# Electron-Hose Instability in the Ion-Focused Regime

David H. Whittum [(a)]
*Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720*

William M. Sharp and Simon S. Yu
*Lawrence Livermore National Laboratory, University of California, Livermore, California 94550*

Martin Lampe and Glenn Joyce
*Naval Research Laboratory, Washington, D.C. 20375*
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A relativistic electron beam propagating through an unmagnetized, underdense plasma exhibits a transverse instability due to the coupling of the beam centroid to plasma electrons at the "ion-channel" edge. The transverse wake field corresponding to this "electron-hose" effect is calculated in the "frozen-field" approximation for a low-current, cylindrical beam in a radially infinite plasma. The asymptotic growth of beam-centroid oscillations is computed, and the growth length is found to be very rapid, indeed much less than the betatron period of the beam. Results for a radially finite plasma and for a slab beam are noted. Damping and saturation mechanisms are discussed.

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In recent years, the demands of the TeV-energy electron-positron collider [1] have spurred considerable interest in the transport of intense relativistic electron beams in the "ion-focused regime" (IFR). Proposed applications include the plasma lens [2], the continuous plasma focus [3,4], the plasma emittance damper [5], and plasma wake-field acceleration [6,7]. At the same time, coherent radiation from intense beams in the IFR has also been the subject of much theoretical [8–11] and experimental [12] work. These novel applications draw on a large body of work in beam-plasma physics [13–15] and extensive application of the IFR in accelerator and radiation research [16,17].

Typically the IFR refers to propagation along a narrow plasma channel which is "underdense" (i.e., with charge density much less than that of the beam) and in addition has total plasma charge per unit length less than that of the beam. In this limit, all plasma electrons are ejected radially to large distances. However, for many novel applications, the plasma may initially extend to large radii, or a broad plasma may be created by beam and secondary ionization. In this Letter, we show that propagation in such a regime suffers from a previously unrecognized hose instability, similar in character to the "transverse two-stream" instabilities [18,19] (e.g., the "ion-hose" instability [15]). This instability results from the electrostatic coupling of transverse beam displacements to plasma *electrons* at the boundary between the ion channel and the surrounding quasineutral plasma, beyond the beam volume. We show that the growth length for the "electron-hose" instability is so short that IFR transport in this regime is problematic at best.

To compute this growth length, we consider first equilibrium propagation of a relativistic electron beam in a uniform, unmagnetized, preionized plasma of density $n_e$, and infinite radial extent. We assume unperturbed beam charge density of the form $\rho_{b0}(r,s)=-en_b(s)H(a-r)$, where $H$ is the step function, $-e$ is the electron charge, $n_b$ is the beam density on axis, $a$ is the beam radius (Fig. 1), $s=t-z/c$ is the retarded time, $t$ is time, $z$ is axial displacement, and $c$ is the speed of light. As the beam head propagates through the plasma, it expels plasma electrons from the beam volume on the short time scale of an electron plasma period $\omega_e^{-1}\ll\tau_r$, where $\omega_e=(4\pi n_e e^2/m)^{1/2}$, $m$ is the electron mass, and $\tau_r$ is the current rise time. When the plasma is underdense $(n_e<n_b)$ *all* plasma electrons are adiabatically expelled from a cylindrical volume of radius $b=a(n_b/n_e)^{1/2}>a$. The resulting pure ion channel persists for a time of order $\omega_i^{-1}$, where $\omega_i=(4\pi n_e e^2/m_i)^{1/2}$ is the ion plasma frequency and $m_i$ is the ion mass. We assume $\tau\ll\omega_i^{-1}$, where $\tau$ is the pulse length, so that ion motion can be neglected.

The beam will be strongly focused (ion pinch force dominating over self-fields) if the Budker condition [13], $n_e\gg n_b/\gamma^2$, is satisfied, where $\gamma$ is the Lorentz factor for the beam. In this limit, *all* beam electrons undergo transverse oscillations at a single well-defined "betatron fre-

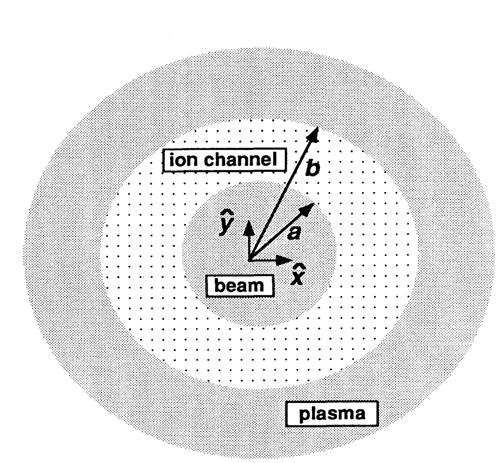


FIG. 1. In equilibrium, a relativistic electron beam of radius $a$ propagates through a channel of unneutralized ions. Plasma electrons have been expelled beyond a radius $b$.

quency" $\omega_\beta = \omega_e/(2\gamma)^{1/2}$. The primary motivation for using the IFR for beam transport is that $\omega_\beta$ is much larger than that achievable with conventional magnets.

We will assume that the collisionless plasma skin depth $c/\omega_e$ is much larger than the channel radius, so that $\omega_e b/c = 2\nu^{1/2} \ll 1$, where $\nu = I/I_0$ is Budker's parameter, $I_0 = mc^3/e \sim 17$ kA, and $I$ is the beam current. Since collisionless plasmas characteristically neutralize magnetic fields on the scale of a skin depth, it is appropriate in this limit to neglect the axial plasma-electron current. In this case, the equilibrium plasma-electron charge density is $\rho_{e0} = -en_e H(r-b)$.

We consider next the effect of a hoselike perturbation which displaces the beam centroid (Fig. 2) by a small amount $\xi(z,s)$ in the $x$ direction. For a pinched-beam equilibrium in which all beam electrons oscillate at the same betatron frequency, it is appropriate to use the "rigid-beam" model [20–23]. The perturbation to the beam charge density is then $\rho_{b1} = -en_b \xi \delta(a-r)\cos\theta$, where $\theta$ is the azimuthal angle in the $x$-$y$ plane.

The perturbed scalar and axial-vector potentials, $\phi_1$ and $A_{z1}$, for a displacement $\xi$ are calculated from Maxwell's equations in the Lorentz gauge,

$$\left\{\nabla_\perp^2 + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right\}\begin{bmatrix}\psi_1\\ A_{z1}\end{bmatrix} = -4\pi\begin{bmatrix}\rho_{e1}\\ \rho_{b1}\end{bmatrix}, \qquad (1)$$

where $\psi_1 = \phi_1 - A_{z1}$, $\rho_{e1}$ is the perturbed plasma-electron charge density, and the transverse gradient is $\nabla_\perp$. We change variables from $(z,t)$ to $(z,s)$ and simplify Eq. (1) with the "frozen-field" approximation, in which the d'Alembertian operators are approximated by $\nabla_\perp^2$ and radiative effects are neglected. This approximation in effect neglects terms of the order $\omega_e b/c\gamma \ll 1$.

The perturbed plasma-electron charge density $\rho_{e1}$ is determined from the potentials through the electron

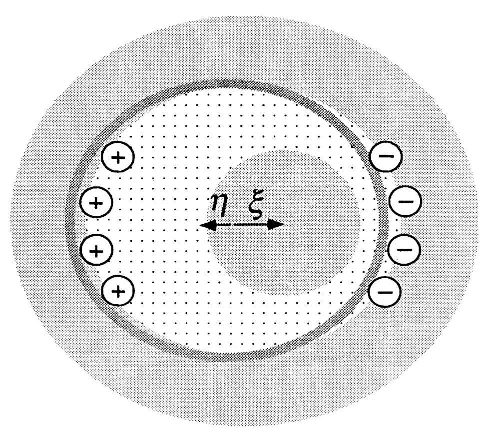


FIG. 2. A beam slice in the ion channel, displaced by an amount $\xi$ in the $x$ direction, induces a displacement $\eta$ of the channel wall, which responds as a simple harmonic oscillator with angular frequency $\omega_0$, deflecting follow-on portions of the beam.

cold-fluid equations.

$$\frac{\partial \rho_{e1}}{\partial s} + \nabla_\perp \cdot (\rho_{e0}\mathbf{V}_{e1}) \approx 0, \qquad (2)$$

$$\frac{\partial \mathbf{V}_{e1}}{\partial s} \approx \frac{e}{m}\nabla_\perp \phi_1, \qquad (3)$$

where $\mathbf{V}_{e1}$ is the plasma-electron velocity. Inspection of Eqs. (2) and (3) shows that $\rho_{e1}$ consists entirely of a surface-charge layer at $r=b$, and may be expressed as $\rho_{e1} = en_e \eta \delta(r-b)\cos\theta$. This perturbation may be thought of as a hoselike rippling of the inner surface of the electron fluid, where $\eta$ represents the hose displacement.

In terms of $\eta$ and $\xi$, the potentials from Eq. (1) are

$$A_{z1} = -2\pi n_b e \xi \cos\theta \left\{\frac{a^2-r^2}{r}H(r-a) + r\right\} \qquad (4)$$

and

$$\psi_1 = 2\pi n_e e \eta \cos\theta \left\{\frac{b^2-r^2}{r}H(r-b) + r\right\}. \qquad (5)$$

Using Eqs. (2)–(5) we find that $\eta$ responds as a simple harmonic oscillator driven by $\xi$,

$$\left[\frac{\partial^2}{\partial s^2} + \omega_0^2\right]\eta(z,s) = \left[\frac{n_b a^2}{n_e b^2}\right]\omega_0^2 \xi(z,s) = \omega_0^2 \xi(z,s). \qquad (6)$$

The characteristic angular frequency is $\omega_0 = \omega_e/2^{1/2}$ and differs from $\omega_e$ because the surface at $r=b$ is the boundary between a region of electron density $n_e$ and a region of zero density.

The Lorentz-force equation for the beam-centroid displacement is

$$\left[\frac{\partial}{\partial z}\gamma\frac{\partial}{\partial z} + \gamma k_\beta^2\right]\xi = \frac{e}{mc^2}\frac{\partial \psi_1}{\partial x} = \gamma k_\beta^2 \eta, \qquad (7)$$

where $k_\beta \sim \omega_\beta/c$ is the betatron wave number. This expression describes the deflection of the beam by the image polarization on the ion-channel wall.

For an infinitely long beam and beam line, perturbations may be taken to vary as $\exp(ikz - i\omega s)$, and Eqs. (6) and (7) lead in this case to the dispersion relation

$$(1 - k^2/k_\beta^2)(1 - \omega^2/\omega_0^2) = 1. \qquad (8)$$

In terms of $k' = k + \omega/c$, Eq. (8) may be rearranged to yield the well-known two-stream instability dispersion relation [24,25]. The result is instability for real $\omega^2 < \omega_0^2$ or real $k^2 < k_\beta^2$, with growth rates diverging as $\omega^2 \to \omega_0^2$ or $k^2 \to k_\beta^2$ from below. In general, such singularities result in an instability which is absolute in both the beam and laboratory frames [24,26,27]. As in the long-pulse, weak-beam limit of the two-stream instability (here, corresponding to $\omega_0 \tau \gg 1$ and $\omega_\beta \ll \omega_0$), it is the resonance at $\omega^2 \to \omega_0^2$ which dominates.

The spatial growth length for the initial-value problem follows immediately from Eq. (8) with the application of well-known results from two-stream theory [26,27]. Nevertheless, it is instructive to explicitly solve the problem up to quadrature for a semi-infinite beam and beam line.

Combining Eqs. (6) and (7) results in an equation for $\xi$ alone of the "beam breakup" (BBU) form [28],

$$\left[\frac{\partial}{\partial z}\gamma\frac{\partial}{\partial z}+\gamma k_\beta^2\right]\xi(z,s)=\int_0^s ds'\,\frac{\omega_0^3}{c^2}\sin\{\omega_0(s-s')\}\xi(z,s') . \qquad (9)$$

From Eq. (9) we may formally identify the electron-hose "dipole wake potential" [29] as $W(s)=W_0\sin(\omega_0 s)$, where $(I/I_0)W_0=\omega_0^3/c^2$ or $W_0=2\omega_0/b^2$. This wake is identical to that of an undamped microwave cavity with a coupling impedance per unit length [30] of $2/\omega_0 b^2$ and resonant frequency $\omega_0$. However, Eq. (9) has the unusual property that the driving term is *independent* of beam current. A smaller current results in a smaller channel radius and a larger geometrical coupling exactly compensating for the decreased current. Such an effect is not to be found in conventional BBU.

Taking $\xi(0,s)=H(s)$, the asymptotic form for the solution of Eq. (9) is obtained by the method of steepest descents [24,26,27],

$$\xi(z,s)\approx\frac{2^{1/2}}{3^{5/4}\pi^{1/2}}\frac{A^{1/2}}{\omega_0 s}\exp(A)\times\sin\{\omega_0 s-3^{-1/2}A-\pi/12\} , \qquad (10)$$

where the exponent is $A=(z/L_g)^{2/3}$. The growth length is

$$L_g=\frac{2^3}{3^{9/4}}\left(\frac{\gamma I_0}{I}\right)^{1/2}\frac{1}{(W_0 s)^{1/2}}=\frac{2^2}{3^{9/4}\pi}\frac{\lambda_\beta}{(\omega_0 s)^{1/2}} , \qquad (11)$$

and $\omega_0 s\gg A\gg 1$ is assumed [31]. This result shows that $L_g\ll\lambda_\beta$, corresponding to the "weak focusing" limit familiar from conventional, microwave BBU theory [32]. However, in contrast to conventional BBU, where $k_\beta$ and $W_0$ may be independently adjusted to achieve strong focusing, no such freedom is seen in Eq. (11). This result is remarkable in that it implies that focusing is rendered *ineffective* by the electron-hose wake field [33].

The severity of this result motivates us to consider mechanisms which might reduce electron-hose growth. From the dispersion relation of Eq. (8) we observe that the instability may in principle be mitigated by weakening the resonance at either $\omega^2\to\omega_0^2$ or $k^2\to k_\beta^2$. From Eq. (9) one may conclude that, since focusing is weak (i.e., since the resonance at $k^2\to k_\beta^2$ contributes only weakly), damping mechanisms relying on a spread in betatron wave number can only be effective if the spread is very large, $\Delta k_\beta/k_\beta\sim 1/k_\beta L_g>1$. Such a large spread in $k_\beta$ is unsuitable for many applications. This class of effects includes Landau damping due to a spread in energy within a beam slice [32], energy-sweep damping [34], and "phase-mix damping" due to nonlinear focusing, arising from a radially nonuniform plasma [15,16].

On the other hand, several mechanisms exist for weakening the electron-hose coupling at $\omega^2\to\omega_0^2$. First, the electron hose is eliminated entirely if the plasma charge per unit length, $Q_p$, is less than the beam charge per unit length, $Q_b=I/c$. In this case, *all* plasma electrons are ejected to large radii, leaving an ion channel with no excess plasma. This situation is in fact fairly typical for IFR channels produced by laser or beam ionization [15,16]. One may well ask how much excess plasma charge, $\Delta Q=Q_p-Q_b$, is tolerable and to answer this question, we have repeated the calculation of Eqs. (1)-(11) for a bounded plasma. Assuming the plasma extends initially from $r=0$ to $r=b+\delta$, we find that the condition $L_g>\lambda_\beta$ requires $\delta<b/\omega_e s$, corresponding to a charge excess of $\Delta Q/Q_p<1/\omega_e s$. Thus very little excess plasma charge per unit beyond the value $Q_b$ is tolerable.

One also expects the resonance at $\omega_0$ to be altered, and growth reduced, by plasma-electron collisions with ions and neutrals. Assuming a phenomenological collision rate $\nu_e$ in Eq. (3), we have found that the peak in growth occurs at $\nu_e s=2A/3$, convecting backward along the beam. This maximum varies as $\xi\propto\exp(z/L_g)$, with [35] $L_g\sim 0.2\lambda_\beta(\nu_e/\omega_e)^{1/2}$. This growth length is somewhat longer than that of Eq. (11), but still short compared to $\lambda_\beta$.

Growth could also be reduced by varying the resonant frequency of plasma oscillations, through the external geometry. For example, if we add a conducting pipe of radius $R$, we find a resonant frequency $\omega_0'\sim\omega_0(1+b^2/2R^2)$, so that the dipole $\eta$ oscillates at a slightly higher frequency which depends on $R$. Thus a variation of the pipe radius on the scale of a growth length could in principle produce an effect analogous to "stagger tuning" [32]. However, such a variation in $R$ would be quite rapid. Alternatively, an axial variation in plasma density, as in the continuous plasma focus [3,4], may produce phase-mix damping. In this case, the plasma density would have to vary appreciably over a length $L_g<\lambda_\beta$.

In addition, growth will be reduced somewhat by the plasma return current, neglected in the approximation $\omega_e b/c\ll 1$. In the low-current limit we have considered, the electron hose is reminiscent of the "image-displacement" [36]. If a conducting boundary or a sufficiently dense plasma were nearby, it would carry a dipole image current, and the combined Lorentz force on the beam due to the image fields would be a factor of $1/\gamma^2$ less than for the electric-field term alone. On the other hand, to achieve even $\omega_e b/c\sim 2$ requires $I\sim I_0$, a current larger than is envisioned for typical applications.

Ultimately, plasma electrons will be heated as a result of hosing and the cold-fluid model of Eqs. (2) and (3) will lose its validity. Computation of the asymptotic form for $\eta$ reveals that the hose oscillation amplitude is in general much larger than that for the beam, $\eta\sim(\omega_0 s/A)\xi\gg\xi$. In this case, one expects saturation when $\eta\sim b$, corresponding to a significant electron temperature $\sim mc^2\nu$, and a beam-centroid amplitude of order $\xi\sim b/\omega_0 s$. Numerical studies are in progress to rigorously examine the approach to saturation.

For TeV collider applications "flat" beams are also of great interest and we observe that in this case there arises a flutelike analog of the electron hose. We find that the minimum growth length for this flute instability is $L_g \sim 0.2\lambda_\beta/(\omega_e s)^{1/2}$, which is just the scaling of Eq. (11). Thus, for the flat beam the situation appears little improved.

In conclusion, we have presented a simple analytical theory describing a rapidly growing cumulative instability in a relativistic beam-plasma system. Further analytic work and numerical simulation are in progress to assess damping and saturation mechanisms, as well as plasma heating, bunching, and radiative effects.

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[a]Present address: Accelerator Division, National Laboratory for High Energy Physics, 1-1 Oho, Tsukuba, Ibaraki, 305, Japan.

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