From QCD Sum Rules to Relativistic Nuclear Physics

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Nucleon self-energies in nuclear matter are studied by analyzing the correlator of nucleon interpolating fields using QCD sum-rule methods. Large Lorentz scalar and vector self-energies arise naturally, and are comparable to the optical potentials of Dirac phenomenology. The key phenomenological inputs are the baryon density and the value of the nucleon σ term.

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In the Dirac phenomenology of proton-nucleus scattering, nucleon propagation is described by a Dirac equation with an optical potential featuring large (several hundred MeV) Lorentz scalar and vector components [1]. This phenomenology provides a simple yet quantitatively accurate model of spin observables over a wide range of energies and target nuclei [1]. However, the connection of this phenomenology to quantum chromodynamics (QCD) has not been established, and the possibility of such a connection has been overshadowed by concerns that the Dirac equation should not be used to describe composite nucleons. In this Letter, we show how large scalar and vector self-energies arise naturally in finite-density QCD due to changes in the scalar quark condensate and the quark density. We use QCD sum-rule techniques to calculate the self-energies of intermediate-energy nucleons in nuclear matter and find them to be comparable to the Dirac optical potentials.

We consider a correlation function of interpolating fields, built from quark fields, that carry the quantum numbers of the nucleon [see Eqs. (2) and (3)]. By applying an operator product expansion (OPE) for large spacelike momenta, the correlator can be expressed as a sum of coefficient functions, calculated in QCD perturbation theory, that multiply matrix elements of composite operators. In the vacuum, these matrix elements are the nonperturbative quark and gluon condensates. On the other hand, a spectral decomposition shows that this correlator describes the propagation of a (virtual) nucleon as well as higher-mass states with nucleon quantum numbers. QCD sum rules equate these two representations of the correlator; then, after assuming a simple phenomenological ansatz for the spectral density, spectral parameters of low-lying resonances can be extracted in terms of QCD Lagrangian parameters and the condensates [2,3]. A Borel transform improves the overlap of the two descriptions: On the QCD side it improves the convergence of the OPE by suppressing the contributions of higher-dimensional operators, while on the phenomenological side it emphasizes the contribution from the nucleon pole [2].

First, we review the zero-density limit of the nucleon sum rule [4]. The phenomenological description (spectral density) is taken to be a nucleon pole plus a smooth continuum that accounts roughly for all higher-mass excitations. Lorentz covariance implies two distinct Dirac matrix structures for the vacuum correlator: a four-vector (corresponding to $\not{q}$) and a scalar (corresponding to the nucleon mass M_N). A sum rule is obtained for each structure. The ratio of the two rules leads to an expression for M_N as a function of M^2, the mass parameter of the Borel transform. The key assumption of the QCD sum rule is that, for a range of intermediate M^2 values, the Borel-transformed correlator is described both by a truncated OPE (which is increasingly valid as $M^2 \to \infty$) and by a dispersion integral that is dominated by the nucleon pole (which is increasingly true as $M^2 \to 0$).

Sum-rule analyses of the nucleon mass have been made by Ioffe [4] and many others [3]. Ioffe concluded that the contributions of higher-dimensional condensates and the continuum are, in fact, sufficiently small for values of the Borel mass in the vicinity of M_N that meaningful predictions can be made. Furthermore, the principal physical content of the full sum rule, that the scale of the nucleon mass is largely determined by the quark condensate, is manifest even in a simplified sum rule in which only the leading contributions from the OPE to each sum rule are kept and the continuum is neglected [3]. In particular, one can obtain a simple expression for the nucleon mass [3],

$$M_N = -(8\pi^2/M^2)\langle\bar{q}q\rangle_{\text{vac}}, \tag{1}$$

where $\langle\bar{q}q\rangle_{\text{vac}} \simeq -(230 \pm 20\ \text{MeV})^3$ is the quark condensate, and the formula is to be evaluated for $M^2 \sim M_N^2$. (Note that in this simple Ioffe formula, M_N is sensitive to the choice of the Borel mass; the optimal choice for the Borel mass must be obtained from a more sophisticated sum rule [3,4].) In this Letter, we generalize the Ioffe formula to finite density.

It is natural to apply QCD sum-rule methods to calculate the scalar and vector self-energies of a nucleon quasiparticle in the nuclear medium. If the quark interactions with the vacuum condensates strongly influence the low-lying structure in the spectrum, which is an essential assumption of the QCD sum-rule approach, then changes in these condensates due to finite baryon density should also be reflected in changes in the nucleon spectrum. Thus, the condensates should set the scale for nucleon self-energies in medium. There are several recent applications of QCD sum rules to finite-density problems [5–7]. In Ref. [7], Drukarev and Levin use finite-density sum rules to describe nuclear-matter saturation properties. In the Drukarev-Levin approach, the distinction between Lorentz scalars and the time components of four-vectors is neglected. In contrast, we emphasize this distinction to test whether the large scalar and vector optical potentials used in Dirac phenomenology are implied by QCD.

Consider the correlation function $\Pi_N(q)$ defined by

$$\Pi_N(q)\equiv i\int d^4x\, e^{iq\cdot x}\langle T[\eta(x)\bar{\eta}(0)]\rangle_{\rho_N}\,, \qquad (2)$$

with $\langle\hat{O}\rangle_{\rho_N}$ denoting the matrix element of $\hat{O}$ taken in the ground state of the nuclear medium. The ground state is characterized by ρ_N, the nucleon density in the rest frame. Following Ioffe, the interpolating field $\eta(x)$ for the proton is constructed from up- and down-quark fields as [4]

$$\eta(x)=\epsilon_{abc}[u^{aT}(x)C\gamma_\mu u^b(x)]\gamma_5\gamma^\mu d^c(x)\,, \qquad (3)$$

where a, b, and c are color indices and C is the charge-conjugation matrix.

Lorentz covariance, parity, and time reversal imply that $\Pi_N(q)$ has the form

$$\Pi_N(q)\equiv\Pi_s(q^2,q\cdot u)+\Pi_q(q^2,q\cdot u)\not{q}+\Pi_u(q^2,q\cdot u)\not{u}\,, \qquad (4)$$

where u^μ is the four-velocity of the nuclear medium; i.e., $u^\mu=(1,\mathbf{0})$ in the rest frame. Note that there are *three* distinct structures: scalar, $\not{q}$, and $\not{u}$, and thus three invariant functions. To generalize the Ioffe formula, we keep only the leading term in the OPE for each invariant function, and match to a simple ansatz with a nucleon quasiparticle pole.

Wilson coefficients for the OPE at finite density can be calculated using standard techniques [3,8] by incorporating all density dependence into the matrix elements. For the present calculation, we need only the most singular terms of the coordinate-space quark propagator in the presence of the condensates [8]:

$$\langle T[q_i^a(x)\bar{q}_j^b(0)]\rangle_{\rho_N}=\frac{i}{2\pi^2}\delta^{ab}\frac{x^\mu}{x^4}[\gamma_\mu]_{ij}-\frac{1}{4N_c}\delta^{ab}[\gamma_\mu]_{ij}\langle\bar{q}\gamma^\mu q\rangle_{\rho_N}-\frac{1}{4N_c}\delta^{ab}\delta_{ij}\langle\bar{q}q\rangle_{\rho_N}+\cdots\,, \qquad (5)$$

where i and j are Dirac indices and N_c is the number of colors. The only change from the vacuum calculation is a new "condensate" $\langle\bar{q}\gamma^\mu q\rangle_{\rho_N}$, which is simply the rest-frame quark density times u^μ (normal ordering with respect to the perturbative vacuum is implied). We neglect current quark masses and gluon condensate contributions, which are numerically small. The correlation function is evaluated by applying Wick's theorem to Eq. (2), using Eq. (5) for each contraction, and then projecting the leading contributions to each invariant function [8].

For convenience, we present the invariant functions of Eq. (4) in the rest frame of nuclear matter, where $q\cdot u\rightarrow q_0$. The leading contributions are [4,8]

$$\Pi_s(q^2,q_0)=\frac{1}{4\pi^2}q^2\ln(-q^2)\langle\bar{d}d\rangle_{\rho_N}+\cdots\,, \qquad (6)$$

$$\Pi_q(q^2,q_0)=-\frac{1}{64\pi^4}(q^2)^2\ln(-q^2)+\frac{1}{6\pi^2}q_0\ln(-q^2)[\langle u^\dagger u\rangle_{\rho_N}+\langle d^\dagger d\rangle_{\rho_N}]+\cdots\,, \qquad (7)$$

$$\Pi_u(q^2,q_0)=\frac{1}{12\pi^2}q^2\ln(-q^2)[7\langle u^\dagger u\rangle_{\rho_N}+\langle d^\dagger d\rangle_{\rho_N}]+\cdots\,, \qquad (8)$$

where we have suppressed terms that are simple polynomials in q^2 (including divergent terms), as they will be eliminated by a subsequent Borel transform. Results for the neutron are obtained by interchanging u and d. Finally, since we focus on isoscalar quantities in nuclear matter, we take

$$\langle\bar{u}u\rangle_{\rho_N}\simeq\langle\bar{d}d\rangle_{\rho_N}\equiv\langle\bar{q}q\rangle_{\rho_N}\,, \qquad (9)$$

$$\langle u^\dagger u\rangle_{\rho_N}=\langle d^\dagger d\rangle_{\rho_N}\equiv\langle q^\dagger q\rangle_{\rho_N}=\tfrac{3}{2}\rho_N\,, \qquad (10)$$

where $\langle q^\dagger q\rangle_{\rho_N}$ is the quark density for one flavor in the nuclear-matter rest frame.

We generalize the conventional zero-density ansatz for the phenomenological side of the sum rule by assuming a quasiparticle pole for the nucleon, with real self-energies independent of energy and momentum; all higher-mass excitations are included in a continuum contribution. These assumptions are consistent with the essential physics of the phenomenological optical potentials, which have relatively small imaginary parts and are not strongly energy dependent in the region of interest [1]. Lorentz covariance dictates the form

$$\Pi_N(q)=-\lambda_N^{*2}\frac{1}{(q^\mu-\Sigma_v^\mu)\gamma_\mu-(M_N+\Sigma_s)}+\cdots\,, \qquad (11)$$

where λ_N^* is the coupling strength of the current $\eta(x)$ to the nucleon quasiparticle in medium and Σ_v^μ and Σ_s are the self-energies. In general, we can write $\Sigma_v^\mu\equiv\Sigma_v u^\mu+\Sigma_v' q^\mu$; we assume Σ_v, Σ_v', and Σ_s are constants and neglect Σ_v' in the discussion here. In the spirit of the Ioffe formula, we neglect contributions from higher-mass excitations in this first simple calculation. With these simplifications, the phenomenological representations of the invariant functions in the nuclear-matter rest frame are

$$\Pi_s(q^2,q_0) = -\lambda_N^{*2}\frac{M_N^*}{q^2-\mu^2}+\cdots, \qquad (12)$$

$$\Pi_q(q^2,q_0) = -\lambda_N^{*2}\frac{1}{q^2-\mu^2}+\cdots, \qquad (13)$$

$$\Pi_u(q^2,q_0) = \lambda_N^{*2}\frac{\Sigma_v}{q^2-\mu^2}+\cdots, \qquad (14)$$

where we have defined $M_N^* \equiv M_N + \Sigma_s$ and $\mu^2 \equiv M_N^{*2} - \Sigma_v^2 + 2q_0\Sigma_v$. We view Eqs. (12)-(14) as following from dispersion relations in q^2, with $q\cdot u$ (effectively) fixed at the quasinucleon energy. This approach suppresses antinucleon contributions [8].

In the case of the vacuum sum rules, a Borel transform with respect to spacelike q^2 improves the overlap of the theoretical and phenomenological descriptions of the correlator [2]. In the finite-density case, the invariant functions depend on both q^2 and $q\cdot u$; we Borel transform with respect to $-q^2$ with $q\cdot u$ held constant. Equating the Borel transforms of the theoretical and phenomenological descriptions yields three relations, one for each invariant function:

$$\lambda_N^{*2}M_N^* e^{-\mu^2/M^2} = -\frac{1}{4\pi^2}M^4\langle\bar{q}q\rangle_{\rho_N}, \qquad (15)$$

$$\lambda_N^{*2}e^{-\mu^2/M^2} = \frac{1}{32\pi^4}M^6 - \frac{1}{3\pi^2}q_0M^2\langle q^\dagger q\rangle_{\rho_N}, \qquad (16)$$

$$\lambda_N^{*2}\Sigma_v e^{-\mu^2/M^2} = \frac{2}{3\pi^2}M^4\langle q^\dagger q\rangle_{\rho_N}. \qquad (17)$$

These sum rules depend on two parameters, the Borel mass (M) and q_0. We take q_0 to be the energy of the quasiparticle we wish to study; μ^2 is equal to the on-shell four-momentum squared of the quasiparticle. We expect the optimal value of M^2 to be around μ^2 (~ 1 GeV^2), which should ensure reasonable convergence of the OPE while suppressing contributions from higher-mass singularities.

To calculate the self-energies from these formulas, we need to know the scalar and vector condensates in the nuclear medium, $\langle\bar{q}q\rangle_{\rho_N}$ and $\langle q^\dagger q\rangle_{\rho_N}$. The vector condensate is related to the nucleon density in Eq. (10). The *change* in the scalar condensate, to lowest nontrivial order in a density expansion, is related to σ_N, the nucleon σ term (at $t=0$) [7,9]:

$$\langle\bar{q}q\rangle_{\rho_N} = \langle\bar{q}q\rangle_{\text{vac}} + \frac{\rho_N\sigma_N}{m_u+m_d}+\cdots, \qquad (18)$$

where m_u and m_d are the current quark masses. This can be understood quite simply in terms of the Hellmann-Feynman theorem [9]. Estimated corrections due to higher-order terms in the density expansion are small ($\sim 10\%$) at nuclear matter densities and below [9]. A recent analysis of the σ term [10] yields a value of approximately 45 MeV, with an uncertainty of order 7–10 MeV.

Taking ratios of Eqs. (15)-(17) leads to simple expressions for the scalar and vector densities,

$$\Sigma_s = -\frac{8\pi^2}{M^2}(\langle\bar{q}q\rangle_{\rho_N} - \langle\bar{q}q\rangle_{\text{vac}}) = -\frac{8\pi^2}{M^2}\frac{\rho_N\sigma_N}{(m_u+m_d)}, \qquad (19)$$

$$\Sigma_v = \frac{64\pi^2}{3M^2}\langle q^\dagger q\rangle_{\rho_N} = \frac{32\pi^2}{M^2}\rho_N, \qquad (20)$$

which should be valid below nuclear matter saturation density. We have neglected the second term in Eq. (16), which is higher order in the operator product expansion, and to derive Eq. (19) we have subtracted the zero-density result [Eq. (1)]. Upon taking the ratio of Eqs. (19) and (20), the explicit dependence on the Borel mass and the density drops out, yielding

$$\frac{\Sigma_s}{\Sigma_v} = -\frac{\sigma_N}{4(m_u+m_d)}. \qquad (21)$$

For typical values of σ_N (45 ± 8 MeV) and the quark masses ($m_u+m_d = 14\pm 4$ MeV), this ratio is close to -1 (-0.8 ± 0.3), indicating a substantial cancellation of Σ_s and Σ_v in the medium. Since Σ_s and Σ_v are essentially the real parts of the optical potential, this *qualitative* result is in agreement with Dirac phenomenology.

The individual magnitudes of the scalar and vector self-energies in Eqs. (19) and (20) depend on the Borel mass. As in the vacuum sum rule, an appropriate choice for the Borel mass cannot be determined with certainty from an analysis as simple as the one presented above; it requires a study of corrections due to perturbative effects, higher-order condensates, and other singularities [8]. However, scales on both the theoretical and phenomenological sides of the sum rules are similar to those in the vacuum problem (e.g., $\mu^2 \sim M_N^2$), so it is reasonable to assume that the optimal Borel mass will be close to the one that works best in the vacuum sum rule, Eq. (1). Taking this value of the Borel mass ($M^2 \sim 1$ GeV^2) in Eqs. (19) and (20) yields $\Sigma_v \sim -\Sigma_s \sim 400$ MeV at nuclear matter density. These magnitudes are consistent with the large and canceling scalar and vector potentials used in the Dirac optical potentials, although they are somewhat larger [1]. The results are also comparable to those obtained in a simple mean-field treatment of the Walecka σ-ω model [11].

Although the 400-MeV values for Σ_v and $-\Sigma_s$ set the scale for nucleon self-energies in the medium, the precise magnitudes and degree of cancellation implied by Eqs. (19)-(21) should not be taken too seriously at this point—there are significant corrections from higher-order terms in the OPE, from the density expansion, and from the continuum. For example, including the second term in Eq. (16) with $q_0 \sim 1$ GeV reduces the prediction for $-\Sigma_s$ by roughly 50%. Thus, it is not obvious that the cancellation of the self-energies, which is essential to Dirac phenomenology, will survive in a more complete treatment of the sum rule. Various corrections will be assessed in Ref. [8] to test the stability of our leading-order results; preliminary calculations suggest that the self-energies may be reduced by as much as 50%, but still exhibit significant cancellation.

An important assumption of the preceding analysis is that the phenomenological side of the sum rule is dominated by the (positive-energy) nucleon pole. Following Ref. [7], we can consider the nuclear medium as an A-body system with four-momentum P^μ, and postulate a dispersion relation in q^2 at fixed $s_A \equiv (q+P)^2$ for each invariant function [12]. For large A, fixed s_A implies that q_0 is fixed and independent of q^2 [except when q^2 is $O(A)$]. Higher-mass baryon and nucleon-meson states will be suppressed by the Borel transform; they can be included in a more complete sum rule with a simple continuum ansatz [7]. Furthermore, we expect that low-energy excitations in the $(A+1)$-nucleon system will simply spread strength from the nucleon pole to nearby q^2. In the optical model such a spreading of strength is described by the imaginary part, which we know on phenomenological grounds to be relatively small [1]. Thus, assuming a sharp quasiparticle pole should not substantially affect our conclusions. In summary, the dominance of the quasinucleon pole is plausible; however, we emphasize that the dispersion relations in medium require further study.

What about relativistic nuclear dynamics? The sum-rule approach emphasizes the role of the condensates and gives limited information about degrees of freedom at the hadronic level. The correlator is studied at relatively large spacelike momenta and thus it is difficult to resolve the hadronic content of the intermediate states. In particular, the large scalar and vector self-energies we find do *not* necessarily imply that there are large contributions from virtual $N\bar{N}$ pairs [1]. The possibility of separate Lorentz scalar and vector potentials follows directly from the Lorentz structure of the correlator [Eq. (2)], and has no necessary connection to virtual pair creation. Furthermore, the connection between our description based on quark interactions with finite-density condensates and a conventional meson-exchange picture is not clear.

Although we use the machinery of the QCD sum rules, we emphasize that our qualitative results do not rely on specific details of the sum rules. In particular, our qualitative picture is ultimately based only on the assumption that the properties of nucleons are largely determined by interactions with the condensates. The key input to our calculation is the value of the in-medium scalar condensate, which we relate to the nuclear density and the nucleon σ term. Using standard values we see that the scalar condensate has a strong density dependence—it is reduced by 30%–40% from its vacuum value at nuclear matter density. This strongly suggests that nucleons must experience a large scalar potential. Given the empirical fact that the total potential, i.e., scalar plus vector, is known to be small (of order −50 MeV), we expect a strong vector potential with the opposite sign. We find that it emerges naturally from interactions with the finite-density medium, depending in leading order only on the total quark density. Scalar attraction and vector repulsion of this sort are the essential ingredients of relativistic nuclear physics. While improved approximations will affect the quantitative results, we expect these basic ingredients to survive.

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[12] In a recent report, Laboratoire de Physique Théorique et Hautes Energies, Orsay, Report No. 90/52, 1990 (to be published), the authors of Ref. [7] advocate dispersion relations in q^2 at fixed $s=(q+p)^2$, where p^μ is the four-momentum of a nucleon in the medium. Using this alternative approach, including a Borel transform in $-q^2$ at fixed s, does not change our qualitative results.