Excitonic Charge-Density-Wave Instability of Spatially Separated Electron-Hole Layers in Strong Magnetic Fields

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We use the Hartree-Fock approximation to investigate the ground state of a system consisting of spatially separated electron and hole layers in strong magnetic fields. When the layer separation is larger than a critical value a novel excitonic-density-wave state is found to have a lower energy than either a homogeneous exciton fluid or a double charge-density-wave state. The order parameters of the state satisfy a sum rule similar to that of a charge-density-wave state in a two-dimensional electron system. A possible connection between the new state and a recent experimental result is discussed.

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Recently the properties of double-quantum-well (DQW) systems in strong magnetic field have received much attention. The evolution of the electronic ground state as the well separation is varied has been investigated both experimentally [1] and theoretically [2–4]. The steps in the Hall conductance at odd integer values of e^2/h are observed to disappear [1] when the barrier thickness is increased. These quantum Hall effect states, which correspond to the filling factor $\nu = n+\frac{1}{2}$ for the average electron density in each quantum well, have been associated with the symmetric-antisymmetric (SAS) gap of the DQW. The suppression of the SAS energy gap as the well separation is increased has been suggested as the cause of the disappearance of these steps [3].

A common feature revealed in the several previous theoretical studies [2–4] of DQW's is that as the layer separations are increased the dispersion relation of the charge-density excitation develops a local minimum at a wave vector on the order of the inverse of the magnetic length. This minimum becomes a soft mode when the separation reaches a critical value d_c of the order of the magnetic length. The system therefore undergoes a phase transition. In this paper we identify this transition as the one to a novel ground state which we call an *excitonic charge-density-wave* (ECDW) state. This new state has the properties of both an excitonic state and a normal two-dimensional charge-density-wave (CDW) state.

The generalized system we study is the two-layer electron-hole system [5], with one layer containing electrons and the other containing the equal number of holes ($\nu_e = \nu_h \equiv \nu$). It can be realized either by the molecular-epitaxy growth of the InAs-AlSb-GaSb heterostructures [6] or by applying a strong electric field to the GaAs-AlGaAs DQW's [7]. The layer width as well as the tunneling between the two layers will be neglected throughout the paper, since they are not essential in the EDCW transition. For $\nu = \frac{1}{2}$, our system is equivalent to the half-filled electron-electron DQW system studied in Ref. [2]. At small layer separations, where the interlayer Coulomb attraction is strong, electrons and holes pair together to form excitons. The excitonically condensed state of the electron-hole pairs is then the preferable ground state [5]. On the other hand, if the layer separation is much larger than the average intralayer distance between neighboring particles, two independent Laughlin states [8] or triangular CDW states [9] will give the lowest energy of the system, depending on the filling factor ν. Between these two limits, that is, when the layer separation is comparable with the intralayer particle separation, the ECDW state appears. In this novel state, both the excitonic condensation and CDW's exist. Furthermore, the condensations of the excitons will occur not only at $\mathbf{K}=0$ (where $\mathbf{K}$ is the wave vector of excitons) but also at the wave vector of the CDW's.

We start with the general Hamiltonian of the electron-hole system in a strong magnetic field, assuming that only the first Landau levels are occupied,

$$\hat{H} = \frac{1}{2L^2}\sum_{i,j,\mathbf{q}} V_{ij}(\mathbf{q})[\hat{\rho}_i(\mathbf{q})\hat{\rho}_j(-\mathbf{q}) - \delta_{ij}e^{-(ql)^2/2}\hat{\rho}_i(0)] - \mu\hat{\rho}_e(0) - \mu\hat{\rho}_h(0)\,, \tag{1}$$

where $i,j=$electron or hole, μ is the chemical potential, $V_{ee}(\mathbf{q}) = V_{hh}(\mathbf{q}) = 2\pi e^2/\epsilon_q$, and

$$V_{eh}(\mathbf{q}) = -2\pi e^2 \exp(-dq)/\epsilon q\,.$$

L is the linear dimension of the system and l $(=\sqrt{\hbar c/eB})$ is the magnetic length. The spin degrees of freedom of electrons and holes are frozen by the magnetic field. The particle density operators $\hat{\rho}(\mathbf{q})$ in the above Hamiltonian are given by

$$\hat{\rho}_e(\mathbf{q}) = \sum_X a^\dagger_{X_+} a_{X_-} \exp[iq_x X - (ql)^2/4] \tag{2}$$

and

$$\hat{\rho}_h(\mathbf{q}) = \sum_X b^\dagger_{X_+} b_{X_-} \exp[-iq_x X - (ql)^2/4]\,, \tag{3}$$

where $X_\pm = X \pm q_y l^2/2$, and $-L/2 \le X = (2\pi l^2/L)j \le L/2$, with j being an integer. $a^\dagger_X$ (a_X) and $b^\dagger_X$ (b_X) are the creation (annihilation) operators of the electron and hole wave functions in the Landau gauge.

In the normal uniform excitonic phase, as a result of electron-hole interaction, electron-hole pairs condense into a state with zero total momentum. The order parameter in this case is just $\langle a_X b_{-X}\rangle$ which is finite for the condensed excitonic state. Following Anderson's treatment [10,11] of the collective excitations in superconductivity, we derived a set of random-phase-approximation (RPA) equations for $\langle\hat{\rho}_e(\mathbf{q})\rangle$, $\langle\hat{\rho}_h(\mathbf{q})\rangle$, and $\langle\hat{d}^\dagger(\mathbf{q})\rangle$ by linearization of the equations of motion, where

$$\hat{d}^\dagger(\mathbf{q}) \equiv \sum_X a^\dagger_{X_+} b^\dagger_{-X_-} \exp[iq_x X - (ql)^2/4] \qquad (4)$$

is the creation operator of an exciton with a total momentum $\hbar\mathbf{q}$. From these coupled RPA equations we can obtain the dispersion relation of the collective modes in the excitonic state [12],

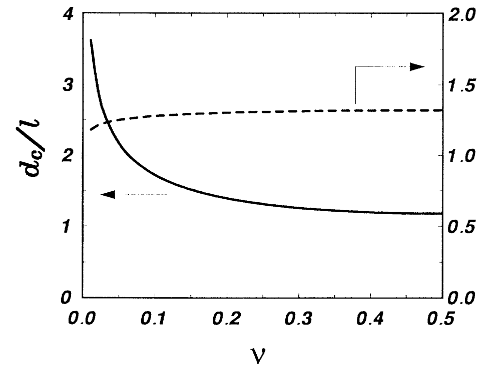

FIG. 1. The critical layer separation d_c (solid line) and the corresponding wave vector q_c (dashed line) as a function of the filling factor ν.

$$\omega^2(\mathbf{q}) = (2\nu-1)^2[E_{eh}(\mathbf{q})-E_{eh}(0)]^2 - 4\nu(1-\nu)[E_{eh}(\mathbf{q})-E_{eh}(0)]\{E_{eh}(0)+E_{ee}(\mathbf{q})-(1/2\pi l^2)[V_{ee}(\mathbf{q})+V_{eh}(\mathbf{q})]e^{-(ql)^2/2}\}\,, \qquad (5)$$

where $E_{ee}(\mathbf{q})$ and $E_{eh}(\mathbf{q})$ are defined as

$$E_{ee}(\mathbf{q}) = \frac{e^2}{\epsilon l}\left(\frac{\pi}{2}\right)^{1/2} \exp\left(-\frac{(ql)^2}{r}\right) I_0\left(\frac{(ql)^2}{4}\right) \qquad (6)$$

and

$$E_{eh}(\mathbf{q}) = -\frac{e^2}{\epsilon}\int_0^\infty dt\, J_0(ql^2 t)\, e^{-(tl)^2/2 - td}\,. \qquad (7)$$

Here $J_0(x)$ and $I_0(x)$ are the Bessel function and modified Bessel function of order zero, respectively. At $\nu=\frac{1}{2}$, the dispersion relation (5) is the same as that obtained by Fertig [2] for a half-filled electron-electron DQW. As has been noticed by several workers [2–4] in the case of the half-filled electron-electron DQW's, $\omega^2(\mathbf{q})$ of Eq. (5) becomes negative at $ql \sim 1.3$ when the layer separation d is increased beyond a critical value $d_c(\nu)$. A plot of d_c as a function of ν is shown in Fig. 1; it defines the phase boundary between the uniform excitonic state and the new state (which we call the ECDW state). Also plotted in Fig. 1 are the values of q_c at which $\omega^2(q)$ first becomes negative, i.e., $\omega^2(q_c)|_{d=d_c}=0$.

The negative values of $\omega^2(q)$ for $d>d_c$ indicate the existence of static CDW distortions in the new ground state of the system. However, because of the coupling of $\langle\hat{\rho}_e(\mathbf{q})\rangle$, $\langle\hat{\rho}_h(\mathbf{q})\rangle$, and $\langle\hat{d}^\dagger(\mathbf{q})\rangle$ in the RPA equations mentioned above, the values of $\langle\hat{d}^\dagger(\mathbf{q})\rangle$ at the wave vectors of the CDW's may also be finite and time independent. We therefore define the order parameters of the ECDW state:

$$\Delta_{\mathrm{CDW}}(\mathbf{Q}) = \frac{2\pi l^2}{L^2}\langle\hat{\rho}_e(\mathbf{Q})\rangle \exp\left(\frac{(Ql)^2}{4}\right) = \frac{2\pi l^2}{L^2}\langle\hat{\rho}_h(\mathbf{Q})\rangle \exp\left(\frac{(Ql)^2}{4}\right) \qquad (8)$$

and

$$\Delta^*_{\mathrm{ex}}(\mathbf{Q}) = \frac{2\pi l^2}{L^2}\langle\hat{d}^\dagger(\mathbf{Q})\rangle \exp\left(\frac{(Ql)^2}{4}\right), \qquad (9)$$

where $\Delta_{\mathrm{CDW}}(0)=\nu$, and $\{\mathbf{Q}\}$ are the wave vectors of the ECDW. In the Hartree-Fock (HF) approximation the Hamiltonian of Eq. (1) is decoupled to

$$\hat{H} = \sum_{X,\mathbf{Q}} \{U_{\mathrm{CDW}}(\mathbf{Q})\Delta_{\mathrm{CDW}}(-\mathbf{Q})(e^{iQ_xX}a^\dagger_{X_+}a_{X_-} + e^{-iQ_xX}b^\dagger_{X_+}b_{X_-}) - U_{\mathrm{ex}}(\mathbf{Q})[\Delta^*_{\mathrm{ex}}(\mathbf{Q})e^{-iQ_xX}a_{X_+}b_{-X_-} + \mathrm{H.c.}]\} - \mu\rho_e(0) - \mu\rho_h(0) - \frac{L^2}{2\pi l^2}\sum_{\mathbf{Q}}[U_{\mathrm{CDW}}(\mathbf{Q})|\Delta_{\mathrm{CDW}}(\mathbf{Q})|^2 + U_{\mathrm{ex}}(\mathbf{Q})|\Delta_{\mathrm{ex}}(\mathbf{Q})|^2]\,. \qquad (10)$$

Here, $X_\pm = X \pm Q_y l^2/2$, $U_{\mathrm{CDW}}(\mathbf{Q})$ is given by

$$U_{\mathrm{CDW}}(\mathbf{Q}) = \frac{e^2 d}{\epsilon l^2}\delta_{\mathbf{Q},0} + \frac{1}{2\pi l^2}[V_{ee}(\mathbf{Q})+V_{eh}(\mathbf{Q})]e^{-(Ql)^2/2}(1-\delta_{\mathbf{Q},0}) - E_{ee}(\mathbf{Q})\,, \qquad (11)$$

and $U_{\mathrm{ex}}(\mathbf{Q})$ is equal to $E_{eh}(\mathbf{Q})$ defined in Eq. (7).

The Hartree-Fock Hamiltonian (10) can be diagonalized by a series of unitary transformations. In this paper we consider only the simplest case, i.e., an unidirectional ECDW state having wave vectors $\{\mathbf{Q}\}=nQ_0$, where $n=0,\pm1,\pm2,\ldots$. Q_0 is the fundmental periodicity of the ECDW. After the diagonalization of $\hat{H}$ in Eq. (10), the order parameters of the ECDW ground state at zero temperature are found to be

$$\Delta_{\rm ex}(nQ_0)=\tfrac{1}{2}\int_0^1 dx\cos(n\pi x)\frac{E_{\rm ex}(x)}{\{[E_{\rm CDW}(x)-\mu]^2+E_{\rm ex}^2(x)\}^{1/2}} \tag{12}$$

and

$$\Delta_{\rm CDW}(nQ_0)=\tfrac{1}{2}\int_0^1 dx\cos(n\pi x)\left[1-\frac{E_{\rm CDW}(x)-\mu}{\{[E_{\rm CDW}(x)-\mu]^2+E_{\rm ex}^2(x)\}^{1/2}}\right]. \tag{13}$$

Here we have assumed that both $\Delta_{\rm ex}$ and $\Delta_{\rm CDW}$ are real quantities. $E_{\rm ex}$ and $E_{\rm CDW}$ are given by the following expressions:

$$E_{\rm ex}(x)=-\sum_{n=-\infty}^{\infty}U_{\rm ex}(nQ_0)\Delta_{\rm ex}(nQ_0)\times\cos(n\pi x) \tag{14}$$

and

$$E_{\rm CDW}(x)=\sum_{n=-\infty}^{\infty}U_{\rm CDW}(nQ_0)\Delta_{\rm CDW}(nQ_0)\times\cos(n\pi x)\,. \tag{15}$$

Equations (12)-(15) are a set of self-consistent equations. To find the ground state of the system for given values of ν and d, we first assume some value for Q_0, solve Eqs. (12)-(15) for $\Delta_{\rm CDW}(nQ_0)$, $\Delta_{\rm ex}(nQ_0)$, and μ, and then minimize the expectation value $E_{\rm HF}(Q_0,d,\nu)$ of the Hartree-Fock Hamiltonian (10) with respect to Q_0. $E_{\rm HF}$ is given by

$$E_{\rm HF}(Q_0,d,\nu)=\frac{L^2}{2\pi l^2}\sum_n[U_{\rm CDW}(nQ_0)|\Delta_{\rm CDW}(nQ_0)|^2+U_{\rm ex}(nQ_0)|\Delta_{\rm ex}(nQ_0)|^2]\,. \tag{16}$$

Since there is an infinite number of order parameters, we introduce a cutoff n_c and set $\Delta_{\rm CDW}(nQ_0)$ and $\Delta_{\rm ex}(nQ_0)$ to zero for $|n|>n_c$. In general, for given values of ν and d, Eqs. (12)-(15) have a number of solutions corresponding to different kinds of states. Among them, three solutions are of particular interest: the uniform excitonic state [$\Delta_{\rm ex}(0)\neq 0$, $\Delta_{\rm CDW}(nQ_0)=\Delta_{\rm ex}(nQ_0)=0$ for $|n|\neq 0$], the double CDW state [$\Delta_{\rm CDW}(nQ_0)\neq 0$, $\Delta_{\rm ex}(nQ_0)=0$], and the ECDW state [$\Delta_{\rm CDW}(nQ_0)\neq 0$, $\Delta_{\rm ex}(nQ_0)\neq 0$]. The self-consistent calculation has been carried out for $n_c=8$ at several different values of the filling factor. In Fig. 2 the energy per electron-hole pair of these three states is shown as a function of the layer separation for $\nu=0.23$. The solution for the ECDW state exists only when the layer separation is larger than some critical value, and it asymptotically approaches the solution for the double CDW state as the separation increases. For $d>d_c$, the ECDW state is energetically more favorable than both the uniform excitonic state and the double CDW state. The first three order parameters of the ground state $\Delta_{\rm ex}(0)$, $\Delta_{\rm CDW}(Q_0)$, and $\Delta_{\rm ex}(Q_0)$ vs d are plotted in Fig. 3 for $\nu=0.23$. Starting from $d=d_c$, as $\Delta_{\rm ex}(0)$ drops rapidly, $\Delta_{\rm ex}(Q_0)$ first increases, then decreases, and exhibits a maximum at $d\sim 1.9l$. From Eqs. (12) and (13) it can be easily shown that the order parameters of the ECDW state satisfy the sum rule

$$\sum_{\mathbf{Q}}[|\Delta_{\rm CDW}(\mathbf{Q})|^2+|\Delta_{\rm ex}(\mathbf{Q})|^2]=\nu\,, \tag{17}$$

which is similar to that for a two-dimensional CDW state [13]. More interesting is that the critical layer separations d_c for the ECDW state and the corresponding wave vector Q_0, obtained from Eqs. (12)-(16), are exactly the same (within 0.1%) as those values given in Fig. 1. At

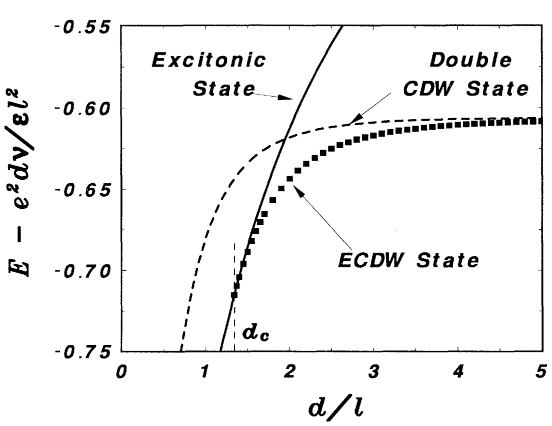

FIG. 2. Energy per electron-hole pair in three different states at $\nu=0.23$ vs the layer separation; $e^2 d\nu/\epsilon l^2$ is the direct Coulomb interaction energy of the system. The vertical coordinates are in units of $e^2/\epsilon l$.

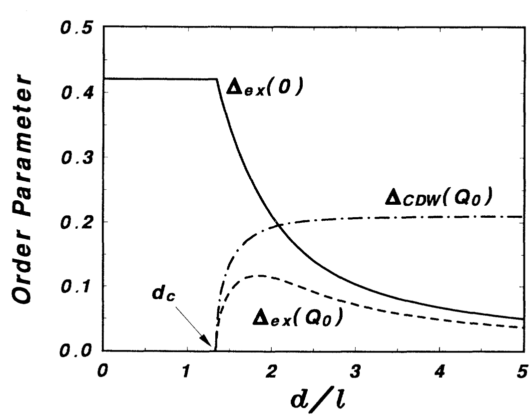

FIG. 3. Variations of the first three order parameters in the ECDW state at $\nu=0.23$ as a function of the layer separation.

$\nu = 0.23$, for example, we found that $d_c = 1.34l$ and the corresponding $Q_0 l = 1.31$. We thus identify the phase transition resulting from the soft mode of the collective excitations in the uniform excitonic state as the transition leading to an ECDW state, most likely a triangular ECDW state. We believe this is also the phase transition discussed in the works by Fertig [2], Brey [4], and MacDonald, Platzman, and Boebinger [3]. Since in the new state the ECDW can be pinned by the impurities in the quantum wells, one should no longer expect the observations of the quantum Hall effects in the system, as the experimental results of Ref. [1] indicated.

In summary, we have found a novel ground state of the electron-hole DQW's under strong magnetic fields, in which an excitonic condensation and crystallization coexist. The transition to such a state when the layer separation is of the order of the magnetic length is consistent with the softening of the collective modes in the uniform excitonic state. The study of the excitation energy spectrum of this novel state as well as the diagonalization of the Hartree-Fock Hamiltonian (10) for a triangular ECDW state are currently under way.

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