[9] Though electromagnetic masses will be treated elsewhere, we sketch how their effects cancel in Eq. (1). The parametrized δB of octet baryons contains four parameters μ,ν,η,ρ. It is $\delta p=\mu+\frac{5}{9}\nu+\eta+\rho$, $\delta n=\frac{2}{3}\mu$, $\delta\Lambda=\frac{2}{3}\mu+\frac{1}{9}\nu$, $\delta\Sigma^+=\mu+\frac{5}{9}\nu+\eta+\rho$, $\delta\Sigma^-=\frac{1}{3}\mu+\frac{1}{9}\nu+\eta+\frac{1}{3}\rho$, $\delta\Sigma^0=\frac{2}{3}\mu+\frac{1}{3}\nu$, $\delta\Xi^0=\frac{2}{3}\mu$, $\delta\Xi^-=\frac{1}{3}\mu+\frac{1}{9}\nu+\eta+\frac{1}{3}\rho$. One checks that $\frac{1}{2}(\delta n+\delta\Xi^0)=\frac{1}{4}(3\delta\Lambda+\delta\hat{\Sigma}^+)$ [here $\delta\hat{\Sigma}^+\equiv 2\delta\Sigma^+-\delta\Sigma^0+2(\delta n-\delta p)$]. Incidentally the Coleman-Glashow relationship $\delta n-\delta p+\delta\Xi^--\delta\Xi^0=\delta\Sigma^--\delta\Sigma^+$ is obviously satisfied by the above expressions of δM. As to the decuplet we find $\delta\Omega=\delta\Sigma^{-*}=\delta\Xi^{-*}=\delta\Delta^-$; so it is $\delta T=0$ and the Okubo decuplet formula when written as $\Omega-\Delta^-=3(\Xi^{*-}-\Sigma^{*-})$ is not affected by electromagnetic effects; we repeat that the above electromagnetic δB's are all parametrized only to zero order in flavor breaking but this should be sufficient at this stage.

[10] Keeping also the term c of Eq. (5) (*but not the third-order flavor breaking* $P_1^\lambda P_2^\lambda P_3^\lambda$), it is $\Xi^{*-}-\Xi^--(\Sigma^{*-}-\Sigma^-)=6E+3c$. Fitting all **8** and **10** masses one gets $E\simeq 3.4$ and $c\simeq 1.17$. This checks that c is small compared to $(a+b)$; now $(a+b+c)/2\equiv -T+c/2=-5.2\pm 0.8$; moreover, c/E, expected to be of order $\Delta m/m_\lambda$, is indeed $\simeq 0.34$. As to the $P_1^\lambda P_2^\lambda P_3^\lambda$ term, it will be determined univocally from the Okubo second-order formula when the Δ^- mass will be known precisely.