Inhibition of Spontaneous Emission Noise in Lasers without Inversion

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I demonstrate a very large degree of quenching of spontaneous emission noise in different laser systems that work on mechanisms other than the population inversion between the bare states of the atom. Such laser systems have much narrower linewidths. I give explicit results for phase diffusion for four different model systems.

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Recently several model systems have been studied from the point of view of laser action without population inversion [1-7]. One has examined the dependence of the gain on various system parameters. So far very little has been done on the quantum features of such laser systems. A question that is of prime concern with all the laser systems is the linewidth. In this Letter I show that lasers without inversion have very considerable quenching of the spontaneous emission noise, resulting in much narrower linewidths as compared with the linewidths of conventional lasers. I demonstrate this explicitly for four different model systems. In each case I give, *without proof, quantitative results* for the noise quenching.

The Schawlow-Townes formula for the laser linewidth depends on two parameters: (i) the mean number $\bar{n}$ of photons at the operating point and (ii) the spontaneous emission by the atoms at the operating frequency of the laser. For a single-mode laser the diffusion coefficient of the phase can be expressed as

$$D_\varphi = S/4\bar{n}\,, \tag{1}$$

where S gives the rate of increase of the mean number of photons due to spontaneous emission, i.e.,

$$\frac{d}{dt}\langle a^\dagger a\rangle = S\langle a^\dagger a + 1\rangle + \text{other terms}\,. \tag{2}$$

The quantity S can be related to the two-time correlation function of the atomic variables. If we write the interaction Hamiltonian between the laser mode and the atoms as

$$H_c = gA^\dagger a + \text{H.c.}\,, \tag{3}$$

where A denotes the atomic dipole moment operator for the laser transition and g is the coupling constant, then one can show that

$$S = N|g|^2 \int_0^\infty [\langle A^\dagger(t+\tau)A(t)\rangle - \langle A^\dagger(t+\tau)\rangle\langle A(t)\rangle] e^{-i\omega\tau}\, d\tau + \text{c.c.} \tag{4}$$

Here N is the number of atoms taking part in the laser action. The two-time correlation function in (4) will depend on the excitation scheme used for the laser action. Thus for a given operating point, i.e., for a given number of laser photons, the linewidth essentially is determined by the parameter S. Thus we will say that noise quenching occurs if the parameter S is less than that for a standard laser.

Λ system with coherent pumping.— Let us first consider the system of Imamoglu, Field, and Harris [7] as shown schematically in Fig. 1(a). The laser transition is

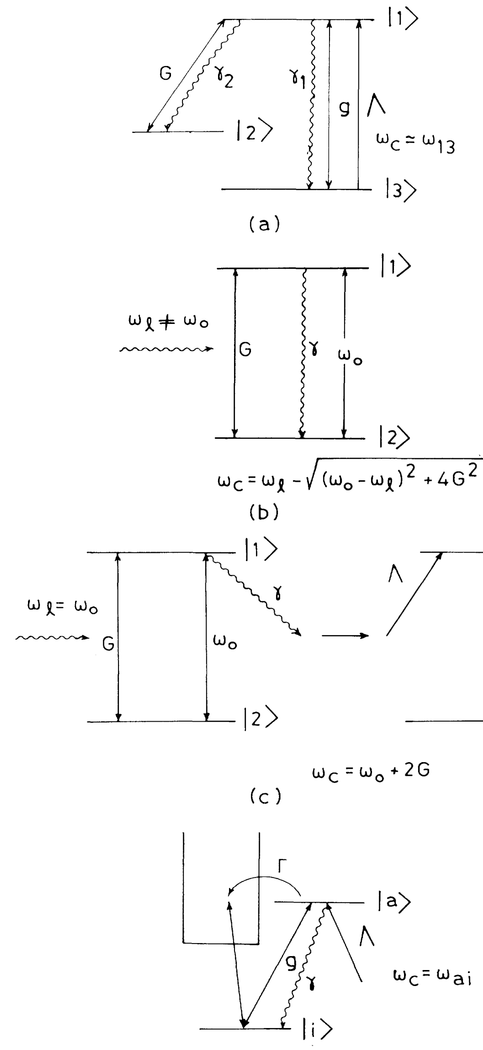

FIG. 1. Schematic illustration of the various pumping and energy-level schemes used for laser action without population inversion.

$|1\rangle \leftrightarrow |3\rangle$ and a coherent field is applied on the transition $|1\rangle \leftrightarrow |2\rangle$. This coherent field (Rabi frequency $2G$) can lead to laser action on the transition $|1\rangle \leftrightarrow |3\rangle$ even if the total excited-state population is less than the ground-state population, when all fields are on resonance and when the field on the $|1\rangle \leftrightarrow |2\rangle$ transition is large. The condition of gain is

$$\gamma_2 > \gamma_1 - \Lambda = \gamma_{10}, \tag{5}$$

where γ_{10} is the decay due to the spontaneous emission alone. We next discuss the spontaneous emission noise. The phase diffusion D or the quantum noise in the present laser model is determined by the spectrum of the spontaneously emitted photons on the transition $|1\rangle \leftrightarrow |2\rangle$. The spectrum $S(\omega)$ is to be calculated from the density-matrix equations for this three-level system in the absence of the probe field and the quantum regression theorem. In terms of the two-time correlation the spectrum [8] is

$$S(\omega) = N|g|^2 \mathrm{Re} \int_0^\infty d\tau\, e^{-i(\omega-\omega_{13})\tau} \lim_{t\to\infty} \langle A_{13}(t+\tau) A_{31}(t) \rangle, \tag{6}$$

where

$$A_{\alpha\beta}(t) = |\alpha\rangle\langle\beta|. \tag{7}$$

Our calculations show that

$$S(\omega) = \frac{[i(\omega-\omega_{13})+\Lambda+\gamma_2]2\Lambda|G|^2}{\{[i(\omega-\omega_{13})+\Lambda][i(\omega-\omega_{13})+\Lambda+\gamma_1+\gamma_2]+|G|^2\}\{|G|^2(2\gamma_1+2\Lambda)+2\Lambda[\gamma_2(\gamma_1+\gamma_2)+|G|^2]\}}. \tag{8}$$

Thus for the case of the cavity mode on resonance, $\omega = \omega_{13}$, and for large Rabi frequency $|G|$, the parameter S (i.e., the diffusion coefficient) becomes

$$S = Ng^2(\Lambda+\gamma_2)\Lambda/|G|^2(\gamma_1+2\Lambda). \tag{9}$$

On the other hand, the diffusion coefficient or S_0 for the standard two-level laser model is obtained from (8) by letting $\gamma_2 \to 0$, $|G| \to 0$,

$$S_0 = \Lambda g^2 N/(\Lambda+\gamma_1)(\gamma_1+2\Lambda). \tag{10}$$

Thus the ratio of the diffusion coefficient is

$$S/S_0 \approx (\Lambda+\gamma_2)(\Lambda+\gamma_1)/|G|^2 \ll 1. \tag{11}$$

The last result follows since Eq. (9) was derived under the condition that the Rabi frequency of the coherent field is much bigger than the pumping and spontaneous emission rates. The result (11) shows that the *Λ-system model* for lasers without inversion has considerable *quenching of spontaneous emission noise.*

Quenching of noise in the system based on gain on the three-photon Mollow sideband.— Let us next consider the laser system [Fig. 1(b)] which works because of gain on the left Mollow sideband [6] even though there is no population inversion between the two bare states of the atom. The details of this model can be found elsewhere [5]. Here the two-level transition frequency ω_0 is pumped coherently by a field of frequency ω_l and the strength is given by the Rabi frequency $2G$. It is known that under appropriate conditions one can have gain in the frequency region $\omega_l - [(\omega_l-\omega_0)^2+4G^2]^{1/2}$. The phase diffusion in this region is determined by the spectrum of spontaneous emission which in turn depends on $|G|$ and $\Delta = \omega_0 - \omega_l$.

In Ref. [5] it was shown that this laser system is equivalent to the standard single-mode laser model provided we make all relaxation times and coupling constants field dependent and provided that we work in the dressed state basis. Calculations show that the ratio of the diffusion coefficient for the present model to that for the standard laser model is

$$\frac{S}{S_0} = \frac{\mu^4}{1+\mu^4+4\mu^2}, \quad \mu = \frac{\Delta-(\Delta^2+4G^2)^{1/2}}{2G}. \tag{12}$$

In writing the ratio (12) we have assumed that the inversion in the usual laser model is the same as the inversion in the dressed state basis for the present model. This has been done so that the noise is compared under identical pumping conditions. By using *different values of Δ and G* one can see that *S/S_0 could be much smaller than 1.* For example, $S/S_0 \approx 1/6$, $1/10$, and $1/20$ for $\Delta = 0$, $G/2$, and G, respectively.

Inhibition of spontaneous noise in a model based on the pumping of one of the dressed states.— We next consider a laser model [4] [Fig. 1(c)] in which the atoms are prepared in one of the dressed states, say $|\psi_+\rangle$ before entering the cavity. The external field is on resonance with the bare atomic transition. The state $|\psi_+\rangle$ is defined to be the eigenstate of the resonant Hamiltonian with eigenvalue G. In such a case one has gain in the frequency region $\omega_l + 2G$. This gain can be used for laser action. The diffusion coefficient depends on the spectrum of spontaneous emission in the frequency range $\omega_l + 2G$. The exact form of the spectrum will depend on the decay rates of the bare states $|1\rangle$ and $|2\rangle$ of the atom. Let Λ be the rate at which atoms in the state $|\psi_+\rangle$ are brought into the cavity and let γ be the rate at which the excited state $|1\rangle$ decays to some other state in the system. When the external field is on resonance with ω_0 and the cavity mode on resonance with $\omega_l + 2G$, then the coefficient S is found to be

$$S = \Lambda |G|^2/\gamma^2. \tag{13}$$

The diffusion coefficient for the standard laser model under similar pumping and decay conditions is

$$S_0 = 4\Lambda |g|^2/\gamma^2, \tag{14}$$

and hence

$$S/S_0 = 1/4. \tag{15}$$

The diffusion coefficient is a quarter of its value for a normal laser system. Thus the present model also *exhibits quantum noise quenching.*

Noise quenching in the system based on the pumping of the autoionizing states.— We finally consider the system [Fig. 1(d)] working on the pumping of the autoionizing states. The spontaneous emission spectrum is determined by the two-time correlation function [9] $\langle A_i^\dagger(t+\tau)A_i(t)\rangle$, where A_i is the atomic operator

$$\int dE\, |i\rangle\langle E| B_{Ea},$$

with

$$B_{Ea} = b_{Ea}\left(1 + \frac{2(E-E_a)}{\Gamma q}\right), \qquad |b_{Ea}|^2 = \frac{\Gamma/2\pi}{(\Gamma/2)^2 + (E-E_a)^2}. \tag{16}$$

The asymmetry parameter q and the autoionization rate Γ are defined by [10]

$$\Gamma = 2\pi |V_{Ea}|^2, \quad q^2 = |V_{ia}|^2/\pi^2 |V_{iE}|^2 |V_{Ea}|^2. \tag{17}$$

This correlation function is to be calculated in the presence of the pumping of the autoionizing state $|a\rangle$. The pumping can be included as in Ref. [2]. For the cavity mode on resonance with the transition $|a\rangle \leftrightarrow |i\rangle$, we find that the parameter S is given by

$$S = g^2\Lambda\left(1 + \frac{1}{q^2}\right)\frac{\gamma}{2\pi}\bigg/\left(\frac{\Gamma}{2} + \frac{\gamma}{2}\right)^2, \qquad \gamma = \frac{4}{3}\frac{|V_{ai}|^2\omega^3}{c^3\hbar}, \quad \omega = \frac{E_a - E_i}{\hbar}. \tag{18}$$

The interference in the Fano system is known to be most prominent in the limit $q \to 0$, whereas in the limit $q \to \infty$ Fano line shapes go over to symmetric profiles. Assuming that the large-q limit is equivalent to a closing of the autoionization channel, then

$$S \to S_0 = 2g^2\Lambda/\pi\gamma. \tag{19}$$

Thus the ratio S/S_0 in the limit of small q becomes

$$\frac{S}{S_0} = \frac{\gamma^2}{(\Gamma+\gamma)^2 q^2} \approx \frac{\gamma}{q^2\Gamma}\frac{\gamma}{\Gamma}, \tag{20}$$

since the small-q limit can be thought of as due to large Γ. Note that $\gamma/q^2\Gamma$ is nothing but the *probability of radiative recombination* [11], $(2\omega^3\pi/3\hbar c^3)|V_{iE}|^2$, and thus $\gamma/q^2\Gamma < 1$. We then get $S/S_0 \ll 1$, i.e., the laser system working due to the pumping of the autoionizing state also leads to a *large amount of quenching of spontaneous emission noise.*

In conclusion, we have shown, by considering various models of lasers without inversion, that the spontaneous emission noise is considerably quenched in all such systems. Clearly lasers without inversion will have much narrower linewidths.

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