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Einstein-Podolsky-Rosen Effects from Independent Particle Sources

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In conventional Einstein-Podolsky-Rosen (EPR) experiments an unstable system decays into an entangled state of two or more particles. When appropriate measurements are made on this entangled state, phenomena are exhibited that run counter to our classical notions of local realism. Using a variant of a gedanken experiment proposed by Greenberger, Horn, and Zeilinger, it is shown that EPR effects can arise even when the particles come from independent widely separated sources.

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Although considerable discussion about the completeness of quantum mechanics followed the publication in 1935 of the gedanken experiment of Einstein, Podolsky, and Rosen (EPR) [1], progress in revealing the true extent of the incompatibility of our ingrained notions of local realism with quantum mechanics has been slow. It was not until Bell's work [2,3] published in 1965 that it was realized that the issue could be rigorously formulated and put to experimental test. This work inspired considerable theoretical and experimental activity [4-9]. An even more provocative demonstration of the incompatibility of quantum mechanics with local realism was recently discovered by Greenberger, Horne, and Zeilinger (GHZ) in a gedanken experiment employing more than two particles [10-13]. Typical of EPR experiments proposed to date, one has a metastable system which decays into n particles which are in an entangled state. Each particle is delivered to its respective detector, consisting usually of a polarization analyzer and a pair of particle counters. Upon performing the appropriate set of experiments stronger correlations are found among the firing patterns of the particle counters than allowed by local realism. Here we show that EPR effects can arise even if the particles do not come from a central unstable source that decays into an entangled state. In particular, EPR effects of the GHZ type can arise when three independent well-separated particle sources are employed. The particles employed could be either fermions or bosons. Possible optical realizations of this experiment are also discussed here.

Our variant of the GHZ experiment is depicted in Fig. 1. Particles are emitted from the three independent particle sources PS1, PS2, and PS3. The outputs of these particle sources are fed respectively through the 50-50 beam splitters S1, S2, and S3. The particles then propagate to their respective detectors. Each detector consists of two phase shifters, a beam splitter, and two particle counters. In particular, detector m, where $m \in \{1,2,3\}$,

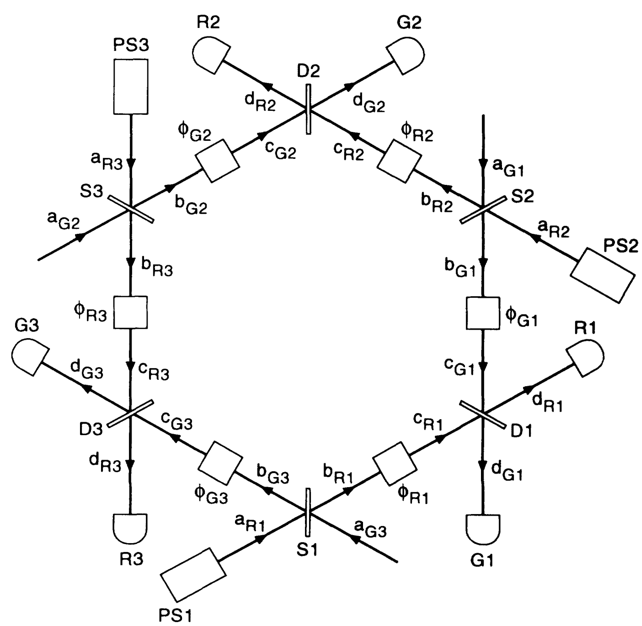

FIG. 1. Schematic of the present EPR experiment. See text for detailed explanation.

consists of phase shifters ϕ_{Rm} and ϕ_{Gm}, 50-50 beam splitter Dm, and particle counters Gm and Rm. The labels G and R have been chosen to be suggestive of the red and green lights employed by Mermin [12] in his version of the GHZ gedanken experiment. Figure 2 shows that our apparatus can be topologically distorted to have the same form as Mermin's gedanken experiment [12]. In addition, it should be noted that GHZ have described a gedanken experiment using the same detector arrangement employed here [11]. Their source, however, was an unstable system that decayed into three separate photons in an entangled state. Other more conventional EPR experiments employing detectors consisting of phase shifters, a beam splitter, and a pair of particle counters have also been proposed [14–17] and even performed [7–9].

The annihilation operators of the modes entering particle counters are labeled $\mathbf{d}_{am}$, where here and throughout the paper $a \in \{R,G\}$ and $m \in \{1,2,3\}$. These operators satisfy usual fermion or boson commutation relations:

$$[\mathbf{d}_{am},\mathbf{d}^{\dagger}_{a'm'}]_{\pm}=\delta_{a,a'}\delta_{m,m'}\,, \tag{1}$$

$$[\mathbf{d}_{am},\mathbf{d}_{a'm'}]_{\pm}=0\,, \tag{2}$$

where "+" denotes anticommutation and "−" denotes commutation. The beam splitters D1, D2, and D3 perform the mode transformation

$$\begin{pmatrix}\mathbf{d}_{Rm}\\ \mathbf{d}_{Gm}\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}\begin{pmatrix}\mathbf{c}_{Rm}\\ \mathbf{c}_{Gm}\end{pmatrix}. \tag{3}$$

The phase shifters perform mode transformations of the form

$$\mathbf{c}_{am}=e^{i\phi_{am}}\mathbf{b}_{am}\,. \tag{4}$$

Finally, the mode transformations performed by the beam splitters S1, S2, and S3 are, respectively,

$$\begin{pmatrix}\mathbf{b}_{R1}\\ \mathbf{b}_{G3}\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}\begin{pmatrix}\mathbf{a}_{R1}\\ \mathbf{a}_{G3}\end{pmatrix}, \tag{5}$$

$$\begin{pmatrix}\mathbf{b}_{R2}\\ \mathbf{b}_{G1}\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}\begin{pmatrix}\mathbf{a}_{R2}\\ \mathbf{a}_{G1}\end{pmatrix}, \tag{6}$$

$$\begin{pmatrix}\mathbf{b}_{R3}\\ \mathbf{b}_{G2}\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}\begin{pmatrix}\mathbf{a}_{R3}\\ \mathbf{a}_{G2}\end{pmatrix}. \tag{7}$$

Each of these beam splitters entangles the particle entering one input port with the vacuum entering the other input port [17].

The mode transformations Eqs. (3)–(7) can be solved to express the modes entering the beam splitters S1, S2, and S3 in terms of the modes entering the particle counters. In particular, one finds

$$\mathbf{a}_{R1}=\tfrac{1}{2}[e^{-i\phi_{R1}}(\mathbf{d}_{R1}-i\mathbf{d}_{G1})-ie^{-i\phi_{G3}}(-i\mathbf{d}_{R3}+\mathbf{d}_{G3})]\,, \tag{8}$$

$$\mathbf{a}_{R2}=\tfrac{1}{2}[e^{-i\phi_{R2}}(\mathbf{d}_{R2}-i\mathbf{d}_{G2})-ie^{-i\phi_{G1}}(-i\mathbf{d}_{R1}+\mathbf{d}_{G1})]\,, \tag{9}$$

$$\mathbf{a}_{R3}=\tfrac{1}{2}[e^{-i\phi_{R3}}(\mathbf{d}_{R3}-i\mathbf{d}_{G3})-ie^{-i\phi_{G2}}(-i\mathbf{d}_{R2}+\mathbf{d}_{G2})]\,. \tag{10}$$

For the moment it will be convenient to think of the experiment being operated in the following manner. At a particular instant of time each of the three independent sources emits a single particle. One then records which particle counters fire. By performing an ensemble of such experiments one can accumulate data on the firing statistics of the particle counters. The state vector for the system is the direct product of the state vector for each individual source. In second-quantized notation the state vector is given by

$$|\psi\rangle=\mathbf{a}^{\dagger}_{R1}\mathbf{a}^{\dagger}_{R2}\mathbf{a}^{\dagger}_{R3}|0\rangle\,. \tag{11}$$

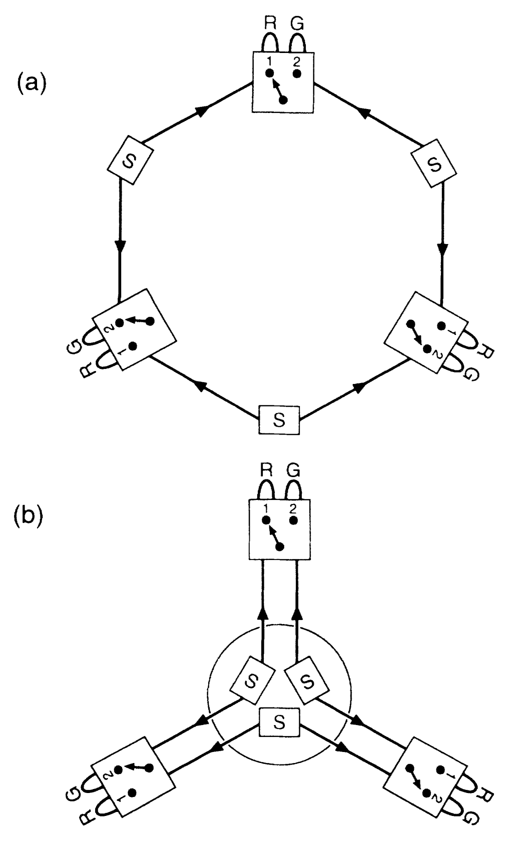

FIG. 2. Topological distortion of the present EPR experiment. The particle sources are denoted by S. The boxes with red R and green G light bulbs represent the detectors. Each detector has two switch settings corresponding to the two settings of the detector phase shifters. By moving the detectors in (a) toward the center, the device can be distorted into a form (b) which is similar to that employed in previous gedanken experiments.

By substituting Eqs. (8)-(10) into Eq. (11) it is straightforward to read off the probability amplitudes and, consequently, the probabilities for the various particle counter firing patterns. Let $P(\alpha_1\alpha_2\alpha_3)$ denote the probability that event α_1 occurs at detector 1, event α_2 occurs at detector 2, and event α_3 occurs at detector 3. The events α_m are elements of the set $\{0,R,G,R^2,G^2,B\}$, where 0 represents the event in which none of the particle counters of the detector fired, R denotes the event in which the particle counter labeled R counts a single particle, G denotes the event in which the particle counter labeled G counts a single particle, R^2 denotes the event in which the particle counter labeled R counts two particles, G^2 denotes the event in which the particle counter labeled G counts two particles, and B denotes the event when both the R and the G counters count a single particle. Let A denote the set of firing patterns where each detector counts a single particle and an odd number of R counters fire:

$$A=\{RRR,RGG,GRG,GGR\}. \tag{12}$$

Let B denote the set of firing patterns where each detector counts a single particle and an even number of R counters fire:

$$B=\{GGG,GRR,RGR,RRG\}. \tag{13}$$

Let C denote the set of firing patterns in which both counters of one detector fire and, as a consequence, none of the particle counters of one of the other detectors fires. There are 12 possible events of this kind, consisting of all the possible permutations of $0BR$ and $0BG$. Finally, let D denote the set of events for which one of the particle counters counts two particles and, as a consequence, the particle counters of one of the other detectors fails to fire. This set has 24 elements consisting of all permutations of $0RR^2$, $0GR^2$, $0RG^2$, and $0GG^2$. For fermions, the Pauli exclusion principle prevents a given mode from being doubly occupied and, hence, events in which one particle counter counts more than one particle (events belonging to D) do not occur. For bosons, because of a destructive interference effect, events for which both particle counters of a given detector fire (events belonging to the set C) cannot occur. The probabilities for the particle counter firing patterns are given by

$$P(\alpha_1\alpha_2\alpha_3)=\begin{cases}\frac{1}{16}\sin^2\phi \text{ if } \alpha_1\alpha_2\alpha_3\in A,\\ \frac{1}{16}\cos^2\phi \text{ if } \alpha_1\alpha_2\alpha_3\in B,\\ \frac{1}{16} \text{ for fermions when } \alpha_1\alpha_2\alpha_3\in C,\\ \frac{1}{32} \text{ for bosons when } \alpha_1\alpha_2\alpha_3\in D,\\ 0 \text{ otherwise,}\end{cases} \tag{14}$$

where

$$\phi=\tfrac{1}{2}[\phi_{R1}-\phi_{G1}+\phi_{R2}-\phi_{G2}+\phi_{R3}-\phi_{G3}]. \tag{15}$$

Let ϕ_m be the difference in the phases of the phase shifters associated with detector m,

$$\phi_m=\phi_{Rm}-\phi_{Gm}. \tag{16}$$

Then, provided one restricts ϕ_m to the two values 0 and $\pi/2$, the present apparatus behaves in essentially the same manner as that used in the gedanken experiment of Mermin [12].

It is easiest to demonstrate the violation of local realism by considering the topological distortion of the apparatus as shown in Fig. 2(b). In this configuration the apparatus is most similar to that employed by Mermin [12] and we can simply follow his arguments with minor modifications. Having demonstrated that the configuration of Fig. 2(b) violates local realism, it follows that the unfolded configuration Fig. 1 or Fig. 2(a) also violates local realism. Otherwise, one would have a local realistic model for the operation of the device of Fig. 2(b). Let the switch position 1 of the detectors depicted in Fig. 2 correspond to a detector phase ϕ_m of 0 and let the switch position 2 denote a detector phase setting of $\pi/2$. If only one of the three detectors is set to position 1, i.e., the detector settings are 122, 212, or 221, it follows from Eq. (14) that the only events in which all detectors fire are those for which an odd number of R's occur. In contrast, if the three detector positions are set to 111, the only events in which all detectors fire are those in which an even number of R's occur. We will show that this behavior is incompatible with the existence of a local realistic model for the behavior of the device. First of all, note that regardless of the switch settings, if one knows that two of the detectors counted only one particle, one knows that the third detector has also counted one particle. If one restricts oneself to the switch settings 111, 122, 212, and 221, one sees that, if in addition one knows which particle counter in each of the two detectors fired, one can predict which particle counter of the third detector will have fired. For example, if the switch setting is 122 and detector 1 fires R and detector 2 fires G, one knows that detector 3 will fire G because there must be an odd number of R's. One thus concludes that a definite message was sent to the third detector telling it which counter to fire. Further, since the switch settings can be changed up to the instant before the particles enter their respective detectors, one concludes that a definite message was sent to each detector telling it which detector to fire when the switch setting is set to 1 and which detector to fire when the switch setting is set to 2. Consequently, one concludes that when all three detectors have fired it is because an instruction set has been sent which can be written in tabular form as

$$\begin{matrix}\beta_{11}\beta_{12}\beta_{13}\\ \beta_{21}\beta_{22}\beta_{23}\end{matrix},$$

where $\beta_{ij}\in\{R,G\}$. The first index i indicates the detector switch setting and the second index j specifies the detector number. Considering only the switch settings

122, 212, and 221, one deduces that the legal instructions sets are

$$\begin{array}{llll} RRR & RGG & GRG & GGR \\ RRR & RGG & GRG & GGR \end{array}$$

$$\begin{array}{llll} RRR & RGG & GRG & GGR \\ GGG & GRR & RGR & RRG \end{array}$$

One sees from this listing that, when the switch positions are set to 111, one should still only see firing patterns in which an odd number of R's occur when all the detectors fire. In contrast, Eq. (14) shows that this never occurs. Instead, only an even number of R's will occur in the firing pattern. Hence, after performing a long series of runs with the switches set to 122, 212, and 221, local realism will be refuted at the first instance when all the detectors fire with the switches set at 111.

We now describe how the state vector equation (11) can be realized. In a realistic experiment particle sources emitting a steady beam of particles would most likely be used. A rigorous treatment of this situation requires a wide-band analysis [15] in which the multitime correlation functions for the firing of the six particle counters are evaluated over the density matrix describing the statistics of the particle sources. We have performed such an analysis for the case when the particles consist of a thermal beam of fermions that has been momentum selected [18]. Here, however, we will confine ourselves to a qualitative discussion.

First, consider the fermion case. For simplicity, let the particle sources PS1, PS2, and PS3 have the same intensity and the same coherence time τ. Let T denote the mean time between the arrival of successive particles. If all three detectors fire in coincidence, one can conclude, because of the Pauli exclusion principle, that the state vector must have been of the form given by Eq. (11). The probability that a particle arrives in a time interval τ is τ/T. The probability that three particles arrive sufficiently in coincidence so that Eq. (14) applies is of the order of $(\tau/T)^3$. Hence, if one has three independent free running fermion sources and particle counters that can resolve the arrival times of the fermions to within the coherence time τ, all one has to do is wait patiently to accumulate the statistics that exhibit violation of local realism.

Photons are perhaps the most convenient particles to use in carrying out an actual realization of the boson experiment. Hence, for the sake of discussion, the boson case will be described in the context of optical experiments. For bosons, one no longer has an exclusion principle to take advantage of. However, the apparatus would still work, provided one had three independent light sources that were sufficiently antibunched that one could certify that only one photon leaves a given source in a coherence time τ. Alternatively, one could construct suitable sources from parametric down-converters for which the signal and idler mode can be separated [19,20]. Since photons are emitted as a signal-idler pair from such sources, by monitoring the idler mode with a photodetector one could determine when a photon is in the signal mode. Hence, if all three detectors of the EPR apparatus and all three photodetectors monitoring the idler modes fire in coincidence and each reports the arrival of only a single photon, one could certify that the state vector for the system was Eq. (11). The experiment will even work if the photodetectors are not able to distinguish between the arrival of a single photon or the arrival of two or more photons in coincidence, provided the parametric down-converters are operated at an intensity level that is low enough so that events in which more than one photon occupy a signal mode during the coherence time τ are sufficiently rare that they can be ignored.

[1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
[2] J. S. Bell, Physics (Long Island City, N.Y.) **1**, 195 (1965).
[3] J. S. Bell, in *Foundations of Quantum Mechanics*, edited by B. d'Espagnat (Academic, New York, 1972).
[4] W. De Baere, Adv. Electron. Electron Phys. **68**, 245 (1986).
[5] J. F. Clauser and A. Shimony, Rep. Prog. Phys. **41**, 1881 (1978).
[6] A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982).
[7] Z. Y. Ou and L. Mandel, Phys. Rev. Lett. **61**, 50 (1988).
[8] Y. H. Shih and C. O. Alley, Phys. Rev. Lett. **61**, 2921 (1988).
[9] J. G. Rarity and P. R. Tapster, Phys. Rev. Lett. **64**, 2495 (1990).
[10] D. M. Greenberger, M. A. Horne, and A. Zeilinger, in *Bell's Theorem, Quantum Theory and Conceptions of the Universe*, edited by M. Kafatos (Kluwer Academic, Dordrecht, 1989), pp. 69–72.
[11] D. M. Greenberger, M. A. Horne, A. Shimony, and A. Zeilinger, Am. J. Phys. **58**, 1131 (1990).
[12] N. D. Mermin, Am. J. Phys. **58**, 731 (1990).
[13] R. K. Clifton, M. L. G. Redhead, and J. N. Butterfield, Found. Phys. **21**, 149 (1991).
[14] M. D. Reid and D. F. Walls, Phys. Rev. A **34**, 1260 (1986).
[15] P. Grangier, M. J. Potasek, and B. Yurke, Phys. Rev. A **38**, 3132 (1988).
[16] J. D. Franson, Phys. Rev. Lett. **62**, 2205 (1989).
[17] S. M. Tan, D. F. Walls, and M. J. Collett, Phys. Rev. Lett. **66**, 252 (1991).
[18] B. Yurke and D. Stoler (to be published).
[19] C. K. Hong, Z. Y. Ou, and L. Mandel, Phys. Rev. Lett. **59**, 2044 (1987).
[20] R. Graham, Phys. Rev. Lett. **52**, 117 (1984).