Kaon Pictures of QCD Plasma Droplets

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We demonstrate that emission of bosons from a dispersed collection of droplets will produce a two-tiered correlation function reflecting the size of the droplets as well as their spatial distribution. We argue that this two-tiered correlation can signal the existence of a mixed phase of QCD plasma and hadronic gas in relativistic nuclear collisions, and propose that positive kaons will be the best indicators of this structure.

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A hallmark of a first-order phase transition is the appearance of a mixed phase in which both states are found in spatially separated regions. Experiments on relativistic nuclear collisions hope to observe a phase transition of hot, dense hadronic matter to an unconfined state in which colored constituents move within a substantial volume. This state is widely expected to hadronize via formation of a mixed phase of dense plasma and dilute hadronic gas [1]. Implications of the mixed phase for observations of reaction products have been discussed extensively, especially with regard to the consequences of the mixed phase for the hydrodynamic evolution of the system [2]. A more microscopic approach by Seibert [3] looks for the granularity of the mixed phase reflected in fluctuations of the velocity distributions of the final hadrons. We propose a complementary microscopic probe of the system's phase space via boson interferometry, which can display the spatial inhomogeneities of the bosons' sources. The interference of K^+ mesons seems the most promising case.

During most of its lifetime the mixed phase must consist of well-separated droplets of plasma [4]: At a given temperature, the plasma is much denser than the hadronic gas (which has only three low-mass constituents instead of at least forty species of light quarks and gluons for the plasma) and thus needs only a small volume to accommodate its share of the energy and entropy. The distribution of plasma droplets is characterized by at least three independent length scales: the mean droplet radius a, the mean separation d between the droplets, and the overall radius R of the mixed-phase system. The existence of size scales much less than R is actually typical of a mixed phase, except very near a critical point where scale-free behavior emerges. In the case of QCD plasma droplets, the size scale is expected [4] to be determined by the temperature T_c, the surface tension σ, and the dynamics of their evolution.

Each of the three spatial scales is reflected in a corresponding characteristic of the two-boson correlation function $C(\mathbf{K},\mathbf{k})$, which is a function of the mean momentum $\mathbf{K}$ and the momentum difference $\mathbf{k}$ of the observed bosons. For an incoherent source characterized by a single length scale, and ignoring for the moment the effects of final-state interactions, C peaks at $C(\mathbf{k}=0)=2$, then falls off to unity over a momentum interval of the order of the inverse source size. For our case of multiple time scales, the largest distance R describes the falloff of C for small $\mathbf{k}$. Conversely, the smallest size scale a determines the maximum value of $\mathbf{k}$ for which C deviates from its asymptotic value of $C(\mathbf{k}\to\infty)=1$. The mean separation d appears only indirectly in the correlation function, since the droplets are distributed randomly. The number of droplets $n\sim(R/d)^3$ determines the relative strength of the slowly falling component, which is diminished by a factor $1/n$ since it arises only from pairs in which both bosons are emitted by the same droplet.

These features emerge clearly in a simple analytical model. The two-particle correlation function of a distributed source is given by

$$C(\mathbf{K},\mathbf{k})=\int P_2(\mathbf{p},\mathbf{p}',\mathbf{r},\mathbf{r}')|\phi_{\mathbf{k}}(\mathbf{r}-\mathbf{r}')|^2 d\mathbf{r}\,d\mathbf{r}'\Big/\int P_2(\mathbf{p},\mathbf{p}',\mathbf{r},\mathbf{r}')d\mathbf{r}\,d\mathbf{r}', \tag{1}$$

where $\mathbf{K}=(\mathbf{p}+\mathbf{p}')/2$, $\mathbf{k}=\mathbf{p}-\mathbf{p}'$, $P_2(\mathbf{p},\mathbf{p}',\mathbf{r},\mathbf{r}')$ is the joint probability of emission of bosons of momentum $\mathbf{p}$ and $\mathbf{p}'$ at positions $\mathbf{r}$ and $\mathbf{r}'$, respectively, and $\phi_{\mathbf{k}}(\mathbf{r}-\mathbf{r}')$ is the symmetrized wave function of the relative motion of the boson pair. For an incoherent composite source of n instantaneous subsources,

$$P_2(\mathbf{p},\mathbf{p}',\mathbf{r},\mathbf{r}')=\sum_{i\neq j}^{n}\int d\mathbf{R}_i\,d\mathbf{R}_j\,P(\mathbf{R}_i)P(\mathbf{R}_j)p(\mathbf{p},\mathbf{r}-\mathbf{R}_i)p(\mathbf{p}',\mathbf{r}'-\mathbf{R}_j)+\sum_{i}^{n}\int d\mathbf{R}_i\,P(\mathbf{R}_i)^2 p(\mathbf{p},\mathbf{r}-\mathbf{R}_i)p(\mathbf{p}',\mathbf{r}'-\mathbf{R}_i)\,, \tag{2}$$

where $p(\mathbf{p},\mathbf{r})$ describes the emission from an individual subsource about its center, and $P(\mathbf{R})$ is the normalized distribution of the centers of the sources. If $p(\mathbf{p},\mathbf{r})$ depends slowly on $\mathbf{p}$ so that its variation may be neglected over the range $\mathbf{k}$,

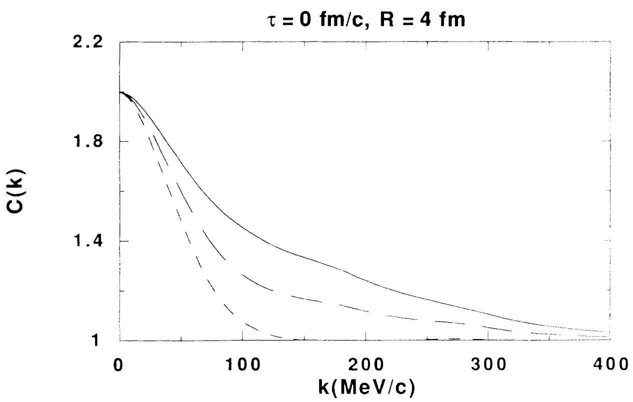

FIG. 1. Correlation function for a collection of n small Gaussian sources of radius $a=1$ fm Gauss distributed over a volume of radius $R_0=4$ fm, for various n; see Eq. (5). Final-state interactions are neglected. The solid, long-dashed, and short-dashed lines represent $n=2$, 4, and infinity.

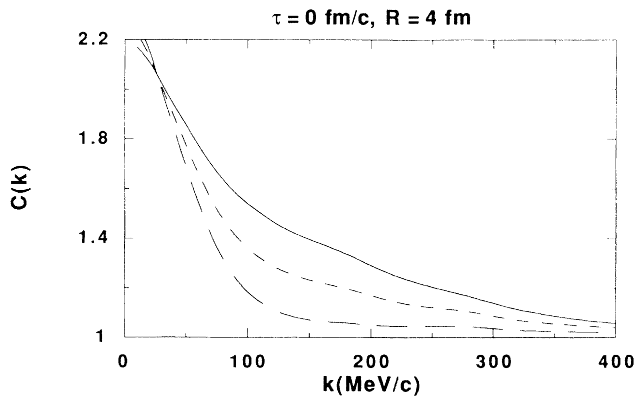

FIG. 2. Correlation function for the same source as Fig. 1, corrected for the final-state Coulomb interaction of K^+ mesons via the exact Coulomb wave functions $\phi_{\mathbf{k}}(\mathbf{r}-\mathbf{r}')$, and then divided by the Gamow factor for clarity of presentation. The solid, long-dashed, and short-dashed lines represent the number of sources $n=2$, 4, and infinity.

it is easy to see that Eqs. (1) and (2) imply

$$C(\mathbf{K},\mathbf{k})=c(\mathbf{K},\mathbf{k})/n+c'(\mathbf{K},\mathbf{k})(n-1)/n\,, \qquad (3)$$

where $c(\mathbf{K},\mathbf{k})$ is the correlation function for emission from a single subsource, and $c'(\mathbf{K},\mathbf{k})$ is the correlation function for the overall one-particle probability distribution $p'(\mathbf{p},\mathbf{r})$, the convolution of P and p,

$$p'(\mathbf{p},\mathbf{r})\equiv\int d\mathbf{R}\,P(\mathbf{R})p(\mathbf{p},\mathbf{r}-\mathbf{R})\,. \qquad (4)$$

The two components c and c' form a two-tiered structure, with the individual-droplet contribution c providing a tail at large $k\sim a^{-1}$, and the overall distribution c' giving a peak at small $k\lesssim R_0^{-1}$.

For example, if $p(\mathbf{p},\mathbf{r})\sim\exp(-r^2/a^2)$ and $P(\mathbf{R})\sim\exp(-|\mathbf{R}|^2/R_0^2)$, and $\phi_{\mathbf{k}}(\mathbf{r})=[\exp(i\mathbf{k}\cdot\mathbf{r})+\exp(-i\mathbf{k}\cdot\mathbf{r})]/\sqrt{2}$ neglecting final-state interactions, then

$$C(\mathbf{k})=1+\exp(-\tfrac{1}{2}k^2a^2)/n+\exp[-\tfrac{1}{2}k^2(R_0^2+a^2)](1-1/n)\,. \qquad (5)$$

This example is shown in Fig. 1 for typical values of a, R_0, and n. The two components are readily distinguishable. Since the physical origin of the two-tiered structure is apparent, we expect it to be found also in more realistic cases with moving, time-dependent sources.

Several other effects can obscure the telltale correlations arising from the granularity of the mixed phase. First, the information will be lost if the bosons are rescattered after their emission from the droplets. To avoid rescattering we need to choose a probe with a long mean free path in hadronic matter. On the other hand, the need for good statistical accuracy means that the bosons need to be emitted copiously, which probably precludes photons. In addition, the interpretation of the correlations will be complicated if the bosons can be produced via long-lived resonances [5]. Finally, the final-state interactions of the observed pair need to be understood, since they appear in the distorted wave $\phi_{\mathbf{k}}(\mathbf{r}-\mathbf{r}')$. K^+ mesons appear to fit all these criteria. Figure 2 shows the effect of final-state Coulomb interactions on kaons, demonstrating that the two-tiered structure can still be found.

Of course, actual nuclear collisions do not provide instantaneous sources of kaons. The time dependence of the source can obscure the spatial information [6] by providing an additional dependence of $C(\mathbf{K},\mathbf{k})$ on the component of $\mathbf{k}$ along $\mathbf{K}$. The effect of finite source lifetime is shown in Fig. 3, which shows that averaging over the

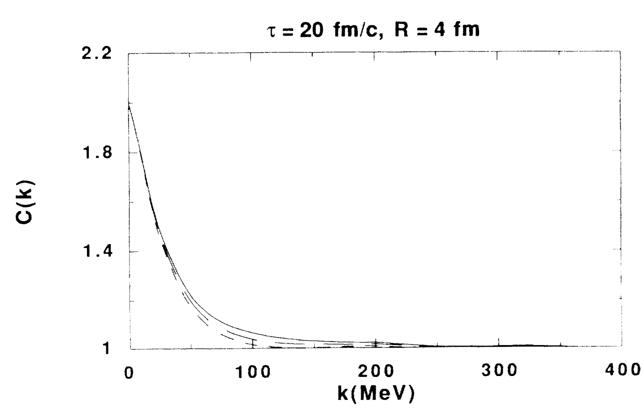

FIG. 3. Correlation function for the same source as Fig. 1, but with a nonzero lifetime $\tau=20$ fm/c. The correlation function is averaged over the direction of the relative momentum k. By choosing the kaon pair's relative momentum perpendicular to the pair's total momentum, the results from Fig. 1 would be recovered. The solid, long-dashed, and short-dashed lines represent the number of sources $n=2$, 4, and infinity.

direction of **k** obscures the desired information about the source granularity. The dependence of $C(\mathbf{K},\mathbf{k})$ on the components of **k** transverse to **K** will not be affected by the source lifetime, so it will be necessary to avoid averaging observations over the direction of **k**. However, the observation of a long lifetime would help to distinguish the long-lived plasma droplets, whose decay is delayed by the need to conserve entropy [1], from possible huge resonances that decay into many mesons [7]. In each case, the mesons would be expected to be emitted incoherently even though the sources are small, in view of the experience in e^+e^- and pp collisions where the source sizes are minimal and fewer degrees of freedom are excited.

In addition, the motion of the sources affects the observed correlations [5,8]. The effect of the moving sources may not be very great in the stopping regime, which is apparently the case at the Brookhaven Alternating Gradient Synchrotron, but will surely be essential at the ultrarelativistic energies of CERN and the Brookhaven Relativistic Heavy Ion Collider. Complementary information may then be obtained from fluctuations in the velocity distributions, which for a mixed phase will also have at least three scales, analogous to the three spatial scales: the overall rapidity interval $y_{\text{target}} - y_{\text{projectile}}$, the intercluster rapidity y_{ij}, and the velocity dispersion of the products of an individual droplet, which will be proportional to the temperature T_c. K^+ mesons appear advantageous also for observing the velocity fluctuations, not only because of their minimal rescattering and their ample production rates, but also because the velocity interval populated by an individual droplet will be small, since the mass of the kaon is greater than the temperature. We expect that analysis of these experiments may fruitfully be aimed at revealing these scales, rather than attempting to demonstrate scale-free behavior. Model simulations are currently in progress for the ultrarelativistic regime, and will be reported elsewhere [9].

We conclude that measurements of the correlations of K^+ mesons from relativistic nuclear collisions can show the granularity of their sources, providing evidence for a first-order phase transition. The interferometry will provide information about several size scales in the late stages of the nuclear collisions. Discovery of the droplet size scale could provide strong evidence for the occurrence of a first-order phase transition in nuclear collisions.

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