Dynamical Control of a Chaotic Laser: Experimental Stabilization of a Globally Coupled System

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A multimode, autonomously chaotic solid-state laser system has been controlled by the technique of occasional proportional feedback, related to the control scheme of Ott, Grebogi, and Yorke. We show that complex periodic wave forms can be stabilized in the laser output intensity. A detailed model of the system is not necessary. Our results indicate that this control technique may be widely applicable to autonomous, higher-dimensional chaotic systems, including globally coupled arrays of nonlinear oscillators.

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The possibility of obtaining complex periodic wave forms from a chaotic system has inspired much recent theoretical and experimental work [1-7]. The basic concept involved is that a chaotic attractor has a large number of unstable periodic orbits embedded in it. It should therefore serve as a rich source of complex periodic wave forms, if an appropriate dynamical control technique can be implemented to stabilize the system. The control algorithm proposed by Ott, Grebogi, and Yorke [1] (OGY) was a breakthrough in this direction and has been applied experimentally to a periodically driven magnetoelastic ribbon [2]. In the OGY scheme, an unstable periodic orbit is stabilized through the application of small, carefully computed perturbations to a system parameter; the perturbations are proportional to the deviation of the system from the unstable fixed point. Other systems studied experimentally with the aim of establishing control over chaos have been a thermal convection loop [3], a yttrium iron garnet (YIG) oscillator [4], and most recently, a diode resonator [5]. Control of the periodically driven diode resonator was achieved by the technique of occasional proportional feedback (OPF) and many higher-order periodic orbits were successfully generated. While Ref. [6] is concerned with the control of chaos through sinusoidal modulations of a control parameter, Ref. [7] clearly shows the relation of the OPF technique to the OGY algorithm.

In this Letter, we describe the application of the OPF technique to an autonomously chaotic multimode laser, a higher-dimensional system for which the chaotic attractor is not characterizable by a two-dimensional map. Further, the application of the control signal is performed on relatively fast time scales of the order of a few microseconds. This limits the application of schemes involving digital computations. In these circumstances, the OPF scheme provides an attractive means to attempt control of an autonomously chaotic laser. Though the OPF scheme does not require any detailed model of laser operation, a knowledge of the characteristic time scale of energy exchange between the active medium and the light in the laser cavity was important for its implementation in this system.

The multimode laser with an intracavity crystal [8] is an example of a system of globally coupled nonlinear oscillators; each longitudinal mode is a relaxation oscillator coupled to all the others [9-12]. Such globally coupled arrays have been found to be of relevance and interest in the study of Josephson-junction networks [13], models of chemical turbulence [14], and heartbeat rhythms [15]. The technique described here could possibly be applied to a wide range of physical, chemical, and biological systems, including arrays and networks of coupled nonlinear elements.

The laser used in our experiments is a diode-laser-pumped solid-state Nd-doped yttrium aluminum garnet (Nd:YAlG) system that contains a KTP (potassium titanyl phosphate) doubling crystal within a cavity of length ~ 3.5 cm. We have previously studied this system extensively and the equations that describe the laser operation in several longitudinal modes are well known [9-12]. In the experiments reported here, it was not necessary for us to utilize a detailed model of the system. The laser was pumped at 60 mW, about 3 times above threshold (~ 20 mW), and the chaotic operation was observed for a given rotational orientation between the YAlG and KTP crystals [9]. At this level of excitation, the laser operates in anywhere from five to ten longitudinal modes, depending on the rotational orientation of the crystals and the length of the laser cavity.

In a periodically driven system it is convenient to sample a system variable at the driving frequency or its submultiples. In the autonomously chaotic laser, there is no modulation applied, and hence one looks for a natural periodicity characteristic of the system. Such a periodicity is present in the form of relaxation oscillations in the laser with intracavity nonlinear crystal [11,12], representative of the fundamental periodicity in the exchange of energy between active atoms and light in the laser cavity. The frequency of the relaxation oscillations ν_r increases as the square root of the excitation level above threshold, and depends also on other parameters characterizing the laser, such as the cavity loss, fluorescence decay time of the active atoms, and the nonlinearity coefficient of the KTP crystal [12]. ν_r is in the range 20-150 kHz, for levels of excitation of the laser up to 5 times above threshold. The source of chaotic behavior in this laser is the coupling of the longitudinal modes through the nonlinear process of sum-frequency generation. This process destabilizes the relaxation oscillations which are normally heavily damped in the system without the intracavity crystal. Autocorrelation functions of the total intensity reveal clear oscillations at the relaxation-oscillation frequency even when the laser is chaotic [11]. Alternatively, one may discern a peak at the relaxation-oscillation frequency in the fast Fourier transform (FFT) of the total intensity fluctuations.

The basic technique for achieving dynamical control is as follows. A system variable (the total laser output intensity) is sampled within a window of selected offset and width. The sampling frequency is related to the relaxation-oscillation frequency of the system. A signal proportional to the deviation of the sampled intensity from the center of the window is generated and applied to perturb a system parameter from its ambient value. This control signal repeatedly attempts to bring the system closer to a periodic unstable orbit that is embedded in the chaotic attractor, resulting in a realization of the periodic orbit with accuracy limited by the frequency and extent of feedback as well as by the positive Lyapunov exponent characterizing the orbit.

A block diagram of the laser system and controller is shown in Fig. 1. The fundamental 1.06-μm radiation is monitored by a photodiode, the output from which serves as the input to the control circuit. A stable oscillator is used to generate the synchronizing frequency with which the output from the chaotic laser is sampled. A variable offset is added to the laser signal to bring it within a window of adjustable width. The window comparator is activated when the wave form makes a transit through the window. When the synchronizing input is coincident with this event, the sample and hold acquires the wave-form voltage. The sampled signal is output through the gate only for time periods short compared to the period of the synchronizing oscillator. A typical time period for application of the correction signal is less than 10 μsec. An inverting amplifier with variable offset and gain delivers the control signal to the diode-laser driver.

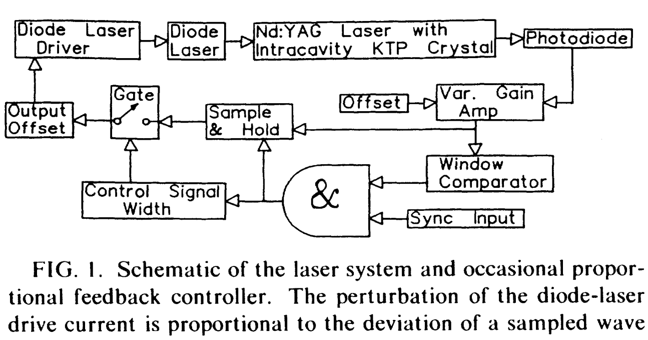

FIG. 1. Schematic of the laser system and occasional proportional feedback controller. The perturbation of the diode-laser drive current is proportional to the deviation of a sampled wave form value from the window center. The synchronizing oscillator frequency, wave-form offset, and control-signal width are varied to optimize stability of the periodic wave form.

The rotational orientation of the YAlG and KTP crystals was adjusted such that the laser was clearly chaotic in operation, but only a small amount (less than a few μW, typically) of green light was generated in the doubling crystal. This indicates that the effect of the nonlinearity is small, and that the laser is in a "weakly" chaotic regime. With the laser in chaotic operation, control is attempted by adjustment of the synchronizing frequency. The synchronizing oscillator frequency is varied near the relaxation-oscillation frequency observed for the laser, and the wave-form offset and window width are adjusted to initiate control. Further adjustments of the control signal width and gain, as well as the wave-form offset and window width, are necessary to optimize the stability of the wave forms obtained. The most effective adjustments were found to be the synchronizing frequency, the wave-form offset, and the control-signal width.

It is very easy to make these adjustments and to obtain many higher-order periodic wave forms of the laser intensity, a few of which are illustrated in Fig. 2. The wave form of the chaotic laser without any control signal is shown first. The FFT of this wave form shows a broad relaxation-oscillation peak centered at about 118 kHz. The time scale shown is 0–500 μsec. Period-1, -4, and -9 wave forms are also shown together with the control signals; a rich variety of wave forms can be obtained in practice and maintained in stable operation for many minutes. The time scale for these figures is 0–200 μsec. The sharp spikes on the control signal that are visible particularly in Fig. 2(d) are an artifact of the sample and hold circuit. The FFTs for the intensity fluctuations are shown below the intensity time traces. The range of frequencies shown is 0–500 kHz in all the figures. For the low-period orbits, control can be established with small perturbations applied near the relaxation-oscillation frequency or its submultiples [Figs. 2(b) and 2(c)]. In this manner we have observed orbits up to period 8. In Fig. 2, it is seen that the wave forms generated align themselves with the correction signal such that the wave form peaks are sampled. There is a range of frequencies of the synchronizing signal over which this alignment is noticeable, and robust, successful control is maintained. For the period-1 orbit, control was retained over a 110–165 kHz synchronizing frequency range. For higher-period orbits such as the period-9 orbit shown in Fig. 2(d), we often found that the synchronization frequency had to be adjusted to a simple rational fraction of the relaxation frequency. The nature of the control signal grows progressively more complex as higher-order periodic orbits are captured [Fig. 2(d)]. We note that in the example wave forms shown here, the control signal consists of only positive-going or negative-going corrections. However, many cases have been observed for complex wave forms in which the corrections are both positive and negative relative to the ambient bias. The perturbations applied to

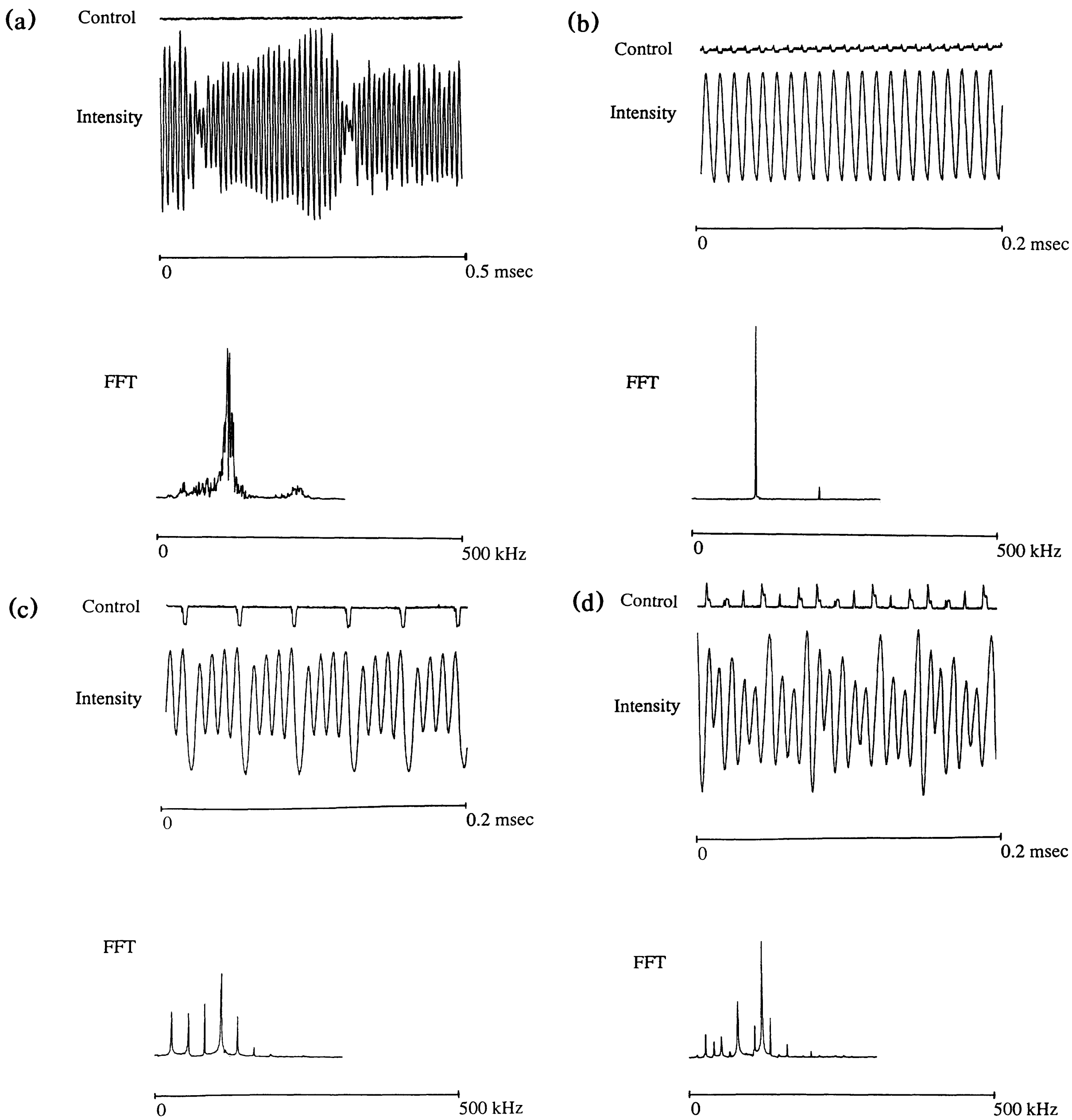

FIG. 2. Time traces of the laser intensity and the corresponding FFTs. The control signal is shown at the top, and then the intensity time trace and FFT. (a) The chaotic intensity fluctuations of the laser output without any applied control signal. The FFT below the wave form shows the broad relaxation-oscillation peak. (b) A period-1 orbit obtained by adjusting the synchronizing oscillator frequency to approximately the relaxation-oscillation frequency. (c) A period-4 orbit obtained by adjusting the synchronizing oscillator frequency to approximately $\frac{1}{4}$ of the relaxation-oscillation frequency. (d) A period-9 orbit obtained with the synchronizing oscillator frequency at $\frac{6}{9}$ of the dominant frequency shown in the FFT. The complex nature of the control signal is clearly visible. The spikes seen in the control signal are an artifact of the sample and hold circuit.

the drive current of the diode laser are only a few percent of the ambient-bias current for the low-order periodic orbits. Even for the higher-order orbits, the maximum perturbations observed were less than 10%. At these levels of control-signal magnitude, however, the original attractor may be modified to some extent by the feedback.

We have found experimentally that wave forms very similar to those obtained from the laser with dynamical control can be produced without control if operating parameters (the relative orientation of the crystals and the pump excitation) are changed. However, these uncontrolled wave forms are typically very unstable and cannot be maintained except for very short periods of time. With control, they can be maintained for many minutes. We have also picked out segments of unstable periodic orbits from chaotic intensity time traces that are similar to the low-order periodic orbits that we have generated with dynamical control under the same operating conditions. Experimental results demonstrating this correspondence and a comparison of wave forms generated numerically from a detailed model of the laser [11,12] will be published separately.

We have also examined the effect of a simple periodic modulation of the pump. Though the laser is stabilized on low-order orbits by the modulation, the modulation amplitude necessary is often much larger than 10% for such stabilization. Further, the wave forms are stable for only short periods of time, and environmental changes such as air currents and temperature drifts tended to have a highly destabilizing influence on the system. In contrast, with OPF stabilization, the laser stays locked in a given orbit for much longer lengths (many minutes) of time.

In conclusion, we have demonstrated dynamical control of an autonomously chaotic, higher-dimensional optical system for the first time on microsecond time scales. The multimode laser system studied by us is an example of a globally coupled system of nonlinear oscillators. The proportional control signal applied to the pump excitation results in an ordered, periodic state of the originally chaotic ensemble of oscillators. The results reported here indicate that the technique of occasional proportional feedback should be widely applicable to a variety of physical, chemical, and biological systems, including arrays and networks of coupled elements.

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