Size-Dependent Analysis of the Metal-Insulator Transition in the Integral Quantum Hall Effect

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The critical exponent ν of the localization length in the integral quantum Hall regime can be measured directly using small Hall-bar geometries with different sizes. We obtain a universal behavior for the three lowest Landau levels. This is in agreement with the universality prediction of the field-theoretic approach to the metal-insulator transition in the quantum Hall effect. The value of $\nu=2.3\pm0.1$ agrees with recent numerical studies for the lowest Landau level. The measured value of the temperature exponent of the inelastic-scattering rate is found to range from $p=2.7\pm0.3$ to $p=3.4\pm0.4$.

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The metal-insulator transition in a two-dimensional electron gas (2DEG) in the quantum Hall regime has recently attracted considerable interest both experimentally [1-3] and theoretically [4-6]. In the center of an impurity-broadened Landau level (LL), the localization length ξ of electron states in a random potential diverges according to $\xi \propto |B-B_c|^{-\nu}$ with B the magnetic field and B_c the critical field [4]. Pruisken has suggested universality for the critical exponent ν independent of LL and the specific random potential [4]. However, numerical studies indicate different values for ν in the two lowest LLs [6]. Experimentally, this exponent has up to now only been accessible indirectly via the temperature dependence of the longitudinal resistivity ρ_{xx} and the Hall resistivity ρ_{xy}. The half-width ΔB of the ρ_{xx} peak between adjacent Hall plateaus and the maximum slope of ρ_{xy} as a function of magnetic field show a scaling behavior: $\Delta B \propto T^{\kappa}$ and $(\partial\rho_{xy}/\partial B)^{\max} \propto T^{-\kappa}$. The temperature exponent κ is related to the critical exponent ν via $\kappa=p/2\nu$ with p the temperature exponent of the inelastic-scattering rate, where diffusive quantum transport is assumed [7]. The exponent p is not known for high magnetic fields. Wei *et al.* [1] have determined the temperature exponent κ and found a value of $\kappa=0.42\pm0.04$ in an InGaAs/InP heterostructure for the two lowest LLs. They proposed that this value of κ should be universal. Such a universal behavior is neither observed in Si metal-oxide-semiconductor field-effect transistors [2] nor in AlGaAs/GaAs heterostructures [3]. Instead, if one analyzes the AlGaAs/GaAs data in the experimentally accessible temperature range $T>25$ mK, different exponents κ are obtained, where κ increases monotonically up to a value of $\kappa=0.81\pm0.03$ with decreasing mobility [3]. This result can be explained if one assumes that the temperature exponent of the inelastic-scattering rate, p, depends on the impurity configuration. Another explanation of the observed nonuniversal behavior is based on the assumption that the long-range potential fluctuations in AlGaAs/GaAs systems shift the temperature range for universal behavior to such low values that it becomes unobservable.

In order to provide new information about the metal-insulator transition at half-filled LLs, we describe a novel experimental approach to scaling in the integral quantum Hall regime. In contrast to previous work [1-3] the exponents involved can be measured *separately.* We thus report the first direct experimental determination of the localization-length exponent ν in the quantum Hall regime. We use small Hall-bar geometries with widths ranging from 10 to 64 μm. In these geometries, the width of the ρ_{xx} peak and the slope of the Hall resistance are found to no longer depend upon temperature below a certain characteristic temperature T_c which increases with decreasing sample width. Instead, they scale with the width of the sample. By comparing the results for samples of different size the exponent ν can be determined. Moreover, by analyzing the characteristic temperatures for the respective sample widths it is possible to obtain the temperature exponent $p/2$ of the inelastic-scattering length.

The two samples used in the present work are AlGaAs/GaAs heterostructures grown by molecular-beam epitaxy. Details of the heterostructure growth have already been reported [3]. After the growth process, the samples were etched into structures with four Hall bars with widths of $W=10$, 18, 32, and 64 μm and a length-to-width ratio of 3. The sample layout is such that, e.g., the 64-μm Hall bar is in every respect twice as large as the 32-μm Hall bar. Each Hall bar is thus characterized by a single parameter only (the width). The sample was mounted in the glass tail of a ^{3}He-^{4}He dilution refrigerator and fully photoexcited using a red-light-emitting diode, in order to further increase the homogeneity of the sample. The electron concentrations and mobilities thus obtained in the two samples are given in Table I. The samples were studied in the temperature range from 25 mK to 1 K. The resistance values were obtained with a standard low-frequency lock-in technique. It was carefully checked that neither noise nor the measuring current of 0.5 nA lead to electron heating. We have eliminated

TABLE I. Electron concentration n_e and mobility μ of the samples after illumination. Localization-length exponents ν, inelastic-scattering-rate exponents p, and maximum conductivity $\sigma_{xx}^{\max}(B_c)$ (in units of e^2/h) for the three lowest Landau levels.

Sample No.	n_e (10^{15} m^{-2})	μ (m^2/V s)		LL 0↓	1↑	1↓	2↑	2↓
1	4.85	15.5	ν	⋯	2.3±0.1	2.4±0.2	2.2±0.2	2.3±0.2
			p	⋯	3.4±0.4	3.2±0.3	2.9±0.4	3.0±0.4
			$\sigma_{xx}^{\max}$	⋯	0.09	0.20	0.17	0.21
2	4.0	5.8	ν	2.3±0.1	2.4±0.2	2.3±0.2	6.5±0.6	
			p	3.3±0.3	2.8±0.3	2.9±0.3	2.7±0.3	
			$\sigma_{xx}^{\max}$	0.17	0.16	0.40	0.75	

noise heating by carefully shielding all connections to the sample and by using low-pass filters both at room temperature and at low temperature. As a further check, we have simulated the effect of electron heating by studying the effect of current heating (with currents $I=1$ to 200 nA) at base temperature. The results are different for different LLs and in different samples. This is in striking contrast to the universal behavior of the localization-length exponent ν to be reported below.

In Fig. 1(a) we show the measured longitudinal resistance R_{xx} in LL $N=1\downarrow$ (the arrow stands for the spin direction), i.e., for the region of filling factors between 3 and 4 for sample 1, normalized to the maximum value as a function of magnetic field B close to the critical magnetic field B_c. The curves are given for the four widths, at a temperature of 25 mK. The half-width of the R_{xx} peak increases with decreasing bar width. In Fig. 1(b) we show the Hall resistance R_{xy} in the same LL. Clearly, the maximum slope decreases with decreasing width. In this low-temperature region, the transport coefficients are no longer temperature dependent, as can be seen from Fig. 2. In Fig. 2 we show the half-width of R_{xx} in LL $N=1\downarrow$ for sample 1 as a function of temperature for the four different widths. At low temperatures, the respective half-width saturates for a given sample width. The saturation values are indicated by the horizontal dashed lines. The saturation starts at increasing temperatures with decreasing sample size. The corresponding characteristic temperatures T_c are indicated by the vertical dashed lines. At high temperatures, the measured values practically coincide, independent of width. The solid line is a fit to the high-temperature data and corresponds to a temperature exponent κ of 0.68 ± 0.04. Nonequilibrium effects described by the edge-state picture of the quantum Hall effect do not play a role in our experiment. We have checked experimentally that nonlocal effects (as, e.g., described in Ref. [8]) are not present in our samples. This is not surprising because, first, the mobility of the samples

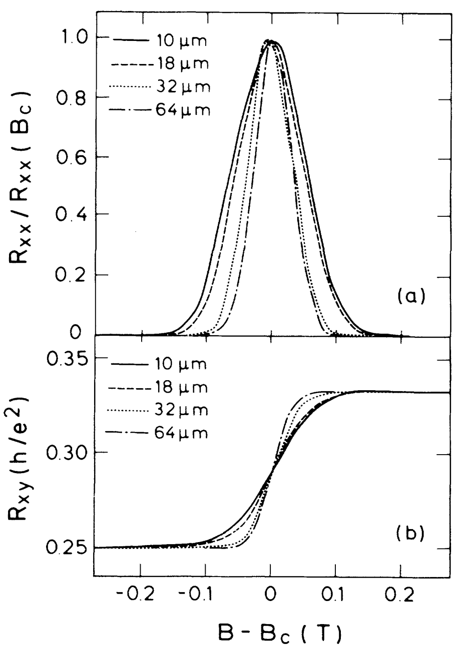

FIG. 1. Transport data for sample 1 in Landau level $N=1\downarrow$ at a temperature of $T=25$ mK. (a) Normalized longitudinal resistance R_{xx} as a function of magnetic field B close to the critical field B_c. (b) The Hall resistance R_{xy} in units of h/e^2 as a function of $B-B_c$.

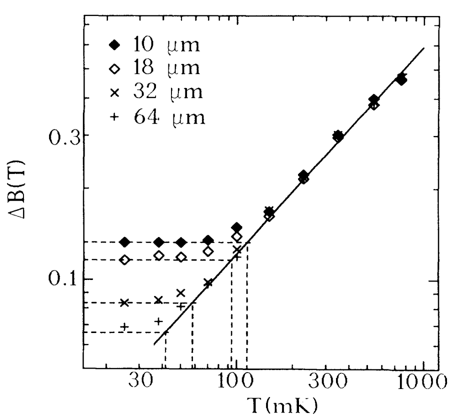

FIG. 2. The half-width ΔB as a function of temperature for sample 1 in Landau level $N=1\downarrow$ for the four sample widths. Horizontal dashed lines: extrapolation to zero temperature. The vertical dashed lines correspond to the characteristic temperatures T_c. Solid line: high-temperature fit.

is low and, second, the length-to-width ratio of 3 is small in comparison to experiments with long samples [8].

We explain the observed behavior by assuming that the temperature of $T \approx 25$ mK is already low enough so that the inelastic-scattering length is larger than the sample width under these conditions. Then the magnetic-field range of the extended states, and thus ΔB and $(\partial\rho_{xy}/\partial B)^{\max}$, is only determined by the sample width. Electron states with a localization length larger (smaller) than the width do (do not) contribute to a finite R_{xx}. This size dependence can be used to obtain the critical exponent ν of the localization length $\xi \propto \Delta B^{-\nu}$. In Fig. 3 we show the low-temperature saturation values of ΔB for the four widths on a double-logarithmic plot. Within experimental error these values lie on a straight line with a slope corresponding to the value of the exponent ν. The values are given in Table I. The exponent $\nu_{N=0}$ for the lowest LL ($N=0\downarrow$) could not be obtained in sample 1 due to the mobility of $\mu=15.5$ m^2/Vs, which leads to indications of the fractional quantum Hall effect. However, in sample 2 this problem did not occur due to the low mobility, so that $\nu_{N=0}$ could be determined to $\nu=2.3\pm0.1$. This value coincides with the result in LL $N=1$ in both samples. On the other hand, in this sample the spin splitting of the LL $N=2$ is not resolved (due to the low mobility), so that we can study the behavior of a non-spin-split LL. The data of Fig. 2 also provide a way of determining the temperature exponent $p/2$ of the inelastic-scattering length [9] $L_{\text{in}} \propto T^{-p/2}$. We extrapolate the low-temperature value of ΔB for a given width to higher temperatures (horizontal dashed line) and the high-temperature fit (solid line) to low temperatures. These lines intersect at a temperature T_c. We argue that at the temperature T_c the inelastic-scattering length is approximately equal to the sample width. In this way, we obtain four values of L_{in} for four temperatures. From $W=L_{\text{in}} \propto T_c^{-p/2}$ we obtain values between $p=2.7$ and 3.4. The detailed results are summarized in Table I.

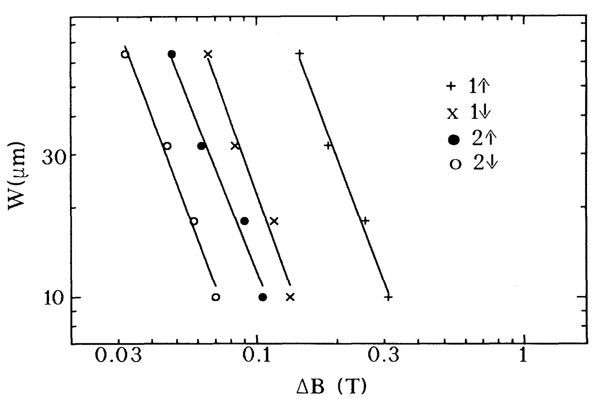

FIG. 3. The sample width W as a function of the saturation half-width ΔB for sample 1 in Landau levels $N=1$ and $N=2$. Straight lines: fits.

We also show in Table I the values of the maximum conductivity $\sigma_{xx}^{\max}(B_c)$ at the critical magnetic field B_c. The values are calculated from the measured ρ_{xx} and ρ_{xy} and are given for the 64-μm Hall bar at $T=25$ mK (which is closest to the critical point). The results for different Hall-bar widths obtained in a given LL differ by approximately 20% or less. We observe different values of $\sigma_{xx}^{\max}$ for the two samples and in different LLs. Thus we conclude that the actual random potential present in the respective sample significantly influences the behavior of the maximum conductivity, in contrast to the localization-length exponent ν.

At this point, we want to compare the absolute values for the inelastic-scattering length and the localization length with experimental and theoretical results of other authors. de Vegvar *et al.* [10] have obtained an inelastic-scattering length of $L_{\text{in}} \approx 6$ μm at $T=280$ mK (at magnetic fields $B \lesssim 0.6$ T). This is of the same order of magnitude as our result $L_{\text{in}} \approx 10$ μm at 115 mK for sample 1 in LL $N=1\downarrow$ (see Fig. 2). A further comparison is possible for our experimental results for the localization length in LL $N=0\downarrow$ in sample 2 with the numerical results of Huckestein and Kramer [5]. If we assume a Gaussian density of states (as a simple form of a LL), we can calculate the energy separation $E-E_c$ corresponding to the measured magnetic-field separation $B-B_c=\frac{1}{2}\Delta B$ for a given sample size. E_c (B_c) is the critical energy (magnetic field). We thus obtain the same order of magnitude of experimental and theoretical localization lengths. These comparisons show that at the sizes and temperatures we have used we can indeed expect the observed saturation effect.

With our present results, we can make a *direct* comparison of experimental and theoretical results for the localization-length exponent ν. Pruisken has suggested universality for ν independent of LL and the details of the random potential [4]. Within experimental error, our values obtained for the three lowest (spin-split) LLs coincide, with an average value of $\bar{\nu}=2.3\pm0.1$. Furthermore, this result is found in two samples with different mobilities. This is strong support for the universality statement. Moreover, our results support the value $\nu=\frac{7}{3}$ which is obtained from the percolation picture including quantum tunneling [11], which coincides with the numerical result $\nu=2.34\pm0.04$ in the lowest LL [5]. There have also been attempts to numerically study higher LLs, but for the first LL $N=1$ these studies have not yet been able to conclusively obtain scaling [6].

In sample 2 the spin splitting in LL $N=2$ is not resolved. Here we find a value of $\nu=6.5\pm0.6$ and a value of $\kappa=0.21\pm0.02$ which is considerably higher, respectively, lower than the result for the spin-split LLs. The inelastic-scattering-length exponent, obtained from the characteristic temperatures T_c, is comparable to the value for LL $N=1$. A non-spin-split LL has been studied before by Wei *et al.* [12] in large InGaAs/InP samples.

They have found a temperature exponent κ that was half the value obtained for a spin-split LL. Within their work, it could not be clarified whether this was due to a different scattering mechanism or to a different delocalization phenomenon. Our results clearly demonstrate that this difference arises from the delocalization properties. However, the localization-length exponent ν is not simply twice as large as for the spin-split case, as might be concluded from Ref. [12].

The present experimental results show conclusively why in low-mobility AlGaAs/GaAs heterostructures the temperature exponent κ is not universal (as has been demonstrated in Ref. [3]). We recall the relation between the localization-length exponent ν, the scattering-length exponent $p/2$, and the half-width exponent κ: $\kappa = p/2\nu$. In Ref. [3] the exponent κ has been found to vary in differently doped samples, caused by the presence of Coulomb scatterers close to the 2DEG. Because of the fact that ν is universal, p has to change from $p=1.3$ to 3.8, using the results from Ref. [3]. This is in disagreement with the commonly assumed values of $p=1$ ("dirty metal limit" [13]) and $p=2$ (obtained from Fermi-liquid theory). Our results thus clearly demonstrate that the inelastic-scattering-rate exponent can be substantially larger in high magnetic fields compared to the theoretical zero-magnetic-field results. The actual value of p is determined by the specific disorder present in the respective sample, in our case a random distribution of Coulomb scatterers. The effect of Coulomb scatterers in sheet-doped samples has been studied by Haug *et al.* [14], who showed that the center of the Hall plateau is shifted to higher (lower) magnetic fields in the case of repulsive (attractive) Coulomb scatterers. In a model calculation, this could be attributed to non-Born scattering. We suppose that this mechanism also determines the actual value of the scattering-length exponent $p/2$.

In summary, we have reported the *direct* measurement of the localization-length exponent ν in the integer quantum Hall regime. A universal behavior of ν in the three lowest Landau levels is obtained, with a value of $\nu = 2.3 \pm 0.1$. This value agrees with the result from the percolation picture including quantum tunneling and the result of recent numerical studies. The inelastic-scattering-rate exponent p at high magnetic fields can be measured in the same experiment. This exponent is determined by the specific form of the random potential, and therefore the temperature exponent $\kappa = p/2\nu$ is not universal.

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