Self-Organized Critical Behavior in Pinned Flux Lattices

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(Received 28 May 1991)

We study the response of pinned flux lattices, under small perturbations in the driving force, below and close to the pinning-depinning transition. For driving Lorentz forces below F_c (the depinning force at which the whole flux lattice slides), the system has instabilities against small force increases, with a power-law distribution characteristic of self-organized criticality. Specifically, $D(d) \sim d^{-1.3}$, where d is the displacement of a flux line after a very small force increase. We also study the initial stages of the motion of the lattice once the driving force overcomes the pinning forces.

PACS numbers: 74.60.Ge, 05.40.+j

Transport phenomena typically deal with either independent or collective motion of a large number of particles. In the latter, there is a high degree of coherence and the forces between the particles are as important as the applied driving force.

In this paper, we will study a highly simplified model of the onset of collective transport in a disordered medium. These systems are typically modeled by a large collection of objects (in our case flux lines), interacting via some potential, and moving in a random static medium under the influence of a uniform applied force. Collective transport arises when the interaction forces are as important as the applied and random forces. Flux pinning in superconductors and charge-density waves are two examples of these types of phenomena, which sometimes are grouped together under the general classification of "hysteretic phenomena." Randomly distributed defects are responsible for the pinning of both flux lattices in type-II superconductors and charge-density waves. Since this problem has proven to be extremely difficult to study, we will focus on a highly simplified model that we believe has some of the essential ingredients. Also, we will use zero-temperature deterministic dynamics without inertial effects. In spite of the apparent simplicity of these models, they are far more difficult to tackle than the usual cellular-automata toy models. For a detailed analysis of more realistic approaches to the flux-pinning problem, the reader is referred to the excellent presentations in Refs. [1-3].

We are interested in the stick-slip dynamics exhibited by simple models of flux lattice motion in a type-II superconductor. When a bias current is applied to the system, a Lorentz force acts on the flux lines, some of which are confined inside potential wells, which we model to be parabolic. We have monitored the response of particles (from now on to be called vortices) when an externally applied force (e.g., Lorentz) is slightly increased. Of particular interest is the response (i.e., displacement of the vortices) of the system at the threshold of instability.

Since our model is one dimensional (1D), it may appear to be more appropriate for charge-density waves than for flux transport in type-II superconductors. However, 1D models have been fruitfully applied for many years to describe the essential features of pinning and critical current densities in superconductors [4]. Examples of these confined-geometry superconductors include microbridges, constrictions, wires connecting two superconducting grains, and long Josephson junctions [1].

Bean [5] and de Gennes [6] have used a sandpile analogy to describe the critical state of a type-II superconductor with sites that pin the flux lattice. Extending this analogy, we wish to explore the possibility that a superconductor in the critical state could exhibit self-organized criticality (SOC), as has been suggested for real sandpiles [7]. In our model, we perturb the system and watch its collective (bundle motion of flux lines) response. We study the change in the equilibrium positions of the flux lines in two regimes: when the applied driving force is well below the value where the whole lattice is in the free-flow (sliding) mode and when the force is below but close to this value. The presence of power-law behavior over several decades would be an indication that the system is in the SOC regime.

Previous numerical models of pinned flux lattices [8] have considered Gaussian forms of the potentials describing the vortex-vortex and the vortex-pinning-site interactions. Here we will go further in the simplification of the model by using parabolic potentials. The principal advantage of this is that it greatly speeds up the calculations, enabling the use of simple relaxation techniques for the dynamics. The enhanced speed also permits the performance of hundreds of simulations which allows us to do an ensemble average over a very large number of different configurations.

The pinning centers will act as parabolic traps for the flux lines, and the potential acting on the ith vortex, V_{pi}, is approximated to be

$$V_{pi} = \sum_j -\frac{\xi_p}{2} + \frac{1}{2\xi_p}(x_i^v - x_j^p)^2, \text{ for } |x_i^v - x_j^p| \le \xi_p , \quad (1)$$

and constant for $|x_i^v - x_j^p| > \xi_p$. Here x_i^v, x_j^p, and ξ_p represent the position of the vortex, the position of the pinning site, and the range of the pinning force, respectively. The force exerted by a single well of this kind is such that the applied force needed to take a vortex out of the well is equal to unity. This will define an upper bound for F_c, the externally applied Lorentz force that will bring the system into collective free-flow motion. The vortex-vortex potential felt by a particular vortex i is approximated by

$$V_{vi}=\sum_{j,j\neq i} A_v(|x_i^v-x_j^v|-\xi_v)^2,\ \text{for}\ |x_i^v-x_j^v|\le\xi_v\,, \qquad (2)$$

and constant for $|x_i^v-x_j^p|>\xi_v$. We will choose the vortex-vortex interaction range ξ_v equal to 1 and the prefactor $A_v=20$ to prevent vortices from jumping over other vortices. If the driving force F_L is applied in the positive x direction, and friction is neglected, the total force that acts on particle i is

$$\mathcal{F}_i=-\nabla_i V_{pi}-\nabla_i V_{vi}+F_L\,. \qquad (3)$$

Exploiting the fact that $\mathcal{F}_i$ is linear in x_i^v, successive configurations of the flux lattice can be calculated. This is performed by updating x_i^v in such a way that $\mathcal{F}_i$ vanishes, where we take as input the x_j^v calculated in the previous step. Another parameter involved in the model is the initial distance between vortices, a, that will determine the number of neighbors considered in the interaction. We work in the range $0<a<\xi_v$, in order to always have interacting vortices. If L denotes the system size, and N_v the number of vortices, then $aN_v=L$. In type-II superconductors, a will be determined by the externally applied magnetic field.

The simulations are performed by randomly distributing N_p pinning centers, with $F_L=0$, and letting the vortices relax. The applied force is then increased by some very small amount ΔF_L (here we will take $\Delta F_L=0.005$), and the lattice is again allowed to relax. This procedure is repeated until the lattice starts to flow (i.e., the positions of all the vortices change by a large amount). This happens when the driving force has reached the depinning or critical value F_c. All our runs have been done in samples with periodic boundary conditions.

For a given choice of N_v and N_p, the system exhibits two regimes. When the Lorentz force is sufficiently large, the vortices move freely. This can be denoted as the free-flow or the "sliding" regime. For sufficiently low values of the externally applied force, the system does not move ("stick" regime), since the pinning sites anchor the vortices in place. However, a sudden and very small increase in the externally applied force will produce a shift in the location of the vortex lines. We monitor the distribution of these displacements for a wide range of parameters $(F_L,N_v,N_p,a,\xi_p,\ldots)$. Part of the difficulty in studying this type of system is the large number of parameters that need to be varied in order to explore the huge phase space available. We have only fixed $\xi_v=1$ (thus our unit of length becomes ξ_v), $A_v=20$, and the individual depinning scale, $F_c=1$, for one vortex in one pinning site. One of the signatures of SOC is the multiscale response of the system when a small perturbation is applied to the system. We have computed a very large number of such response distributions and a few of them will be presented and discussed below.

Figure 1 shows F_c^{eff}, which is the critical force averaged over 500 different configurations of the pinning lattice. The error bars are the standard deviations in the effective force. The plot shows how the collective depinning force varies as a function of the number of pinning sites. We observe that the depinning force increases monotonically when the density of pinning sites increases. These curves provide the phase boundary between the pinned and depinned regions as a function of the model parameters.

Let us now focus on the regime below the depinning transition, where a very small increase in the applied driving force produces small displacements in the vortex locations. In this regime, we have computed the magnitude of these displacements (Fig. 2) and the distribution of these magnitudes. This distribution exhibits a power-law behavior over several decades. More importantly, we have found that this power-law dependence exists over a wide range of parameters. Power-law distributions of this type are a signature of SOC. As in other studies of this general nature, limitations in the model result in upper and lower cutoffs in the power-law dependence. The upper cutoff corresponds to the distance between vortices a (recall that our choice of $A_v=20$ did not allow crossing of vortices past each other). The lower cutoff corresponds to the maximum allowed margin of tolerance used in the relaxation runs in order to obtain the desired convergence (0.0009 in this work). Previous simulations [7] in systems that exhibit SOC were performed by introducing a local perturbation. Here the perturbation (a very small increase in the force) is global, and it is the presence of disorder and interactions (vortex-vortex and vortex-pinning) that makes the system evolve to a state

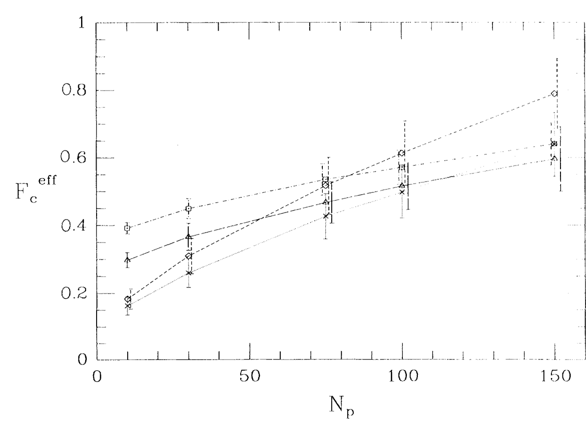

FIG. 1. Effective critical force (F_c^{eff}) vs number of pins (N_p) for different parameters [$a=0.9$ (×), $a=0.6$ (◇), $a=0.3$ (△), and $a=0.2$ (□)]. N_v is constant and equal to 30 and the pinning range is set to $\xi_p=0.25$. The lines are a guide to the eye and a few error bars are slightly shifted (horizontally only) to avoid overlap.

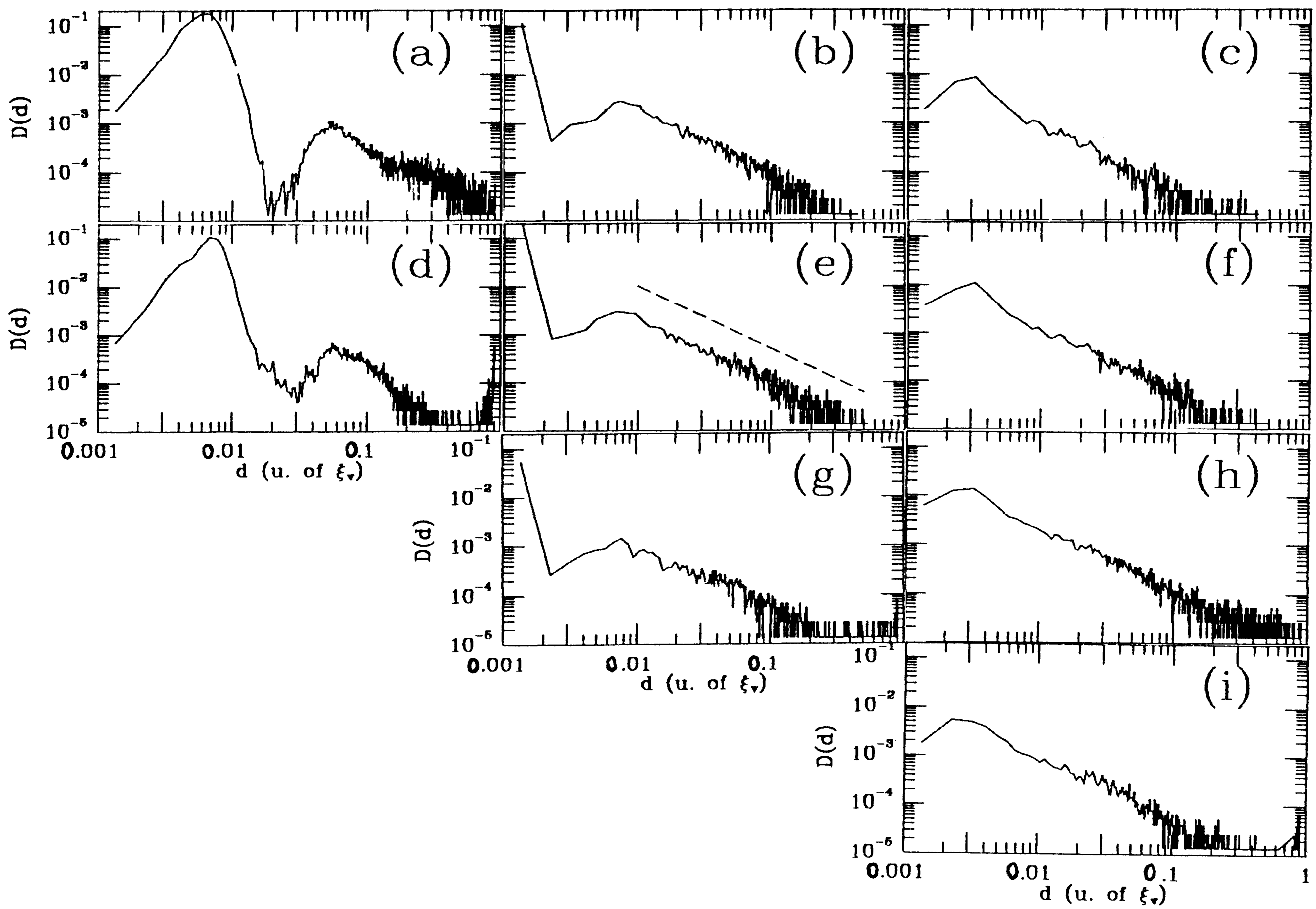

FIG. 2. Evolution of the normalized distribution of displacements [$D(d)$ vs d] during the force increase for $a=0.9$, $\xi_p=0.50$, and various pinning site numbers. In order to improve the statistics the results have been averaged over a range of applied forces, given below. Note that for weak pinning, SOC behavior is observed in only a portion of the range of displacements. (a) $F_L=0.09$–0.11 and $N_v=10$, (b) $F_L=0.09$–0.11 and $N_v=75$, (c) $F_L=0.09$–0.11 and $N_v=150$, (d) $F_L=0.14$–0.16 and $N_v=10$, (e) $F_L=0.14$–0.16 and $N_v=75$ (the dashed line has a slope of -1.3), (f) $F_L=0.14$–0.16 and $N_v=150$, (g) $F_L=0.39$–0.41 and $N_v=75$, (h) $F_L=0.39$–0.41 and $N_v=150$, (i) $F_L=0.59$–0.61 and $N_v=150$.

where very small perturbations generate responses at many length scales.

Though we do observe the power-law dependence over a wide range, it is not observed over the whole range of parameters inspected. For very low pinning densities, there is a wide peak in the distribution corresponding to very small displacements that we believe is due to free-moving (unpinned) vortices. These (recall that we are still below the depinning transition) are a subset of the total number of vortices, N_v, and can only move a short distance before they are stopped by other pinned flux lines. At higher densities of pinning sites, these free vortices become scarce.

The dynamics of the vortex current when the entire lattice begins to flow is monitored by counting, at each iteration of the equations of motion, the number of votrices that have crossed a given reference point. In most of the situations examined, the number of vortices that cross the reference is a periodic function of the number of iterations.

In summary, we present a highly simplified model for the response under very small perturbations of pinned flux lattices in type-II superconductors in restricted geometries. We have made extensive numerical simulations over a wide range of parameters, and we observe that our model exhibits self-organized critical behavior over several decades. Furthermore, this feature is robust over a wide range of values of the model parameters. We believe this model describes features that could be observed in superconducting microbridges, constrictions, wires connecting two superconducting grains, and long Josephson junctions [1]. We hope that our calculations will motivate experiments in this direction.

We thank R. Richardson for his critical reading of the manuscript. This work has been supported, in part, by NSF Grants No. DMR-90-01502 and No. PHY-89-

04035. O.P. is also supported by the Ministerio de Educación y Ciencia, Spain, and by Comisión Interministerial de Ciencia y Technología (CICYT) under Project No. MAT88-0211.

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