Isospin-Dependent Effective Interaction in Nucleon-Nucleus Scattering

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We study the isospin-dependent component of the effective nucleon-nucleon interaction which causes the $\Delta T=1$ (p,p') and (p,n) reactions off nuclei. It is shown that, at intermediate energies, the modification to the impulse approximation comes from the g-matrix-type correction and the rearrangement term. They are numerically estimated with the isospin-asymmetric nuclear-matter reaction matrix approach. The isobaric-analog transitions ${}^{42}\mathrm{Ca}(p,n){}^{42}\mathrm{Sc}$ and ${}^{48}\mathrm{Ca}(p,n){}^{48}\mathrm{Sc}$ are analyzed.

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The nucleon direct reaction of a few hundred MeV projectile energy has become one of the preferred tools to extract nuclear structure information, complementing reactions with electromagnetic probes. Its success crucially depends on a good command of the effective nucleon-nucleon interaction utilized in the calculations based on the distorted-wave Born approximation (DWBA). From the theoretical point of view, there is a possibility of performing a convergent calculation of the Watson expansion [1] or its variants [2,3], since the impulse approximation is expected to be a reasonable starting point at these energies. In the practical analysis with currently available computational resources, however, the treatment based on the *density-dependent effective interaction* [4,5] seems to be the only workable scheme to handle the correction to the impulse approximation or the medium correction.

For elastic scattering, the optical-model potential obtained by folding the Brueckner reaction matrix (g matrix) [6] is known to represent the leading terms in the Watson expansion [7], and the g-matrix approach has been reasonably successful in describing the elastic-scattering experiments [4,5,8,9]. For inelastic scattering, we have shown for the simplest case of the isoscalar natural-parity transitions that the effective interaction is the sum of the g matrix and the rearrangement term expressible as the density derivative of the g matrix [10,11]. Attempts to describe experimental data along this line have been pursued in recent years, and have met with great success. The set of density-dependent effective interactions constructed by Kelly and co-workers [12–14] has been proven quantitatively accurate and practically useful in the description of the elastic scattering and the isoscalar natural-parity excitations. In this background, we believe it quite urgent to identify the medium correction in other components of the effective interaction than the spin-isospin–independent part. That would enable us to take full advantage of the nucleon projectile with its capacity to excite a variety of nuclear modes not accessible by other probes.

Although the isobaric-analog excitation [15] has been one of the best-studied nuclear excitation modes, the isospin-dependent part of the effective interaction has been given relatively little theoretical attention [16]. This unsatisfactory state of affairs needs to be altered before any systematic empirical analysis can be performed. In this Letter, we intend to identify the leading medium correction to the isospin-dependent component of the effective nucleon-nucleon interaction which induces the isovector nuclear excitations. In the formulation, the g matrix appears in *isospin-asymmetric nuclear matter*, where the proton Fermi momentum differs from the one for the neutron. This is necessary even for isospin-symmetric nuclei in the evaluation of the rearrangement term for the isovector transitions. We report the result of a numerical example at the incident energy $E_p=150$ MeV. It is applied to the isobaric-analog transitions ${}^{42}\mathrm{Ca}(p,n){}^{42}\mathrm{Sc}$ and ${}^{48}\mathrm{Ca}(p,n){}^{48}\mathrm{Sc}$ at $E_p=135$ MeV. We hope that this work will revitalize the microscopic analysis of *all the components* of the effective interaction for intermediate-energy nucleon scattering.

The g matrix in isospin-asymmetric nuclear matter satisfies the Bethe-Goldstone equation formally identical to the one in symmetric nuclear matter, namely,

$$g=v+vG_0Qg\,, \tag{1}$$

where v is the bare two-nucleon interaction, G_0 is the free two-nucleon propagator, and Q is the Pauli exclusion operator. The self-energy correction to the propagator which plays a minor role at sufficiently high energy is neglected. The difference of proton and neutron densities ρ_p and ρ_n generates the *isovector component* in Q in addition to the isoscalar component present in symmetric nuclear matter [17]. As a result, the effective interaction g obtains four independent components g_{TT_z} specified by the total isospin T of the two interacting nucleons and its projection T_z. Featuring the tensor property in the isospin space, we write the g matrix in the form

$$g=g^0+g^\tau\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2+g^\alpha(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2)+g^\beta\sqrt{2/3}[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^{(2)}\,, \tag{2}$$

where $\boldsymbol{\tau}_1$ and $\boldsymbol{\tau}_2$ are the isospin operators of two interacting nucleons. The four coefficients in Eq. (2) can be written in terms of g_{TT_z} as

$$g^0=\tfrac{1}{4}(g_{11}+g_{10}+g_{1-1}+g_{00}), \tag{3a}$$

$$g^\tau=\tfrac{1}{12}(g_{11}+g_{10}+g_{1-1}-3g_{00}), \tag{3b}$$

$$g^\alpha=\tfrac{1}{4}(g_{11}-g_{1-1}), \tag{3c}$$

$$g^\beta=\tfrac{1}{6}(g_{11}-2g_{10}+g_{1-1}). \tag{3d}$$

In the isospin-symmetric limit, g^α and g^β naturally disappear. From Eqs. (3a) to (3d), one recognizes that g^0, g^τ, and g^β are invariant with respect to the exchange of protons and neutrons in the medium $\rho_p \leftrightarrow \rho_n$, while g^α changes its sign. These properties put restrictions on the possible form of the density dependence of each term. A better view is obtained by defining the isoscalar density $\rho^s \equiv \rho_p+\rho_n$ and the isovector density $\rho^v \equiv \rho_p-\rho_n$. The exchange of proton and neutron is now represented by $\rho^v \leftrightarrow -\rho^v$. At the isospin-symmetric limit, therefore, the following odd functions of ρ^v disappear:

$$\frac{\partial}{\partial\rho^v}g^0=\frac{\partial}{\partial\rho^v}g^\tau=\frac{\partial}{\partial\rho^v}g^\beta=0. \tag{4a}$$

Taking the leading density dependence of g^α and g^β, one obtains

$$g^\alpha \approx \rho^v\frac{\partial}{\partial\rho^v}g^\alpha \text{ and } g^\beta \approx \tfrac{1}{2}(\rho^v)^2\frac{\partial^2}{\partial\rho^{v2}}g^\beta. \tag{4b}$$

We consider the elastic and inelastic scattering in order. The optical-model potential U of a nucleon in isospin-asymmetric nuclear matter is given by folding the interaction g with the asymmetric nuclear-matter density. It is convenient to split the resultant optical-model potential into isoscalar and isovector parts *à la* Lane as [18]

$$U=U^s+U^v\tau_z, \tag{5}$$

where each term is given by

$$U^s=g^0\rho^s+g^\alpha\rho^v \approx g^0\rho^s, \tag{6a}$$

and

$$U^v=g^\alpha\rho^s+(g^\tau+g^\beta)\rho^v \approx g^\alpha\rho^s+g^\tau\rho^v$$
$$\approx \left[\rho^s\frac{\partial}{\partial\rho^v}g^\alpha+g^\tau\right]\rho^v. \tag{6b}$$

In the derivation of U^s and U^v, we have dropped all terms that are of second or higher orders in powers of ρ^v. In the last line of Eq. (6b), the relation Eq. (4b) is used. The above expressions show that the optical-model potential U is obtained by folding

$$v_{\rm el}=g^0+\left[\rho^s\frac{\partial}{\partial\rho^v}g^\alpha+g^\tau\right]\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$$
$$\equiv v_{\rm el}^0+v_{\rm el}^\tau\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2, \tag{7}$$

with the nuclear densities ρ^s and ρ^v. One can therefore regard the interaction $v_{\rm el}$ defined by Eq. (7) as the effective interaction for elastic scattering.

For inelastic scattering, both through diagrammatical analysis of Watson expansion and the macroscopic collective excitation model, one can equate the inelastic transition potential with the density derivative of the optical-model potential U [10,11]. Assuming that the transition of the target nucleus is described by the isoscalar and isovector transition densities $\rho_{\rm tr}^s$ and $\rho_{\rm tr}^v$, one can express the transition potential $U_{\rm tr}$ which describes inelastic scattering as

$$U_{\rm tr}=\rho_{\rm tr}^s\frac{\partial}{\partial\rho^s}U^s+\rho_{\rm tr}^v\frac{\partial}{\partial\rho^v}U^v\tau_z. \tag{8}$$

This yields, for $N=Z$ nuclei ($\rho^v=0$), the expression

$$U_{\rm tr}=\rho_{\rm tr}^s\left[g^0+\rho^s\frac{\partial}{\partial\rho^s}g^0\right]+\rho_{\rm tr}^v\left[g^\tau+\rho^s\frac{\partial}{\partial\rho^v}g^\alpha\right]\tau_z, \tag{9}$$

which shows that the transition potential can be obtained from the effective interaction

$$v_{\rm in}=v_{\rm in}^0+v_{\rm in}^\tau\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2, \tag{10}$$

where the isospin-independent and -dependent components, $v_{\rm in}^0$ and $v_{\rm in}^\tau$, are given by

$$v_{\rm in}^0=g^0+\rho^s\frac{\partial}{\partial\rho^s}g^0 \tag{11a}$$

and

$$v_{\rm in}^\tau=g^\tau+\rho^s\frac{\partial}{\partial\rho^v}g^\alpha. \tag{11b}$$

The second term in Eq. (11a) is the so-called rearrangement term for the isoscalar transition. Its relation to the full multiple-scattering theory [3] is diagrammatically analyzed in Ref. [10]. Equation (11b) is one of the main results of this Letter. The analysis of its microscopic content similar to the case of Eq. (11a) is possible though slightly cumbersome. Its second term is analogous to the one in Eq. (11a), and is no less remarkable since it shows up at $\rho^v=0$ despite that g^α itself vanishes at this limit. For $N\neq Z$ nuclei, there are corrections to Eqs. (7), (11a), and (11b). They can be safely neglected in the case of $N\approx Z$, since they are quantities of order ρ^v. Comparing Eq. (7) with Eq. (11b), one finds

$$v_{\rm in}^\tau=v_{\rm el}^\tau \tag{12}$$

for the isospin-dependent effective interactions. Notice that the rearrangement *factor* which connects the elastic and inelastic effective interactions in the isospin-independent channel [10] is absent here. We stress that this *does not* mean the absence of the rearrangement term. It is achieved by natural but formal redefinition of the elastic effective interaction, Eq. (7). In principle, therefore, with Eq. (12), the isospin-dependent empirical interactions can be constructed from the combined analysis of the elastic scattering *and* the $\Delta T=1$ inelastic and charge-exchange reactions of $N\approx Z$ nuclei in a manner similar to that for the isospin-independent counterpart [12–14].

We now turn to the numerical assessment of the medium correction to the isospin-dependent component of the effective interaction. The detailed description of the technique to solve the isospin-asymmetric Bethe-Goldstone equation, Eq. (1), and the full numerical results will be published elsewhere. The method is a straightforward extension of the one developed for the solution of the symmetric nuclear-matter g matrix without the "angle-average" approximation [19]. Here, we show the relevant result of an example of such a calculation. At the projectile energy of $E_p = 150$ MeV, the Reid soft core potential [20] as the input free two-nucleon interaction, we obtain the following numbers for the *volume integrals* (or the strength of the interaction in zero-range approximation) of isospin-dependent components of the g matrix in the leading order of nuclear densities:

$$g^{\tau} = [\{22+75i\} + \{-24-53i\}\rho^{s}/\rho_0] + [\{8+22i\} + \{17-17i\}\rho^{s}/\rho_0]\mathbf{L}\cdot\mathbf{S}\,, \qquad (13)$$

$$g^{a} = [\{6+38i\}\rho^{v}/\rho_0] + [\{-1+1i\}\rho^{v}/\rho_0]\mathbf{L}\cdot\mathbf{S}\,, \qquad (14)$$

where the numbers are in units of MeV fm^3, and $\rho_0 = 0.17$ fm^{-3} is the nuclear-matter saturation density. The g matrices of Eqs. (13) and (14) include both the density dependence arising from the Pauli blocking and the one induced by the nonlocality (see Ref. [19]). The density dependence of g^{τ} of Eq. (13) is comparable to the previous results [21,22] in sign and order, but is stronger in magnitude. The inclusion of the nonlocality, which has been omitted in earlier g-matrix calculations, accounts for a substantial part of the stronger density dependence found here, but more careful cross checking will be needed. The antisymmetrization is already taken into account in Eqs. (13) and (14), and only their direct amplitudes are to be included in DWBA calculations. One observes that in the effective interaction $v_{\rm in}^{\tau}$ (or equivalently in $v_{\rm el}^{\tau}$), the density dependence of g^{a} tends to cancel that of g^{τ}, especially in the imaginary part of the central force. More to the point, these two density-dependent corrections are of the same order. This clearly displays that partial inclusion of the medium correction, only with the g matrix, for example, cannot be very meaningful.

In order to show how the density-dependent effective interactions, Eqs. (13) and (14), work in inelastic scattering, we show the result of DWBA calculations [23] for the isobaric-analog transitions ^{42}Ca$(p,n)^{42}$Sc and ^{48}Ca$(p,n)^{48}$Sc at $E_p = 135$ MeV. To facilitate the calculation, we adopt the following two approximations: First, we take the empirical optical-model potential of Schwandt *et al.* [24] instead of calculating it microscopically. Second, we replace the density-*independent* part of Eq. (13) by the Yukawa parametrization of the free two-nucleon scattering matrix by Franey and Love [25] at $E_p = 140$ MeV (FL140). The tensor part of the FL140

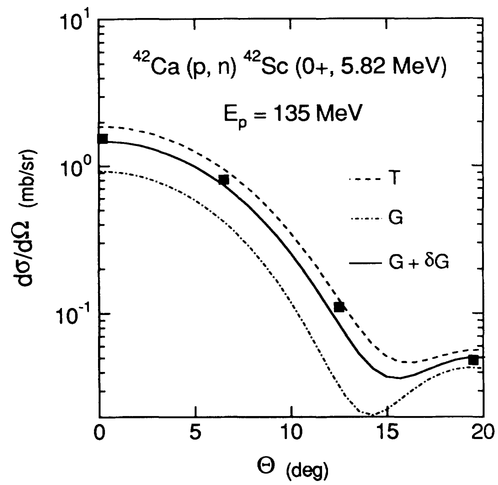

FIG. 1. The differential cross section for the reaction ^{42}Ca$(p,n)^{42}$Sc. The dashed, dot-dashed, and solid lines, respectively, are the results of the impulse approximation, g matrix, and the full calculation.

interaction is used without additional density dependence. The density-dependent portion of the above effective interactions is treated as a Yukawa force of very short range. The calculated differential cross sections are shown in Figs. 1 and 2 where the dashed lines correspond to the impulse approximation with the FL140 interaction, the dot-dashed lines, to the results with only g^{τ} included, and the solid lines, to the results with the full medium correction included. We make the following observation: First, the impulse calculations without the density dependence slightly overestimate the experimental cross sections [26]. Second, g-matrix calculations lower the cross section, but undershoot the data. Finally, with both medium corrections (g matrix and rearrangement term)

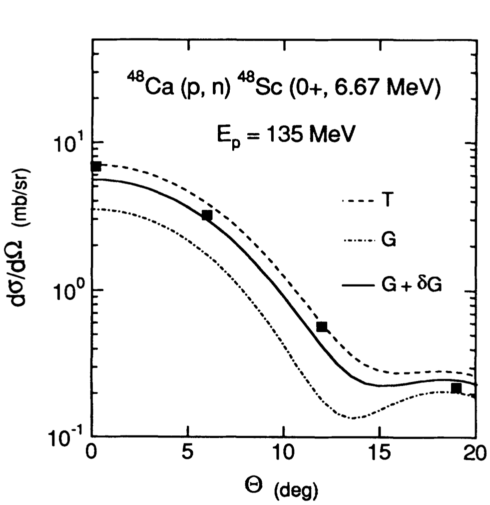

FIG. 2. The differential cross section for the reaction ^{48}Ca$(p,n)^{48}$Sc. See the caption of Fig. 1.

included, the calculations hit the experimental data points strikingly well. A warning is due, however, to the overemphasis on the favorable comparison with the experimental numbers at this point, since the uncertainty of the isospin-dependent component of the free scattering matrix is already very large. Also, the approximations in the current calculation leave large space for further improvements. Rather, these calculations are to be looked at as examples to support our basic point; that is, for microscopic calculations of the proton reactions, the medium modification should be and can be estimated with the proper inclusion of the rearrangement term.

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