Direct Measurement of Velocity-Space Transport in a Fully Ionized Plasma

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Direct measurements of velocity-space transport coefficients, which may be interpreted as the Fokker-Planck coefficients, in the direction of the confining magnetic field are reported. The measurements were made using the techniques of laser-induced fluorescence and optical tagging, which allow ions to be followed in phase space. The results show that in a plasma with nearly only thermal fluctuations the data agree well with test particle calculations made by N. Rostoker for classical collisional processes. In the presence of larger-amplitude drift-wave fluctuations there is a pronounced enhancement of the velocity diffusion coefficient at a velocity less than the thermal speed.

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In this Letter we report the direct measurement of velocity-space transport coefficients in the direction of the confining magnetic field. The coefficients may be interpreted along the lines of the Fokker-Planck formalism. The measurements were made using the techniques of laser-induced fluorescence (LIF) [1] and optical tagging [2]. The tagging technique allows ion trajectories to be followed in phase space. When only thermal fluctuations are present we expect transport to be due to classical Coulomb collisions only. Measurements taken under this condition agree well with test-particle calculations made by Rostoker [3]. In the presence of enhanced fluctuation levels due to drift waves, however, there is a pronounced increase in the velocity diffusion coefficient at a velocity less than thermal speed. There is a large body of work on the theoretical aspects of velocity-space transport [4] and considerable work has been done at University of California at Irvine and elsewhere on the measurement of real-space diffusion [5,6]. However, the measurements reported here are unique in that they allow direct comparison with detailed theoretical calculations such as the Fokker-Planck coefficients.

A schematic of the experiment, which was performed in the UC Irvine *Q*-Machine [7], is shown in Fig. 1. Two laser beams were used: one, the tagging beam, pumped particles from the ground state of the barium ion into a long-lived metastable state, and the other examined the same metastable state and thus identified the tagged particles. The two beams were superimposed by a beam splitter and directed through the collector end of the plasma column. The photon emission from the tagged particles was picked up by lens systems that directed the optical signal to photomultipliers (PMT). The output of each laser was pulsed by means of an acousto-optical modulator.

As explained in Ref. [1], the frequency of the laser light is related to the ion velocity through the Doppler shift. Sweeping the laser frequency amounts to scanning the ion velocity. This allows the determination of the ion velocity distribution, from which the ion temperature and other information may be determined.

To take the tagging measurements an initial velocity scan of the plasma ions was made with the tagging laser. This yielded a calibrating scan that allowed us to determine the zero velocity point in the plasma reference frame (the plasma drifts toward the cold collector at nearly twice the thermal speed). The laser is then set to the desired velocity. The output from the lasers was pulsed as described above except that the tagging laser was pulsed at half the repetition rate of the search laser. This allowed alternate pulses of the search laser to be used to subtract out both the background distribution function and any background light present. Pulse sequences are shown in Fig. 1. The frequency of the search beam was then swept using the pulse sequence I in the figure. The velocity distributions obtained were continuous distributions of only the tagged particles averaged over 10-μsec intervals. The distributions show that particles, originally at a specific velocity, have diffused to other velocities over the 10-μsec interval. A duration of 10 μsec is long enough for significant diffusion in velocity space to have taken place, but much shorter than the time

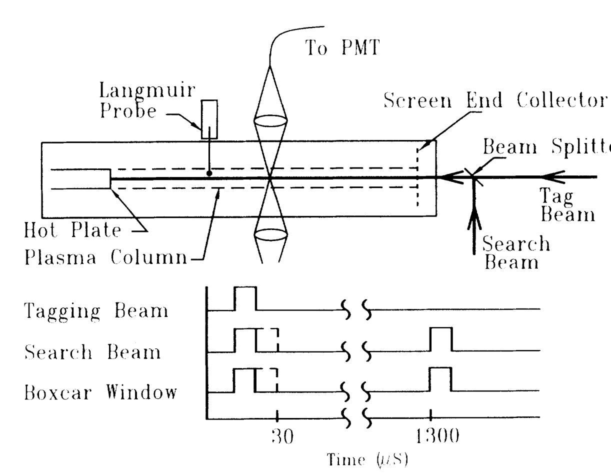

FIG. 1. Schematic of the experimental arrangement and timing sequences. The solid line in the inset indicates sequence I, the dashed lines sequence II.

required for the tagged particles to equilibrate with the background.

In principle this should yield enough information to enable the calculation of a velocity-space diffusion coefficient if one knows the initial frequency width of the laser line. However, the laser linewidth is affected by the intensity in a way not easily measured. Thus, it was decided to use this as an initial distribution, and then compare it to a second distribution taken using the pulse sequence labeled II in Fig. 1. The moments of each distribution were calculated for many conditions and then compared with the calculation of the Fokker-Planck coefficients as explained below.

The 10-μsec window size is the shortest window that could be used due to noise considerations. To make the interpretation of the results simpler it is necessary to use the shortest possible window. The reason for this is simple: As the tagged particles are created, the transport begins. Particles originally at a specific velocity start to move away from that velocity. As they move away from the initial velocity their trajectory becomes more dependent on what the current velocity is and not the initial velocity. Thus a longer window would result in a distribution convoluted by the differences in the transport rate at different velocities.

The initial distribution of tagged particles can be represented to a good approximation by a drifting Maxwellian

$$f(v,t=0)=\frac{n_T}{(2\pi\sigma_T^2)^{1/2}}\exp\frac{(v-v_T)^2}{2\sigma_T^2}, \tag{1}$$

where T refers to the tagged particles, σ_T is the standard deviation of the initial tagged particle distribution, and n is the density.

The general problem of velocity transport may be more easily understood by first considering a continuity equation for velocity space. Then the evolution of Eq. (1) is given by

$$\frac{\partial f(\mathbf{v})}{\partial t}+\nabla_v\cdot\Gamma=0.$$

If one assumes that the flux Γ in velocity space may be expressed as a convection term plus a diffusion term, with critical times t_C and t_D, respectively, then one may write in one dimension,

$$\Gamma=C_vf(v)+D_v\frac{\partial}{\partial v}f(v),$$

where

$$C_v\equiv\frac{\langle\delta v\rangle}{\tau_C},\quad D_v\equiv\frac{\langle\delta v\delta v\rangle}{\tau_D},$$

and $\delta v=v_T(t)-v_T(0)$.

This yields

$$\frac{\partial f(v,t)}{\partial t}+\frac{\partial}{\partial v}\left[C_vf(v,t)+D_v\frac{\partial}{\partial v}f(v,t)\right]=0.$$

For times short enough to consider the diffusion independent of velocity, the solution of this equation is

$$f(v,t)=\frac{n_T}{\sqrt{2\pi}}\frac{1}{\sigma_T^2+2Dt}\exp-\left[\frac{(v_T-v-C_vt)^2}{2(\sigma_T^2+2Dt)}\right].$$

Thus the change in the first and second moment of the measured tagged particle's distribution gives the velocity-space convection and diffusion rates, respectively.

Having established a conceptual basis, the experimental results can now be compared with the three-dimensional theory for ion-ion collisions in a fully ionized plasma given by the Fokker-Planck equation [8]:

$$\left.\frac{\partial f}{\partial t}\right|_{\text{coll}}=\frac{\partial}{\partial\mathbf{v}}\cdot\left\langle\frac{\delta\mathbf{v}}{\delta t}\right\rangle f(v,t)+\frac{1}{2}\sum_{i,j}\frac{\partial^2}{\partial v_i\,\partial v_j}\left[\left\langle\frac{\delta v_i\delta v_j}{\delta t}\right\rangle f(v,t)\right].$$

The first and second terms are interpreted as the convection and diffusion terms, respectively. When $\langle\delta\mathbf{v}/\delta t\rangle$ and $\langle\delta v_i\delta v_j/\delta t\rangle$ are known, the future of the distribution is known to second order in δv.

Calculation of the Fokker-Planck coefficients for this case was done by Rostoker [3]. The equations for parallel convection and diffusion, assuming thermodynamic equilibrium, are given by

$$C_{v_z}=\left\langle\frac{\delta\mathbf{v}}{\delta t}\right\rangle=\frac{2\pi e^4n\ln\Lambda}{m_i^2}\frac{1}{v_{\text{th}}^2}\frac{\partial}{\partial x}\frac{\text{erf}(x)}{x} \tag{2}$$

and

$$D_{v_zv_z}=\left\langle\frac{\delta v_i\delta v_j}{\delta t}\right\rangle=\frac{-8\pi ne^4\ln\Lambda}{m^2v_{\text{th}}}\frac{1}{4\sqrt{2}x}\frac{\partial}{\partial x}\frac{\text{erf}(x)}{x}, \tag{3}$$

where $\Lambda=\frac{3}{2}(k^3T^3/\pi n)^{1/2}/e^3$, $v_{\text{th}}=(kT/m_i)^{1/2}$, and $x=v/\sqrt{2}v_{\text{th}}$. Here k is the Boltzmann constant, T is the temperature, n is the plasma density, m_i is the ion mass, and z represents the coordinate axis aligned with the magnetic field.

The convection term is odd in velocity. Thus particles with velocities on either side of $v=0$ are "pushed" toward zero. At $v=0$ there is an equal number of particles moving to the right and to the left; hence convection is zero at this point. In the rest frame of a particle with a velocity greater than the average speed of the distribution there are more particles with negative speed than positive and thus collisions tend to reduce the particle's velocity. The reverse is true for particles moving at speeds less than the bulk average velocity. Diffusion, on the other hand, is even in velocity. The particle velocity random walks in velocity and thus tends to move away from the bulk of the distribution. The combination of the effects of convection pushing particles toward the bulk and diffusion pushing particles away from the bulk maintains the Maxwellian shape.

The results are shown in Figs. 2–4. All data are shown

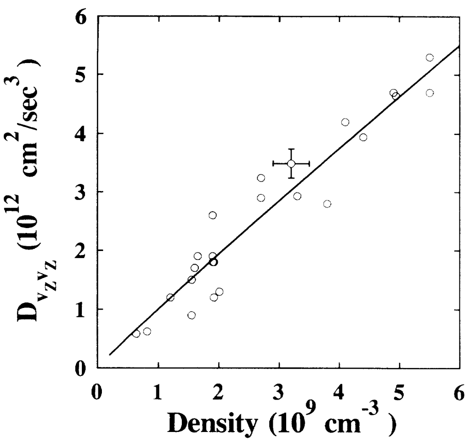

FIG. 2. Plot of the longitudinal velocity diffusion coefficient as a function of plasma density for $v_z = 0$. The circles are data points; the solid line is the theory from Eq. (3).

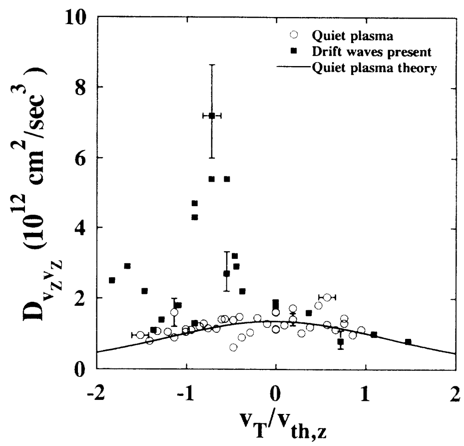

FIG. 3. Plot of the longitudinal velocity diffusion coefficient as a function of the normalized tagged-ion velocity for a quiet plasma (open circles) and one in which drift waves are present (solid squares). The solid line is the theory for quiet conditions from Eq. (3).

in the plasma-reference-frame coordinates. The measurements referred to as quiet plasma conditions were taken near the axis of the plasma column to avoid complications arising from drift waves and other phenomena associated with the steep density gradients at the edges. The techniques for determining ion density and temperature are described in Ref. [1]. The experimental and theoretical results are not normalized to each other; there are no anomalous or arbitrary coefficients.

Figure 2 shows a plot of the velocity-space diffusion coefficient $D_{v_z v_z}$ as a function of plasma density in a "quiet" plasma for a single value of v_z. By quiet we mean that the density profile was reasonably constant over the area subtended by the plasma source, and that the fluctuation level over the same area was less than 0.1% of the total density ($\delta n/n \leq 0.1\%$) as measured by a Langmuir probe. The data show good agreement with a linear variation with density. Additional measurements of the diffusion coefficient taken with different values of v_z yielded similar results, indicating good agreement with classical diffusion theory.

Figure 3 shows a comparison of the variation of the diffusion coefficient $D_{v_z v_z}$ versus the tagged velocity v_T normalized to the thermal velocity $v_{th,z}$. The open circles were taken in the quiet plasma condition and show good agreement with Rostoker's theory. The solid squares show data taken when the plasma density profile was altered to have a sufficiently large gradient to destabilize drift waves with $\delta n/n \approx 6.0\%$ and a frequency of approximately 1.5 kHz. The dramatic departure from theory is quite apparent; the data show a peak in the axial velocity diffusion coefficient at a velocity about $-0.7 v_{th,z}$. At velocities higher than zero in the plasma frame agreement with classical predictions seems to be restored.

Figure 4 shows a comparison of the convective coefficient C_{v_z} with the normalized velocity $v/v_{th,z}$. Within experimental error the mean value agrees with theory in the quiet case. However, the error bars are too large to draw firm conclusions about agreement with classical theory insofar as the variation with velocity is concerned. When fluctuations are present, the scatter of points does not allow any further conclusions to be drawn. The relatively large error bars are due to the fact that the convective coefficient, being linear in v, is very susceptible to the relative drift of the tagging and search lasers, while the diffusion coefficient, being dependent on the spread in the velocity and not the shift in average velocity, is not as sensitive.

In conclusion, for particles near equilibrium the measured velocity-space diffusion coefficient is in agreement

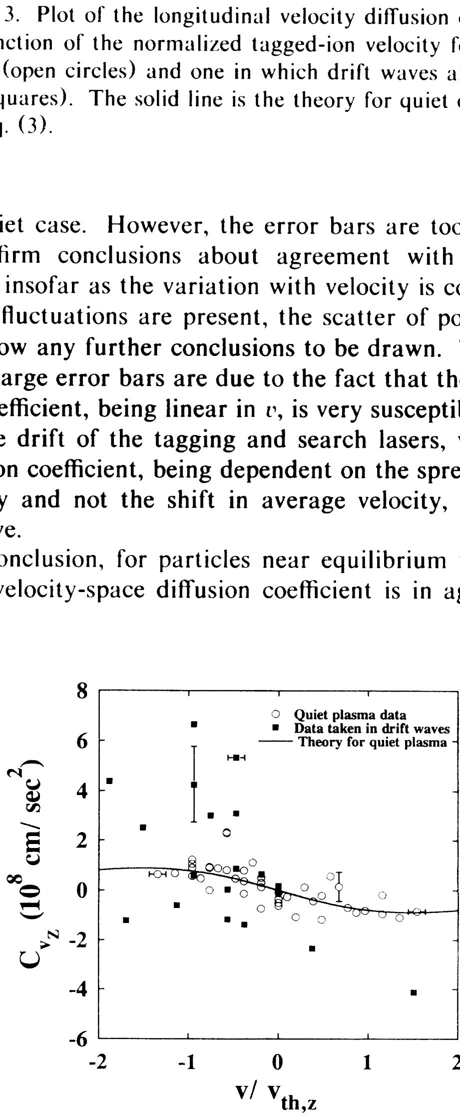

FIG. 4. Plot of the longitudinal velocity convection coefficient as a function of the normalized tagged ion velocity for a quiet plasma (open circles) and one in which drift waves are present (solid squares). The solid line is the theory for a quiet plasma, Eq. (2).

with classical collisional test-particle predictions. Additionally, when the fluctuation level is significantly greater than thermal, classical collisional theory is insufficient to predict results. We are actively pursuing an explanation at this writing. The dependence of the perpendicular spatial diffusion coefficient on the magnitude of density fluctuations has already been discussed by one of the authors [6].

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